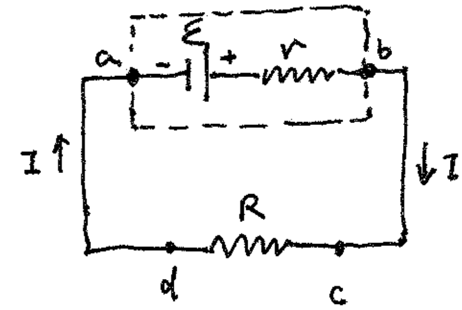
Chap 28 Direct Current Circuit

28.1 Electromotive force (EMF)

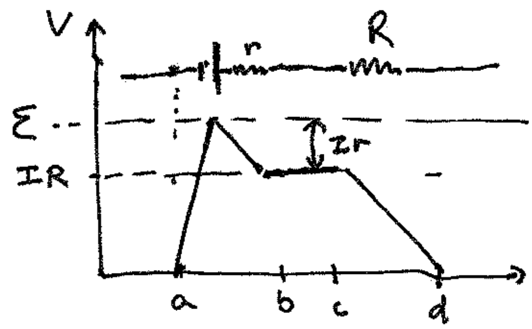
the EMF of a battery is the maximum possible voltage that a battery can provide.



all battery have internal resistance "r".

$$\Delta V = V_b - V_a = \mathcal{E} - \underbrace{Ir}$$

due to internal resistance r



From 28.2b

$$\mathcal{E} = IR + Ir$$

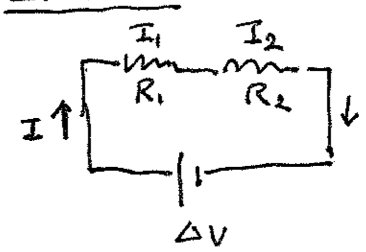
$R \equiv$ load resistance

$$\rightarrow I = \frac{\mathcal{E}}{R+r}$$

or $$I\mathcal{E} = I^2R + I^2r$$
= total power

28.2 Resistans in Series and in parallel

① In Series -



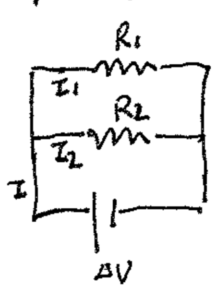
$$I_1 = I_2 = I$$

$$\therefore \Delta V = IR_1 + IR_2 = I\underbrace{(R_1+R_2)}_{R_{eq}}$$

$$\therefore R_{eq} = R_1 + R_2$$

$$\rightarrow R_{eq} = R_1 + R_2 + R_3 + \cdots$$

② In parallel



$$I = I_1 + I_2$$

$$= \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\rightarrow \Delta V = I\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$\therefore R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \quad \text{or} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

28.3 Kirchhoff's Rules

When a circuit is not that simple, one can use Kirchhoff's rule to analyze the circuit

1. Junction rule: $\sum I_{in} = \sum I_{out}$

— the sum of current entering any junction is the same as the sum leaving the junction

— Conservation of electric charges

2. Loop rule:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0$$

— the sum of potential differences across all elements around any closed loop must be zero

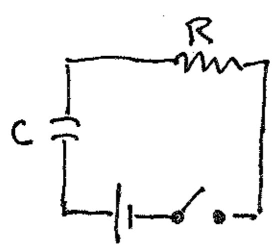
— Conservation of energy

Do example 28.10 multiloop page 873

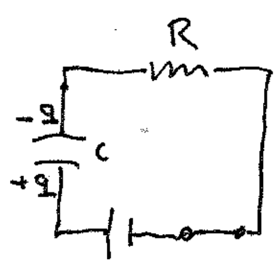
28.4 RC Circuit

— circuit containing Resistor and Capacitor

Charging a Capacitor



$t<0$



$t>0$

Using Kirchhoff's rule

$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad (28.11)$$

1) at $t=0$ (Capacitor is not charged)

$$I_0 = \frac{\mathcal{E}}{R}$$

2) When the capacitor is completely charged, $I=0$

$$Q = C\mathcal{E}$$

Remember $I = \frac{dq}{dt}$

(28.11) $\rightarrow \quad \mathcal{E} - \frac{q}{C} - IR = 0$

$$I = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = -\frac{1}{RC}\, q + \frac{\mathcal{E}}{R}$$

$$\rightarrow \boxed{\frac{dq(t)}{dt} + \frac{1}{RC}\, q(t) - \frac{\mathcal{E}}{R} = 0}$$

1st order differential Equation

$\rightarrow$ Solution

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$
$$= Q\left(1 - e^{-t/RC}\right)$$

or $\frac{dq(t)}{dt} = \frac{C\mathcal{E}}{RC} - \frac{q}{RC}$

$$= -\frac{q - C\mathcal{E}}{RC}$$

$$\frac{dq(t)}{q - C\mathcal{E}} = -\frac{1}{RC}\, dt$$

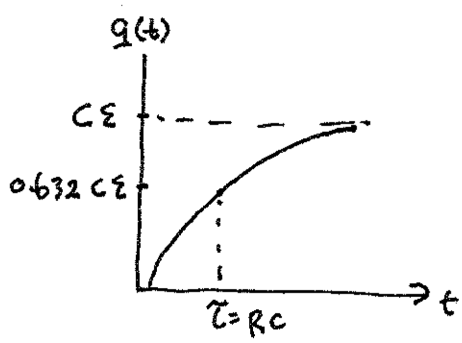
$$\int_0^q \frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\int_0^t dt$$

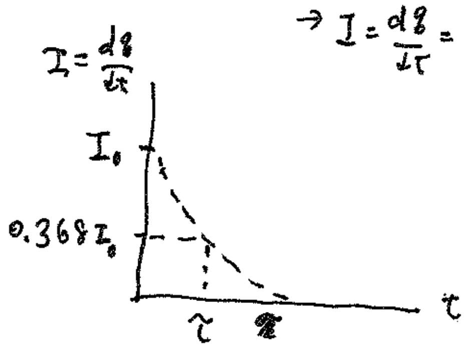
$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = \frac{-t}{RC}$$

$$\rightarrow \quad q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q\left(1 - e^{-t/RC}\right)$$

$RC \equiv \tau$
$= \text{time constant}$

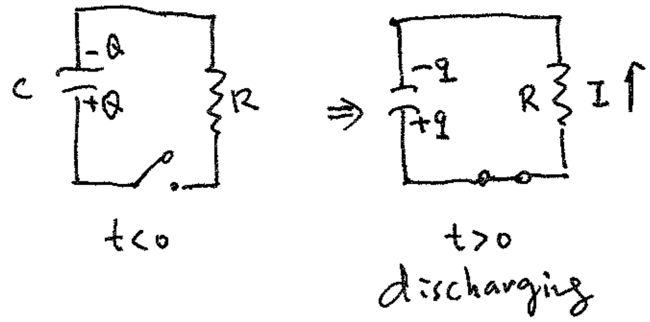
$\rightarrow I = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$





Note: the dimension of τ

Discharging a Capacitor



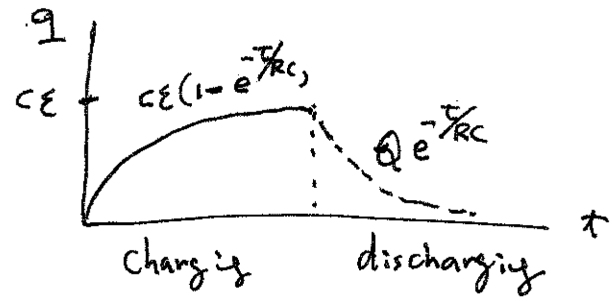
$$-\frac{q}{C} - IR = 0 \qquad I = \frac{dq}{dt}$$

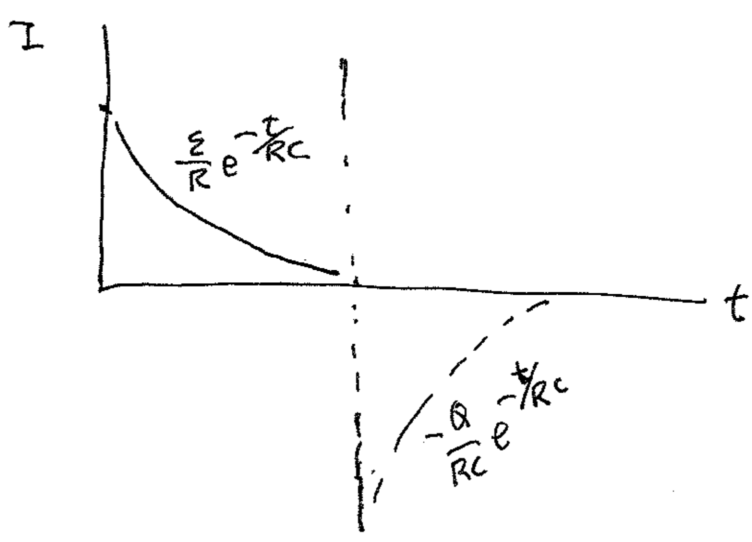
$$-R\frac{dq}{dt} = \frac{q}{c} \quad \rightarrow \quad \frac{dq}{q} = -\frac{1}{RC}\,dt$$

$$\int_Q^q \frac{dq}{q} = -\frac{1}{Rc}\int_0^t dt$$

$$\rightarrow \ln\left(\frac{q}{Q}\right) = -\frac{t}{Rc} \quad \rightarrow \quad q(t) = Q\,e^{-t/RC}$$

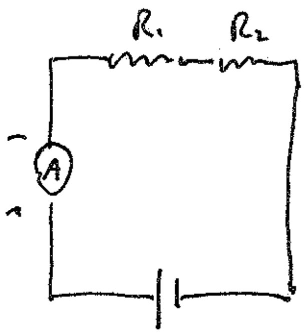
$$\rightarrow I = \frac{dq}{dt} = -\frac{Q}{RC}\,e^{-t/RC}$$

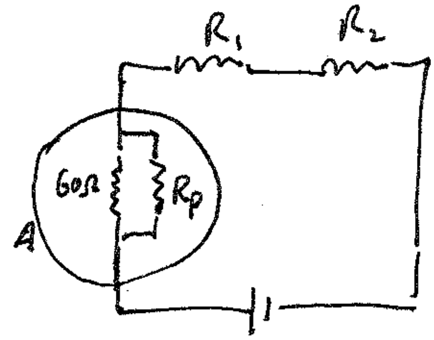




28.5 Electric Meters

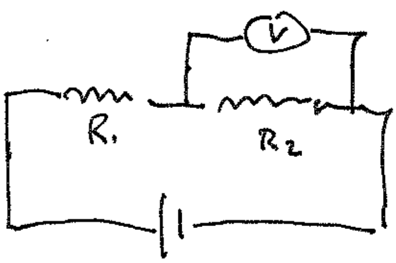
The Ammeter: Ideally, an ammeter should have zero resistance, so that the current being measured is not altered.

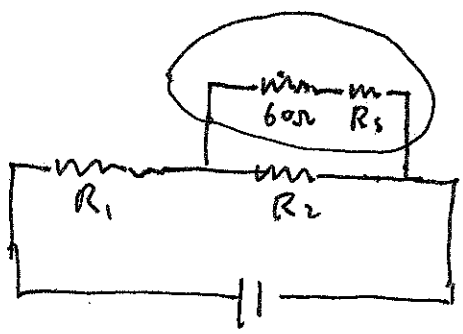




Connect a R_p. $R_p \ll 60\Omega$ when the Galvanometer is used as an Ammeter.

The Voltmeter





Connect a R_s. $R_s \gg 60\Omega$, when use the galvanometer as a voltmeter.