國立東華大學
Department of Physics
National Dong Hwa University, 1, Sec. 2,
Da Hsueh Rd., Shoufeng, Hualien, 974, Taiwan

General Physics I, Midterm 2
PHYS10000AA, AB, AC, Class year 109-2
04-01-2021

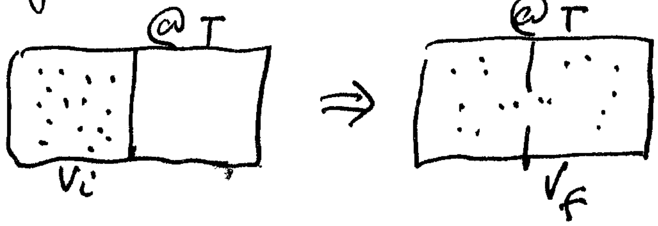
SN:____________________, Name:____________________

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables ***are*** *allowed. Please hand in both solution and this problem sheet.* ***ABSOLUTELY NO CHEATING!***

Problems (20% each, total 6 problems, 120%)

1. **Entropy:** (25%) In an adiabatic free expansion, an ideal gas of volume V_i at original temperature T was free-expanded to a volume of V_f in an isolated system. (a, 5%) Explain what does it mean by "an adiabatic free expansion"? (b, 15%) What is the entropy change of this free expansion process of the gases, and (c, 5%) justify your answer.
2. **Dipole:** (35%) (a, 15%)Derive the electric field set up by a pair of positive and negative charges of charge q. The charges are vertically standing separated by a distance d (in the z-axis direction). If we take the center of mass of the charges systems to be the origin of our coordinate system, and the vertical axis is the z-axis. What is the electric field intensity at a z distance from the origin that can be observed?(b, 10%) Now, if the dipole is making an angle ϕ with respected to the z-axis, and an uniform electric field is applied perpendicular to the z-axis (from left to right) making an angle θ with the dipole, What is the torque that can be generated by the electric field? (c, 10%) If this electric field is to flip the dipole from an initial angle $\theta=90°$ to an angle θ, how much work is needed?
3. **Spherical capacitor:** (10%) A spherical capacitor is consisted of two concentric spheres of radius a and b, where $b>a$. The outer sphere carries a charge -Q, and the inner sphere has a +Q charge. What is the capacitance of this set up?
4. **Gauss law:** (40%) (a, 10%) What is Gauss law? Explain it. (b, 10%) Use Gauss law to calculate the electric field due to a charged spherical shell of radius a. What is the electric field at a distance r (r>a) from the center of the spherical shell? (c, 10%) What is the electric field at a distance r<a? (d, 10%) plot the electric field as a function of distance from the origin?

1. (a) "An adiabatic free expansion" means there is no heat transfer (adiabatic), free expansion means gases are freely to expand. No work is done to or by the gases. Such as



1. (b) for an adiabatic free expansion

$V_i \to V_f$

$dW = 0,$

$dQ = 0$

$dE_{in} = 0$

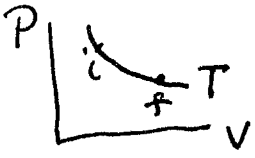
$\Delta T = 0 \quad \therefore T_i = T_f$

We can calculate the entropy change using an isothermal reversible expansion process.

$$\Delta S = \int_i^f \frac{dQ_r}{T} = \frac{1}{T}\int_i^f dQ_r$$

Since $\Delta E_{in} = 0 \Rightarrow \int_i^f dQ_r = \int_i^f dW$

Note: such as



when no heat is given to this system.

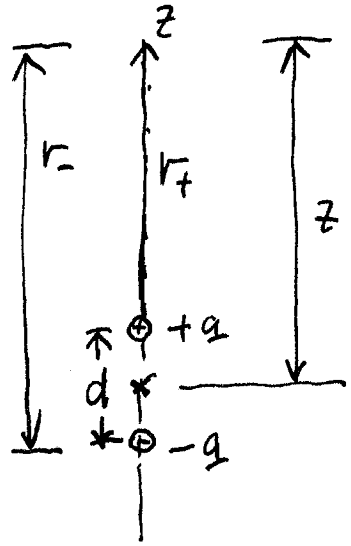
$$\therefore \Delta S = \frac{1}{T}\int dW = \frac{1}{T}\int P dV = \frac{1}{T}\int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$= nR\int_{V_i}^{V_f} \frac{dV}{V} = nR \ln\left(\frac{V_f}{V_i}\right) > 0$$

(c) free expansion is an irreversible process. the isothermal reversible expansion has the same temperature and same initial/final points. So we can use it to calculate the entropy

2. (a) This is the same problem as in the P23-6 of the lecture notes.

Refer to the figure to the right. The electric field is the vector sum of the two charges



$$\therefore \vec{E} = \vec{E}_{(+)} + \vec{E}_{(-)}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_+^2} + \frac{-q}{r_-^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z-\frac{1}{2}d)^2} + \frac{-1}{(z+\frac{1}{2}d)^2}\right]$$, since z is a constant

$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$ But $z \gg d$, $\frac{d}{2z} \ll 1$

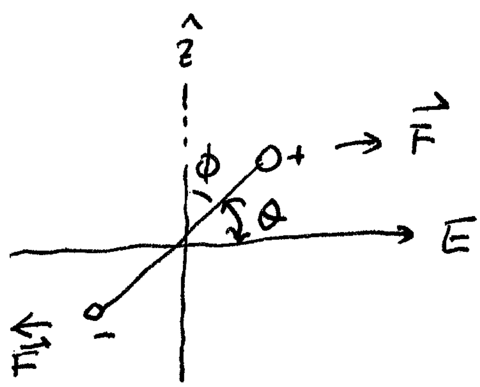
$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{2d}{2z}+\dots\right) - \left(1-\frac{2d}{2z}+\dots\right)\right]$$ So we can expand the polynominal

$$= \frac{q}{4\pi\epsilon_0 z^2}\left[1+\frac{2d}{2z}+\frac{2d}{2z}-1+\dots\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{d}{z^3}$$ if we define $\vec{P} = qd$

Then $E = \frac{1}{2\pi\epsilon_0}\frac{\vec{P}}{z^3}$

(b) the forces are $+\vec{F}$ and $-\vec{F}$. But the both generate a clockwise torque



$$\tau = 2\cdot F\cdot\frac{d}{2}\sin\theta$$

$$= 2qE\frac{d}{2}\sin\theta = PE\sin\theta$$

2 (c)

The work needed is $U_f - U_i$

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_{\theta_i}^{\theta_f} pE \sin\theta \, d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$

$$= pE(\cos\theta_i - \cos\theta_f) \quad , \quad \theta_i = 90^\circ$$

$$\therefore U_f - U_i = \Delta U = -pE\cos\theta_f$$

$$= -\vec{p} \cdot \vec{E}$$

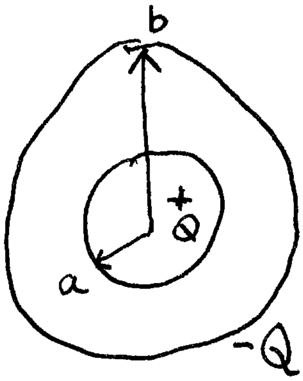
that is, this much energy is needed.

3.

$$V_b - V_a = -\int_a^b E_r \, dr$$

$$= -\frac{1}{4\pi\epsilon_0} Q \int_a^b \frac{dr}{r^2}$$

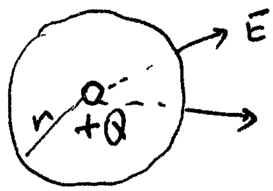
$$= \frac{1}{4\pi\epsilon_0} Q \left[\frac{1}{r}\right]_a^b = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right) \quad \text{let } k_e = \frac{1}{4\pi\epsilon_0}$$



$$C = \frac{Q}{\Delta V} = V_b - V_a = \frac{Q}{k_e Q}\left(\frac{1}{b} - \frac{1}{a}\right)^{-1}$$

4. (a) (in p24-1, and 24.2)

Gauss law gives the relation of the net electric flux and charges enclosed by a closed surface called Gaussian surface.



Example
if we have a charge +Q enclosed by an imaginary closed surface as shown

The electric flux = $E = k_e \frac{Q}{r^2}$

flux $\Phi_E = k_e \frac{Q}{r^2} \cdot 4\pi r^2$

$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} 4\pi r^2 = \frac{Q}{\epsilon_0}$

that is $\Phi_E = \frac{Q}{\epsilon_0}$

(b) Use Gauss law

1) when $r > a$

$E = k_e \frac{Q}{r^2}$ treat it as if the charge is concentrate in the origin like a point charge

(c) when $r < a$

There is no charge within the sphere (the Gaussian surface)

$\therefore E = 0$