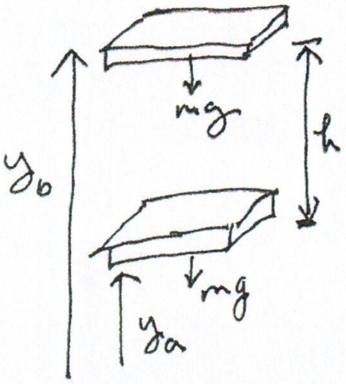
Chap 8 potential energy
Conservation of Energy



The book subject to the same gravitational force mg

The work done by the gravity is

$$\underbrace{mg y_b - mg y_a = mgh}$$

this much energy has the potential to become ~~en~~ kinetic energy

$$\underbrace{mgh} \rightarrow \frac{1}{2} m v^2$$

gravitational potential energy

$$W = (F_{app}) \cdot \Delta r$$

$$= (mg \hat{j}) \cdot [(y_b - y_a)\hat{j}]$$

$$= mg y_b - mg y_a$$

(7-14) $\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$$= (mg y_f - mg y_i)$$

It is similar to the energy transferred to kinetic energy

$$\therefore \quad U \equiv mgy$$

$W = \Delta U_g$ → the gravitational potential energy depends on only the vertical height of the system.

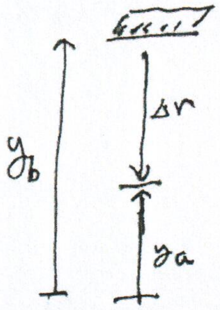
$$W = (F_{app}) \cdot \Delta r = (mg\hat{j}) \cdot [(x_b - x_a)\hat{i} + (y_b - y_a)\hat{j}]$$

$$= mg\, y_b - y_a$$

Note: $\hat{j} \cdot \hat{i} = 0$ orthogonal

8.2 The isolated system – Conservation of Mechanical energy

If the book falls back to its origin



$$W_{\text{on book}} = (m\vec{g})\cdot\Delta\vec{r}$$
$$= (-mg\hat{j})\cdot[(y_a - y_b)\hat{j}]$$
$$= mgy_b - mgy_a$$
$$= \Delta E_K$$
$$= \Delta K_{\text{book}}$$

$$mgy_b - mgy_a = -(mgy_a - mgy_b)$$
$$= -(U_f - U_i)$$
$$= -\Delta U_g$$

$$\therefore \quad \Delta K = -\Delta U_g$$
$$\Delta K + \Delta U_g = 0$$

mechanical energy $\quad E_{mech} \equiv K + U_g$
$$= K + U$$

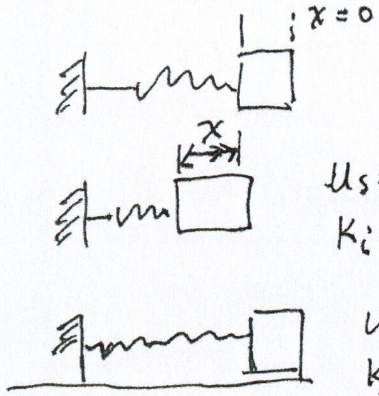
$$\Delta K + \Delta U = (K_f - K_i) + (U_f - U_i) = 0$$
$$\rightarrow K_f + U_f = K_i + U_i$$
$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

— conservation of energy

Elastic potential energy

$$W_{App} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$\therefore U_s \equiv \frac{1}{2}kx^2$ — the energy stored in the deform spring



$U_s = \frac{1}{2}kx^2$

$K_i = 0$

$U_s = 0$

$K_f = \frac{1}{2}mv^2$

disregarding the process

Conservative force (Can be assesed by initial and final conditions)

1) The work done is independent of the path taken
2) The work done through a closed path is zero

Non conservative force

Acting within a system cause a change

$\Delta K = -f_k d$ — the decrease in kinetic energy due to a non conservative force

$$\Delta E_{mech} = \Delta K + \Delta U_g = -f_k d$$

The work done by a conservative force W_c

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

But $\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx$

$$U_f = -\int_{x_i}^{x_f} F_x \, dx + \underbrace{U_i}$$

↑ we ~~only~~ often take it as zero

zero point of the potential

↓

$$dU = -F_x \, dx$$

$$\boxed{F_x = -\frac{dU}{dx}}$$

The negative derivative of the potential energy of the system is the force.

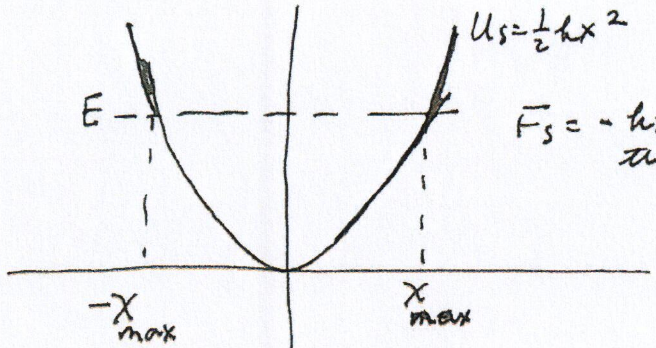
for example $F_s = -kx$

$U_s = \frac{1}{2}kx^2$

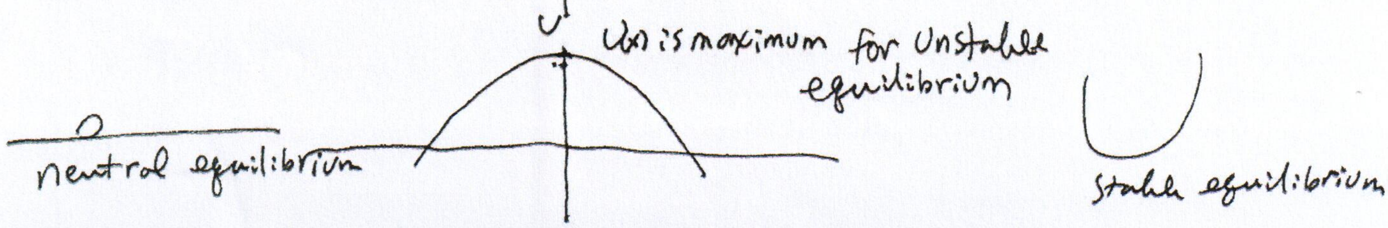
Do example 8.11 (page 237)

$$-\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

plot U_s



$F_s = -kx$ = the slope of the U_s



Chap 8. Conservation of energy -2

Energy is always conserved.

in an nonisolated system: $\Delta E_{system} = \sum T$

T = energy transferred to the environment.

$\rightarrow \Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + \cdots$

all items on each side can be zero

Isolated System: No energy transferred across the boundary of the system.

$\Delta K = -\Delta U_g \rightarrow$ in a book system

$$W_{on\,book} = (m\vec{g}) \cdot \Delta\vec{r} = (-mg\hat{j}) \cdot [(y_f - y_i)\hat{j}]$$

$$= mgy_i - mgy_f$$

$$= \Delta K_{on\,book}$$

$$= -(mgy_f - mgy_i)$$

$$= -\Delta U_g$$

$\therefore \Delta K = -\Delta U_g$

$\rightarrow \underbrace{\Delta K + \Delta U_g}_{= \Delta E_{mech}} = 0$

$\rightarrow \Delta E_{mech} = 0$

Kinetic friction, check Fig 8.7. page 204

$$\sum W_{\text{other force}} = \int (\Sigma \vec{F}_{\text{other force}}) \cdot d\vec{r}$$ page 205

$$\sum W_{\text{other force}} + \underbrace{\int \vec{f}_k \cdot d\vec{r}}_{\text{friction}} = \int m\vec{a} \cdot dr = \int m \frac{dv}{dt} \cdot d\vec{r}$$

$$= \int_{t_i}^{t_f} m\left(\frac{dv}{dt} \cdot \vec{v}\right) dt$$

Note: $\frac{d}{dt}(\vec{v} \cdot \vec{v}) = \frac{dv}{dt} \cdot v + v \cdot \frac{dv}{dt} = 2\frac{dv}{dt} \cdot \vec{v}$

$$\therefore \frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{1}{2}\frac{d}{dt}(\vec{v} \cdot \vec{v}) = \frac{1}{2}\frac{dv^2}{dt}$$

$$\therefore \sum W_{\text{other force}} + \int f_k \cdot dr = \int_{t_i}^{t_f} m \cdot \left(\frac{1}{2}\frac{dv^2}{dt}\right) dt$$

$$= \frac{1}{2} m \int_{v_i}^{v_f} d(v^2) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$$

$$\sum W_{\text{other}} + \underbrace{\int \vec{f}_k \cdot d\vec{r}}_{= -f_k dr} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$$

$$\boxed{\sum W_{\text{other}} - f_k d = \Delta K}$$