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General Physics I, Midterm 3
PHYS10000AA, AB, AC, Class year 109-2
05-11-2021

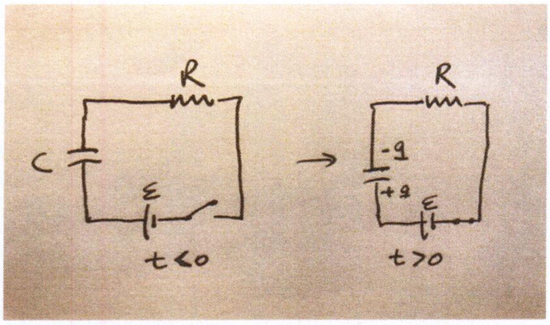
SN:____________________, Name:____________________

*Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables **are** allowed. Please hand in both solution and this problem sheet.*
ABSOLUTELY NO CHEATING!

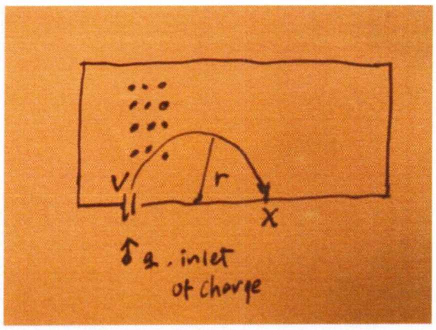
Problems (20% each, total 6 problems, 120%)

1. **Electric current:** (30%) Consider a section of conducting metal with diameter A. In a given time Δt, the charge carriers travels at drift speed V_d and a distance length ΔX. Suppose the number density of mobile charge carrier (with charge q) is n. (a) What is the current generated by the moving of charge carriers in the time period of Δt? (b) As an example, if one mole of copper has 63.5g, and volume 7.09 cm^3. What is the number density n for copper assuming each copper contribute one electron? (c) If we run a 10A current in copper, and the cross section of the copper is 3.31×10^{-6} m^2, What is the drift speed for copper?

2. **RC circuit:** (40%) Consider a simple RC circuit as shown in the figure to the right. At t<0, the circuit was off. At t=0, the circuit is turned on. (a) At t=0 when the circuit is first turned on, what is the current in the circuit? (b) When the capacitor is completely charged, what is the total charge in the capacitor? (c) Use Kirchhoff's rule, write down the first order differential equation and derive the charge q(t) as a function of time. (4) plot the charge as a function of time during charging process.

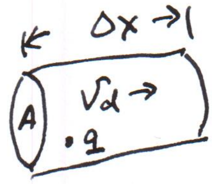


3. **Mass Spectrometer:** (30%) One of the applications of magnetic field is to allow mass/charge of an unknown charged particle be determined through a device called mass spectrometer, as shown in the figure to the right. In the figure, charged particles (or molecules) of mass m with charge q and velocity v is injected into the spectrometer from the left bottom inlet by an acceleration voltage V. An uniform magnetic field B is applied with its direction pointing out of the page as shown. Due to the magnetic force, the charge particle will travel in a circular trajectory with radius r and landed in a place X. What is the mass/charge ration of this unknown charges particle?



1. (a) Let the number density be n.

The total charge in this volume is



$$\Delta Q = (n \underbrace{A \Delta x}_{= \text{Volume}})\, q$$

$$= (n A V_d \Delta t)\, q$$

$$\therefore \; I_{AV} = \frac{\Delta Q}{\Delta t} = n q V_d A \qquad \text{— (1)}$$

(b) For copper $V = \frac{M}{\rho} = \frac{63.5 \text{ g}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3$ (given)

$$n = \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3} \left(\frac{1 \times 10^{6} \text{ cm}^3}{1 \text{ m}^3} \right)$$

$$= 8.49 \times 10^{28} \text{ electrons/m}^3$$ — each copper atom contributes one electron

(c) If we run 10 A in the wire of A = area $= 3.31 \times 10^{-6} \text{ m}^2$

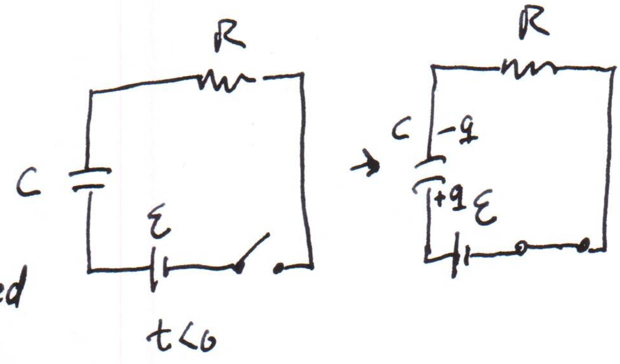
then from eq. (1)

$$V_d = \frac{I}{nqA} = \frac{10 \text{ C/s}}{(8.49 \times 10^{28} \frac{\text{electrons}}{\text{m}^3})(1.6 \times 10^{-19} \text{ C})(3.31 \times 10^{-6} \text{ m}^2)}$$

$$= 2.22 \times 10^{-4} \text{ m/s}$$

2 (a) at $t=0$, Capacitor is not Charge

$$I_0 = \frac{\varepsilon}{R}$$



(b) When the capacitor is completely Charged

$I = 0$

$Q = C\varepsilon$, where C is the capacitance of the capacitor

(c) Kirchhoff's rule

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$\rightarrow I = \frac{\varepsilon}{R} - \frac{q}{RC} \rightarrow \frac{dq(t)}{dt} = -\frac{1}{RC}q(t) + \frac{\varepsilon}{R}$$

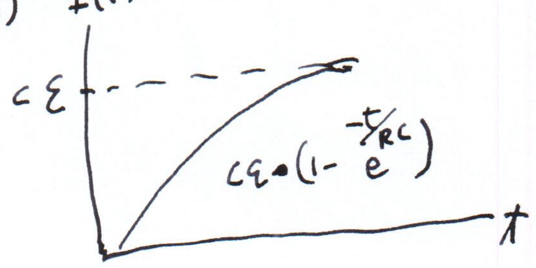
— 1st order differential equation

$$\rightarrow \frac{dq(t)}{dt} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC} \Rightarrow \frac{dq(t)}{q - C\varepsilon} = -\frac{1}{RC}dt$$

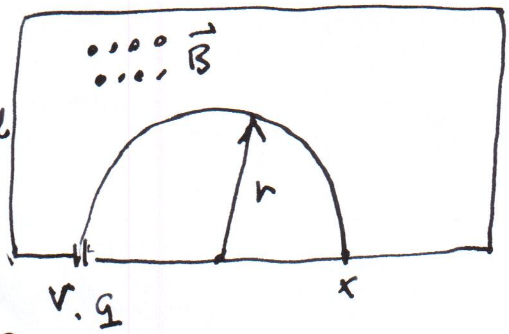
$$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int_0^t dt$$

$$\therefore \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = \frac{-t}{RC} \Rightarrow q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

(*) (d)



3.



The charged particle with velocity $\vec{v}$ injected into the spectrometer after acceleration by the electric field with voltage V will have a kinetic energy $E_K = \frac{1}{2}mv^2$

$$\frac{1}{2}mv^2 = qV \rightarrow v = \sqrt{\frac{2qV}{m}}$$

since $v \perp B$

$$F = ma = \frac{mv^2}{r} = qvB$$

due to the magnetic field

$$\therefore r = \frac{mv}{qB}$$

$$= \frac{m}{qB}\sqrt{\frac{2qV}{m}}$$

But $x = 2r$

$$= \frac{2m}{qB}\sqrt{\frac{2qV}{m}}$$

$$\therefore x = 2r = \frac{2m}{qB}\sqrt{\frac{2qV}{m}}$$

$$m = \frac{B^2 q x^2}{8V}$$

. here both m and q are unknown, therefore we can only obtain m/q value

$$\frac{m}{q} = \frac{B^2 x^2}{8V}$$

All these can be determined by experimental parameters of B, x, and V