



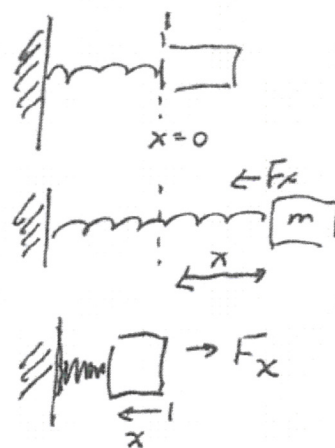
SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are NOT allowed.

Problems (5 Problems, total 100%)

- 1. Thermal energy, molar heat and first law (30%):** (a, 5%) A heating element is rated as "150W". What amount of energy will this heating element provide each minute? (b, 15%) A 150W heater is placed in 1 Kg of methanol and turned on for exactly 1 minute. The temperature increased by 3.54°C . Assuming that all the heat is absorbed by the methanol, calculate the molar heat capacity for liquid CH_3OH . (c, 10%) A $1/4\text{hp}$ (0.25 horse power) motor uses 187W of electric energy while delivering 35J of work each second. How much heat must be dissipated in the form of friction (heat)?
- 2. Simple harmonic Oscillation (20%):** A vertically suspended spring of negligible mass and force constant k is stretched by an amount l when a body of mass m is hung on it. The body is pulled by hand an additional distance y downward and then released. (a, 10%) Show that the motion of the body is governed by the equation $a = -ky/m$, so that the body executes harmonic motion about its equilibrium position, and (b, 10%) show that the period of this motion is the same as that of a simple pendulum of length l .
- 3. Simple Harmonic system (20%):** Refer to the figure to the right for a very simple mass-spring system. The attached mass is m , and the spring has a spring constant k . (a, 5%) Start from Newton's law, if you pull the mass with displacement x , and release it; it will start to oscillate. What is its acceleration a ? (b, 5%) Write down the differential equation that describes the oscillation. (c, 5%) Demonstrate, without solving it, that $x(t) = \cos(\omega t + \phi)$ is one of the possible solutions of the equation. (d, 5%) Solve the period of the oscillation.
- 4. Escape Speed (15%):** Estimate the size of a rocky sphere with a density of 3.0 g/cm^3 from the surface of which you could just barely throw away a golf ball and have it never return. (Assume your best throw is 40 m/s).
- 5. Doppler effect (15%):** If a sound wave has a speed v and frequency f . What is the detected frequency when the source is moving at speed v_s towards the detector and the detector is stationary? Derive this.



1. (a) 150W means 150 J/sec, Therefore for one minute.
it will provide 150 J/sec \times 60 sec = 9000 J

$$(b) \Delta Q = 9000 \text{ J} = n \int_{T_1}^{T_2} C_p dT = n C_p \Delta T$$

$$\therefore 9000 \text{ J} = \frac{1000 \text{ g}}{32 \text{ g/mole}} \cdot C_p \cdot 3.54 \text{ K}$$

$$\rightarrow \underline{\underline{C_p = 81.35 \text{ J/K.mole}}}$$

Note: the molecular weight
for methanol CH_3OH
 $= 12 + 3 + 16 + 1$
 $= 32$

(c) $W = 35 \text{ J}$
Internal energy changes is $\Delta E_{\text{int}} = 187 \text{ J}$

$$\therefore Q = \Delta E_{\text{int}} - W = 187 \text{ J} - 35 \text{ J} = 152 \text{ J}$$

This much heat (152 J) will be dissipated due to friction in the motor.

2. (a) The initial equilibrium is from the weight of the object
 $mg = kl$

now this object is pulled by an addition force a distance y

$$\text{The force } F = mg - k(l+y) = -ky = ma$$

$$\therefore f = ma = -ky$$

$$\rightarrow a = -\frac{ky}{m}$$

↑ the harmonic oscillation by
pulling it y down ward

$$(b) \text{ Since } mg = kl, T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{kl}{g} \cdot \frac{1}{k}} = 2\pi \sqrt{\frac{l}{g}}$$

The period is the same as a simple pendulum.

$$3. (a) \vec{F} = -kx = m a_x \quad \therefore a_x = -\frac{k}{m} x$$

$$(b) a_x = \frac{dv_x}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

$$\rightarrow \underbrace{\frac{d^2 x(t)}{dt^2} + \frac{k}{m} x(t) = 0}_{\text{2nd order differential equation to describe the oscillation of } m \text{ attached to a spring of force constant } k.}$$

(c) To see if $x(t) = A \cos(\omega t + \phi)$ is one of the solutions to differential equation in (b), we simply have to plug in the $x(t)$ into the equation

$$\frac{d^2 x(t)}{dt^2} = \frac{d}{dt} \left[\frac{dx(t)}{dt} \right] = \frac{d}{dt} \left[\frac{d}{dt} (A \cos(\omega t + \phi)) \right]$$

$$= \frac{d}{dt} [-A\omega \sin(\omega t + \phi)]$$

$$= -A\omega^2 \cos(\omega t + \phi)$$

$$= -\omega^2 x(t)$$

$$\rightarrow \frac{d^2 x(t)}{dt^2} + \omega^2 x(t) = 0 \quad \rightarrow \text{Compare this with (b) that is } \omega^2 = \frac{k}{m} \rightarrow \omega = \sqrt{\frac{k}{m}}$$

Therefore it can be a solution of the differential equation.

(d) Let the period of the oscillation be T

$$\text{then } x(t) = x(t+T)$$

$$A \cos(\omega t + \phi) = A \cos(\omega(t+T) + \phi)$$

$$\therefore \omega(t+T) + \phi - (\omega t + \phi) = 2\pi$$

$$\therefore \omega T = 2\pi, \text{ or } \underline{T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}}$$

4. Let the rocky sphere have mass M . $\rho = 3.0 \text{ g/cm}^3$
 From the surface if you want to throw a golf ball and it never return (i.e. it escape the sphere), you use your best to throw, $v = 40 \text{ m/s}$
 from conservation of energy

$$\frac{1}{2} m v_0^2 = \frac{G M m}{R}$$

$$M = \text{mass of the sphere} \\ = \rho \frac{4}{3} \pi R^3$$

then

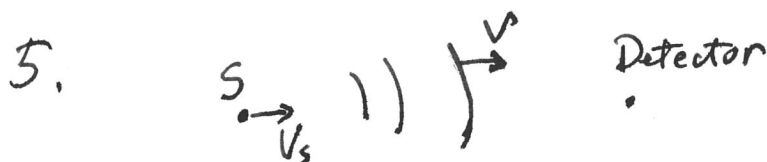
$$\frac{1}{2} m v_0^2 = \frac{G \rho \frac{4}{3} \pi R^3 m}{R}$$

$R =$ the radius of the sphere

$v_0 =$ highest speed you can throw

$G =$ Gravitational constant

$$\rightarrow R = v_0 \sqrt{\frac{3}{8\pi G \rho}}$$



When the source moves toward the detector with a speed v_s , the wavefront moves @ speed v and the wavefront moves a distance vT in one period of time, T (w_1). The source moves $v_s T$ distance for time T . (w_2)

The distance between these two w_1 and w_2 is the wavelength λ' detected

$$\lambda' = vT - v_s T$$

$$\therefore f' = \frac{v}{\lambda'} = \frac{v}{vT - v_s T} = \frac{v}{\frac{v}{f} - \frac{v_s}{f}} = f \frac{v}{v - v_s}$$