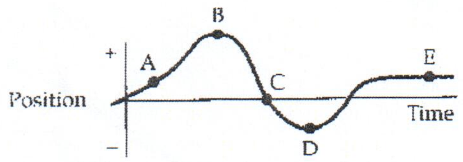
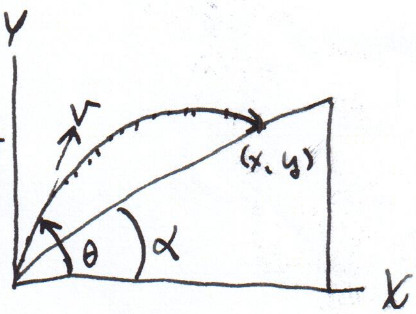
Solution

***Problems (20% each, total 6 problems, 120%)***

1. When the velocity in this figure of this object is minimum, its slope ($= \frac{dx}{dt}$) is zero.
Therefore, the points are B, D, and E.




2. The projectile is shown to the right as a dotted arc.
we can de-compose the displacement in x. and y directions




$$X_p = V\cos\theta \cdot t \quad -①$$
$$Y_p = V\sin\theta \cdot t - \frac{1}{2}gt^2 \quad -②$$

it will subject to gravitational fall

Let the range be R along the slope

$$X_s = R\cos\alpha \quad -③$$
$$Y_s = R\sin\alpha \quad -④$$

from ① and ③ ① = ③ $V\cos\theta\, t = R\cos\alpha$. $t = \frac{R\cos\alpha}{V\cos\theta}$

②, ④ $R\sin\alpha = V\sin\theta \cdot t - \frac{1}{2}gt^2$

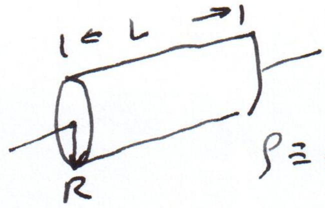
$$R\sin\alpha = \frac{R\cos\alpha\sin\theta}{\cos\theta} - \frac{1}{2}g\frac{R^2\cos\alpha}{V^2\cos\theta}$$

$$\therefore R = \frac{2(\sin\alpha\cos\theta - \cos\alpha\sin\theta)V^2}{g\cos^2\alpha} = \frac{V^2\sin(\theta-\alpha)}{g\cos^2\alpha}$$

Solution

***Problems (20% each, total 6 problems, 120%)***

3. (a) moment of inertia @ page 10-4 of the lecture note




$\rho \equiv$ the density

$$I_{cM} = \int r^2 dm$$
$$= \int r^2 2\pi r dr L\rho$$
$$= 2\pi L\rho \frac{R^4}{4}$$
$$= \frac{1}{2} M R^2$$

(b) when it rolls down, it is not rolling down on it central axis, it is on its edge as indicated in the figure


$$\therefore Mgh = \frac{1}{2} I \omega^2$$
$$= \frac{1}{2}(I_c + MR^2)\omega^2$$
$$= \frac{1}{2}\left(\frac{1}{2}MR^2 + MR^2\right)\omega^2$$

$I = I_{cM} + MR^2$

Parallel axis theorem

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh}$$

But $v = R\omega = R \cdot \frac{1}{R}\sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}gh}$

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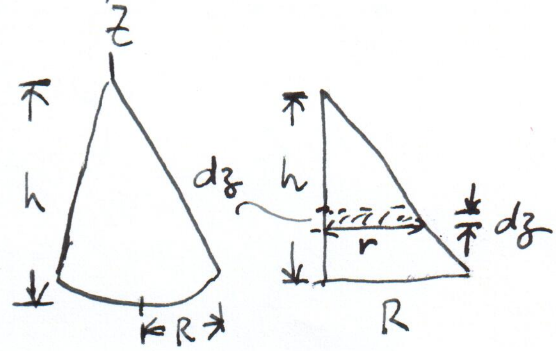
Solution

***Problems (20% each, total 6 problems, 120%)***




4. to calculate the moment of ~~initial~~ inertia

$dI = \frac{1}{2} dm\, r^2$

$dm = \pi r^2 dz\, \rho$

$\therefore I = \int dI = \int_0^R \frac{1}{2}(\pi r^2 dz\, \rho)\, r^2$

$= \frac{1}{2}\pi\rho \int_0^R r^4 dz$     But $\frac{r}{R} = \frac{z}{h}$   $z = \frac{h}{R} r$

$dz = \frac{h}{R} dr$

$= \frac{1}{2}\pi\rho \int_0^R \frac{h}{R} r^4 dr$

$= \frac{1}{2}\pi\rho \frac{h}{R}\frac{1}{5} r^5 \Big]_0^R = \frac{3}{10}\left(\rho \frac{1}{3}\pi R^2 h\right) R^2 = \frac{3}{10} M R^2$

6 (a) centripetal acceleration of the hawk is

$$a_c = \frac{v^2}{r} = \frac{(4.00\ \frac{m}{s})^2}{12m} = 1.33\ \frac{m}{sec^2}$$

(b) the magnitude of the acceleration vector is $|a|$

$$|\vec{a}| = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.33)^2 + (1.2)^2} = 1.79\ \frac{m}{sec^2}$$

$$\theta = \tan^{-1}\left[\frac{a_c}{a_t}\right] = \tan^{-1}\left[\frac{1.33}{1.2}\right] = 48^\circ \text{ inward}$$

5.

$r$ $\qquad W_{on\ book} = (mg)\cdot\Delta r$

$= (-mg\hat{j})\cdot[(y_a - y_b)\hat{j}]$

$a$ $\qquad = mgy_b - mgy_a$

$= \Delta E_K$

$= \Delta K_{book}$

$$mgy_b - mgy_a = -(mgy_a - mgy_b)$$

$$= -(U_f - U_i)$$

$$= -\Delta U_g$$

$$\therefore \quad \Delta K = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$