Price competition when consumer behavior is characterized by conformity or vanity

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Abstract

It has long been recognized that the pleasure of consuming a good may be affected by the consumption choice of other consumers. In some cases, social pressures may lead to conformity; in some others, individuals may feel the need of exclusiveness under the form of vanity. Such externalities have proven to be important in several markets. However, the market implication of these externalities are still unclear. To investigate them, we propose to combine the consumption externality model and the spatial duopoly model. When conformity is present but not too strong, both firms remain in business but price competition is fiercer and results in lower prices. The market share of the large firm increases with the population size; as the population keeps rising, the large firm may serve the entire market and set a price that has the nature of a limit price. When conformity is strong enough, different equilibria may exist. In most of these equilibria, a single firm captures the whole market. At the other extreme, when vanity is at work, price competition is relaxed.

Keywords: Price competition; Consumer behavior; Conformity; Vanity

1. Introduction

Ever since Veblen (1899), it has been recognized that the pleasure of consuming a particular good may be affected by the consumption choice of other consumers.

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In other words, the decision to buy a good depends not only upon the intrinsic utility from consuming it but also upon the social attributes associated with its consumption. The reason is that consumption decisions are made to satisfy both material and social needs. There are at least two types of explanations for such needs. On one hand, society may harshly censure nonconformist attitudes in some activities. As a result, individuals imitate each other because they feel the desire to avoid social ostracism. In this case, social interaction leads to conformity. On the other hand, people may have strong individualistic values. As a consequence, individuals feel the need of exclusiveness in some activities. They want to signal their idiosyncrasies and, to do so, strive to derive prestige from their consumption of positional goods. Social interaction now leads to vanity. A widespread form of vanity lies in the negative impact that congestion has on individual satisfaction. Clearly, what we see is a mixture of behaviors in which particular standards govern some activities while others are free from such constraints.

In market economies, casual observation suggests that conformity characterizes some markets like those for garments or beverages where we see many consumers purchasing similar goods. Likewise, collective passions or fads (which correspond both to a form of conformity) lead to the concentration of the market in the hands of a small number of producers (the ‘superstars’). All of these agree with Frank and Cook (1995) who observe that several industries, including entertainments, sports and the arts, are dominated by a system in which the winners get much more than the others. On the other hand, vanity seems more common in markets for luxury goods like perfumes, sport cars, or ‘haute couture’. Segments of population characterized by conformity or vanity are likely to be different. However it is valuable to have a model that encompasses both types of behavior as special cases.

Consumption externalities are, therefore, important in several markets in which the decision to buy from a particular store is positively or negatively affected by the group of consumers patronizing the store. Leibenstein (1950) has coined the terms bandwagon and snob effects to describe the global impact of such attitudes. However the microeconomic foundations of these effects as well as their market implication are still unclear. Recently, Becker (1991) has discussed a provocative example in which one restaurant eventually captures almost the entire business while its competitor has a negligible market share in a market where no restaurant has an ex ante technological advantage. He suggested that a positive consumption externality, that is, the demand for a store being positively related to its number of customers, may explain why similar stores experience vastly different sales patterns over long time periods. Karni and Levin (1994) have provided a game-theoretic foundation for such an externality by assuming that the (indirect) utility function depends positively on the number of clients. At a deeper level, Bernheim (1994, p. 864) shows that “when status is sufficiently important relative to intrinsic utility, many individuals conform to a single, homogeneous standard of behavior, despite heterogeneous underlying preferences”. Hence, when small
departures from the social norm impair seriously their status, individuals may have similar consumption patterns. In the case of vanity, Corneo and Jeanne (1997) argue that the roots of such a behavior come from the desire of consumers to signal their social status and wealth by purchasing specific products, thus making their status (at least partially) observable to others. Finally, Bagwell and Bernheim (1996) develop a signaling model where consumers gain utility from social status as manifested by a social contract. The social contract of an individual is determined by her income and her choice of quality of a single consumption good. Since an individual’s income is not observed by others, high-income individuals resort to choosing to consume high-quality in order to influence the social contract which enhances their status. In a separating equilibrium, low-income individuals are better off consuming low-quality at a lower price, and hence at a higher quantity level. Thus, status signaling may provide an explanation of the Veblen effect.

In contrast to these contributions, the present paper does not attempt to explain the Veblen effect, but instead to explain firms’ strategic pricing behavior when consumer preferences exhibit either conformity or vanity. Somewhat surprisingly, spatial competition and product differentiation models have disregarded such social influences on consumer behavior by assuming that people always buy from the cheapest store (the cheapest product).

Observe that there is a wide body of literature addressing network goods for which consumers’ preferences depend on the clientele size (see Besen and Farrell, 1994; Katz and Shapiro, 1994, for recent surveys). As in the case of conformity, the willingness to pay for such a good increases with the number of customers who buy it because the good becomes more useful when the number of other consumers connected to the network rises. Though the reasons for this externality are technological rather than social, the corresponding models lead to reduced forms that can be used to study the market impact of the social phenomena described above.

In this paper, we propose to combine the consumption externality model and the spatial models of product differentiation in order to highlight the role of the bandwagon and snob effects in price competition with differentiated products, especially when one firm has an initial advantage over its competitor. As will be shown, this leads to new results which cannot emerge in a symmetric environment. By considering any location pair inside or outside the interval of consumers, we are able to deal with both exogenous horizontal and vertical differentiation. Further, we do not consider the case where individuals react to the consumption of specific individuals but assume that consumer utilities exhibit a consumption externality given by a function of the size of the clientele buying from the same

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1Note that informational cascades provide an alternative explanation to the phenomena that motivate this paper. However, they tend to be more fragile to small shocks than network effects (see, e.g., Bikhchandani et al., 1998).
store. Clearly, this is a simplifying assumption, but it accounts for a substantial part of the phenomena we want to study since we may always control for the population under consideration. Hence, consumers’ choices influence the utility of any particular consumer, thus leading to a formulation where consumers, and not only firms, are involved in a game-theoretic environment in which they must choose which firm to patronize. The elements of the resulting partition may then be viewed as the equilibrium networks of consumers generated by a given price system. By focusing on a differentiated industry, we may illustrate important issues such as the emergence of a single product/standard in equilibrium although the initial advantage of the winner might be very marginal (Frank and Cook, 1995, Ch. 1). Depending on the parameters of preferences, we show that several products/standards may arise as the outcome of competition between firms. Last, although one might prefer a setting in which such an externality would be the outcome of a social process as in Bernheim (1994) and Cole et al. (1992) it seems reasonable to assume here that the social norm enters into the preferences as an externality since our main goal is to study the impact of different social attitudes on the behavior of strategic firms in a differentiated market.

The analysis developed below shows that integrating social effects into product differentiation models may lead to very different outcomes. When bandwagon effects are present but not too strong, both firms remain in business but price competition is fiercer and results in lower equilibrium prices. An interesting feature of the model is that, due to asymmetric product differentiation, the marginal consumer now depends on the population size. In particular, it is shown that the market share of the firm with the initial advantage increases with the population size, a phenomenon which is frequently observed in modern markets. As the clientele keeps rising, the market share of the small firm may vanish while the large firm sets a price which has the nature of a limit price. In other words, we can explain how the increase in the population size leads to the emergence of a common standard. This is not surprising under vertical differentiation but rather unexpected in the horizontally differentiated case. The reason lies in the fact that the externality at work here makes the utility of the product vary with the number of buyers, thus blurring the distinction between horizontal and vertical differentiation.

Since the results above are derived under specific functional forms, one may wonder about their generality. To this end, we investigate under which conditions an increasing, concave externality function leads to the same results.

When bandwagon effects are strong enough, different price equilibria may coexist in which either firm captures the whole market. In other words, the emergence of a monopoly is made endogenous; this depends on the relative importance of the idiosyncratic and social factors. In addition, this also shows that it is difficult to predict the market structure which is going to emerge because no selection mechanism seems to be preferable to another in the present context. At the other extreme, when snob effects are at work, price competition is relaxed and
firms have more market power. Hence the market is characterized by higher equilibrium prices.

It is worth noting that the main distinctive features of network externality models, such as multiplicity of equilibria and upward sloping demands, arise only when network effects are strong. Under weak network effects, the product differentiation effect turns out to be powerful enough to smooth out such features. Still, as noted above, network effects may become themselves strong enough for a single network to emerge in equilibrium once the population size has reached some critical value. Such results show the complexity of the interplay between the various effects at work in our model.

The paper is organized as follows. In Section 2 we present the model in the case of two stores competing for customers distributed along Main Street when network effects are present. Two cases are distinguished: conformity (positive effects) and vanity (negative effects). More fundamentally, the case of conformity must be subdivided into strong and weak conformity. Section 3 deals with the case of vanity and weak conformity, in which the product differentiation effect dominates the network effect. The case of strong conformity turns out to be very different and is tackled in Section 4. Section 5 concludes and discusses various possible extensions.

2. The model

Consider two stores selling a homogeneous product and exogenously located at \( x_A \) and \( x_B \in \mathbb{R} \), with \( x_A \leq x_B \). We assume that production is costless and denote by \( p_i \) the (mill) price charged by store \( i \). There is a continuum of consumers of mass \( n \) uniformly distributed over the interval \([0,1]\). A consumer located at \( x \in [0,1] \) bears a transportation cost of \( t(x - x_i)^2 \) for buying from the store located at \( x_i \) \((i = A,B)\), while \( t > 0 \) is the transportation rate.

The present model differs from standard models of product differentiation because of the introduction of a consumption externality. Let \( n_i \) denote the number of consumers patronizing store \( i \). The externality affecting store \( i \)'s consumers is defined as follows:

\[
E(n_i) = \alpha n_i - \beta n_i^2
\]

in which the sign of \( \alpha \) expresses the type of externality, while \( \beta \geq 0 \) expresses the degree of concavity of the externality function. The case \( \alpha < 0 \) corresponds to

\footnote{It is well known that, for linear transportation costs and in the absence of consumption externality, a price equilibrium in pure strategies fails to exist for some location pairs, while such an equilibrium always exists when these costs are quadratic \( (d'Aspremont et al., 1979) \).}
vanity in consumer behavior since consumers are always worse off as the number of consumers patronizing the same store rises. When $\beta > 0$, this negative effect is magnified by the size of $n_i$. Intuitively, this is because one expects the crowding effect to become more and more important as the clientele increases. By contrast, when $a > 0$, the externality is positive and increasing in the network size up to $n_i = \alpha/2\beta$ (which may be arbitrarily large). In this case, there is conformity in that a consumer is better off when belonging to a (growing) network than by being alone. In the next two sections, we will restrict ourselves to $n_i \leq \alpha/2\beta$ because we wish to focus on the impact of an increasing externality. Since the maximum of $E(n)$ is $\alpha^2/4\beta$, by increasing $\alpha^2/4\beta$ while keeping the ratio $\alpha/2\beta$, we are able to study the impact of the concavity of the externality on the market outcome. The case where $\alpha > 0$ and $n_i > \alpha/2\beta$ leads to different conclusions and will be discussed in Section 5.

If all consumers buy one unit of the product, we have $n_A + n_B = n$. The (indirect) utility of consumer $x$ when buying from $i$ is then defined by

$$V_i(x) = an_i - \beta n_i^2 + K - p_i - \tau(x - x_i)^3$$

in which $K$ stands for the gross, intrinsic utility a consumer derives from consuming one unit of the product. In (2), the first two terms represent the network effect while the last three terms correspond to the stand-alone value of product $i$. In the tradition of spatial models of product differentiation, it is assumed that $K$ is sufficiently large to ensure that all consumers prefer buying rather than dropping out of the market. This assumption is further discussed below.

We now return to the product differentiation interpretation of our model. In order to isolate the different possibilities, we momentarily set $\alpha = \beta = 0$. Then, our location model encompasses both the standard cases of horizontal and vertical product differentiation. If $0 < x_A + x_B < 2$, consumers split their purchase between the two stores when they offer their product at the same price, so that our setting describes horizontal product differentiation. On the other hand, if $x_A + x_B \geq 2$ (resp. $x_A + x_B \leq 0$), then all consumers are closer to store $A$ (resp. $B$) and our setting describes vertical differentiation since firm $A$ (resp. $B$) captures the whole set of consumers for $p_A = p_B$.

When $x_A + x_B = 1$, firms $A$ and $B$ are symmetrically located. Let $x_A \neq x_B$. Firm $A$ (resp. $B$) has a locational advantage if $x_A + x_B > 1$ (resp. $x_A + x_B < 1$). In the absence of network effects, this advantage (together with the above assumption of a uniform consumer density) materializes in firm $A$ having a larger equilibrium market than firm $B$.

Despite its simplicity, the utility (2) therefore captures different types of product differentiation together with various kinds of social externalities. Note that, in the case of vanity, one might want the utility to be a negative function of the fraction, instead of the total mass, of people buying the same good. We believe that both interpretations are meaningful and the relevant one depends on the type of market
considered. By contrast, in the conformity case, it seems more natural to focus on the total mass only.

Although consumers value only the size of the network they belong to, it is worth noting that consumers’ preferences give rise to a ‘peer gathering’ outcome. Indeed the last term in (2) implies that a network is formed by a connected set of consumers. This means that a network is formed by consumers having similar preferences as in Tiebout, although they do not explicitly value the ‘peerage’ of the co-consumers.

The market is modeled as a two-stage game, the solution of which is given by a subgame perfect Nash equilibrium in pure strategies. In the first stage, firms select their price; firm i’s strategy space is \([0, \infty)\). In the second, given any pair of prices \((p_A, p_B)\), consumers allocate themselves between the two stores; the strategy space of a consumer is \([A, B]\). An equilibrium consumer partition of a second-stage subgame is a partition of consumers between the two firms \((N_A^A, N_B^A)\) such that no consumer whose utility is (2) is strictly better off by patronizing the store different from that he is assigned to.

Unlike the standard model of spatial competition, different types of equilibrium consumer partitions may arise here because of the network effects. To begin with, we want to determine for a given location pair \((x_A, x_B)\) all the price pairs for which both firms have a strictly positive demand. In other words, we are interested in finding a consumer \(\hat{x} \in (0, 1)\) such that consumers situated to the left (right) of \(\hat{x}\) patronize store A (B). For that \(\hat{x}\) must satisfy the following equation:

\[
\alpha n_A - \beta n_A^2 - p_A - t(\hat{x} - x_A)^2 = \alpha n_B - \beta n_B^2 - p_B - t(\hat{x} - x_B)^2
\]

For the corresponding partition (denoted \(\hat{C}\)) to be an equilibrium of the subgame generated by \((p_A, p_B)\), the following condition must be met:

\[
n_A = \hat{x}n \quad \text{and} \quad n_B = (1 - \hat{x})n
\]

Condition (4) means that, in equilibrium, the networks expected by each consumer are the actual networks. Substituting (4) into (3) and solving for \(\hat{x}\) yields

\[
\hat{x} = \frac{p_B - p_A + t(x_B - x_A) - (\alpha n - \beta n^2)}{2[t(x_B - x_A) - (\alpha n - \beta n^2)]}
\]

Clearly, the price pairs for which \(\hat{x} \in (0, 1)\) depend on the sign of the denominator of (5).

In the sequel, we say that consumer preferences exhibit

- **vanity** or **weak conformity** if

\[
t(x_B - x_A) > \alpha n - \beta n^2
\]
• **strong conformity** if

\[ t(x_B - x_A) < an - \beta n^2 \]  

(7)

Condition (6) holds if and only if the product differentiation effect \( t(x_B - x_A) \) dominates the externality \( E(n) \) evaluated at the size of the total population. The former is an increasing function of the interfirm distance, while the latter is increasing with the population size. Clearly, (6) covers the cases in which \( \alpha < 0 \) (vanity) and the case in which the externality is positive but not too large, that is \( \alpha > 0 \) and \( 0 < E(n) < t(x_B - x_A) \) (weak conformity).

Condition (7) never holds when firms are sufficiently far apart. More precisely, this is so when the product differentiation effect \( t(x_B - x_A) \) exceeds \( E(n) \) regardless of the population size, that is \( t(x_B - x_A) > \alpha^2/4\beta \). On the contrary, when firms are sufficiently close, (7) is likely to arise when \( \alpha \) is large and/or \( \beta \) small.

3. Market equilibrium under vanity and weak conformity

In this section, we study the case of weak conformity in the cases of a quadratic (Section 3.1) and of a general increasing and concave externality function (Section 3.2).

3.1. The case of a quadratic externality function

Assume throughout this section that (6) holds. We first characterize the price pairs such that both firms share the market. This is done by determining the necessary and sufficient conditions for \( x \) defined by (5) to belong to \((0,1)\), that is,

\[
\begin{align*}
\frac{p_A - p_B}{p_A - p_B} & > an - \beta n^2 - t(x_B - x_A)(2 - x_A - x_B) \\
\frac{p_A - p_B}{p_A - p_B} & < t(x_B - x_A)(x_B + x_A) - an - \beta n^2
\end{align*}
\]  

(8)

It is readily verified that the admissible interval for \((p_A,p_B)\) is nonempty if and only if (6) holds. This implies that the inequalities (8) provide the characterization for \( n_A = \hat{x}n \) and \( n_B = (1 - \hat{x})n \) to hold. We now characterize the price pairs ensuring that a single store serves the whole consumer population. First, \( n_A = n \) and \( n_B = 0 \) (the corresponding partition is denoted \( C_A \)) is an equilibrium consumer partition when \( V_A(x) = V_B(x) \) for all \( x \in [0,1] \) given that \( n_A = n \) and \( n_B = 0 \) in (2). This is true if and only if

\[
\frac{p_A - p_B}{p_A - p_B} \leq an - \beta n^2 - t(x_B - x_A)(2 - x_A - x_B)
\]  

(9)

Similarly, \( n_A = 0 \) and \( n_B = n \) (this partition is denoted \( C_B \)) is an equilibrium consumer partition if and only if

\[
\frac{p_A - p_B}{p_A - p_B} \geq t(x_B - x_A)(x_B + x_A) - an + \beta n^2
\]  

(10)
Since (8), (9) and (10) are mutually exclusive, we see that, under vanity and weak conformity, a unique equilibrium consumer partition is associated with any given price pair. This implies that each firm’s demand is a well-defined function; in particular, in the domain defined by (8), demands are linear and decreasing in own price. We can now study the first-stage game. The profit functions are defined as follows:

\[ I_A = p_A n_A(p_A, p_B) \quad \text{and} \quad I_B = p_B n_B(p_A, p_B) \]

where the demands \( n_A \) and \( n_B \) are as above. This game is solved at a Nash price equilibrium. An equilibrium of the full game is then given by a price pair \((p_A^*, p_B^*)\) and a consumer partition \(C^* = (N_A^*, N_B^*)\) such that

\[ p_A^* \mu(N_A^*) \geq p_A n_A(p_A, p_B^*) \quad \text{for all} \quad p_A \geq 0 \]
\[ p_B^* \mu(N_B^*) \geq p_B n_B(p_A^*, p_B) \quad \text{for all} \quad p_B \geq 0 \]

where \( \mu \) is the Lebesgue measure, while

\[ V_A(x) \geq V_A(x) \quad \text{for all} \quad x \in N_A^* \]
\[ V_B(x) \geq V_A(x) \quad \text{for all} \quad x \in N_B^* \]

It follows from the foregoing that \( C^* \) is given by either \( C^* = ([0, \hat{x}], (\hat{x}, 1]) \), or \( C_A = ([0, 1], 0) \), or \( C_B = (0, [0, 1]) \). Since demands are linear and decreasing in own price, in the price domain where they are positive, a Nash equilibrium in pure strategies exists. Consider first the case of an interior equilibrium. Differentiating \( I_i \) with respect to \( p_i \) and solving the resulting equations for \( p_A \) and \( p_B \) yields the unique solution:

\[ p_A^* = \frac{t}{3}(x_B - x_A)(2 + x_A + x_B) - \alpha n + \beta n^2 \] \hspace{1cm} (11)
\[ p_B^* = \frac{t}{3}(x_B - x_A)(4 - x_A - x_B) - \alpha n + \beta n^2 \] \hspace{1cm} (12)

For (11) and (12) to be a price equilibrium, it remains to check that they satisfy the inequalities (8), which is so if and only if

\[ \alpha n - \beta n^2 < \frac{t}{3}(x_B - x_A) \min\{2 + x_A + x_B, 4 - x_A - x_B\} \] \hspace{1cm} (13)

Observe that (13) reduces to (6) in the case of symmetrically located stores, in which case both equilibrium prices are equal to \( t(x_B - x_A) - \alpha n + \beta n^2 \).

Consider, first, the case of vanity \((\alpha < 0)\). It is readily verified that (13) is always satisfied if \(-2 \leq x_A + x_B \leq 4\); in particular, (13) holds in all cases of horizontal differentiation. Furthermore, (13) also holds outside this interval provided that \(|\alpha|\) be large enough. This means that two vertically differentiated products of very different qualities survive due to the existence of strong enough vanity effects, while typically only the better product gets a positive demand in the
absence of vanity. Thus, vanity could explain why some people patronize distant clubs or restaurants with a small clientele. Finally, observe that both *equilibrium prices increase with the degree of vanity* when (13) holds. This is because consumers’ vanity makes the demand addressed to each firm less elastic, thus reducing the incentives to lower prices.

Note, however, that these results crucially depend on the assumption that all consumers buy the differentiated product. In a more general setting, one might expect strong vanity to lead some consumers to opt out. Depending on the locations of firms, two cases may arise in an uncovered market equilibrium: either the abstaining consumers are located near the market edges or between the two firms’ market segments. In the latter, each firm becomes a local monopolist.

Consider now the case of conformity ($\alpha > 0$). The assumption of a covered market is then less questionable since, as will be seen, conformity leads to lower market prices, thus increasing total utility of purchasing.

In the presence of weak conformity ($0 < E(n) < n(x_B - x_A)$), two cases may arise. In the first one, (13) is satisfied. Without loss of generality, consider the case in which $x_A + x_B > 1$ so that (13) becomes

$$an - \beta n^2 < \frac{1}{3}(x_B - x_A)(4 - x_A - x_B) \tag{14}$$

which can be met only if $x_A + x_B < 4$.

By (14), it is readily verified that market sharing may occur when horizontal differentiation prevails ($x_A + x_B \leq 2$) but also when there is vertical differentiation provided that $x_A + x_B$ remain lower than 4. In both cases, *the equilibrium prices decrease with the degree of conformity.* Last, we have $p^*_A > p^*_B$ if and only if $x_A + x_B > 1$, that is, $A$ is the store with the locational advantage.

In the second case, (13) does not hold so that one store serves the whole market. Assume again $x_A + x_B > 1$. Since (13) is given by (14), monopolization of the market always occurs when $x_A + x_B \geq 4$, that is, at least one store is far away from all consumers. It then follows from (9) that all consumers patronize the closer store (store $A$). However, market monopolization may also be the equilibrium outcome when $x_A + x_B \leq 2$, that is, under horizontal differentiation. As is well known, this cannot happen in the absence of consumption externalities since no firm can capture the entire market in equilibrium. This shows that *weak conformity effects may generate a market outcome in a horizontally differentiated industry, which would otherwise occur only in an industry with vertically differentiated products.* Stated differently, weak conformity is sufficient to destroy the ability for a horizontally differentiated firm to stay in business whatever the behavior of its competitor. This is because the network effect makes the ‘quality’ of a product

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3The intuition is similar to the one given for vanity: consumers’ conformity renders the demand addressed to each firm more elastic, thus increasing the incentives to lower prices.
endogenous through its clientele’s size. This effect benefits more the firm with locational advantage because it attracts more consumers.

Since $x_A + x_B > 1$ and (13) does not hold, it follows from (9) that it is firm $A$ that captures the whole market so that the equilibrium prices are

$$p_A^* = an - \beta n^2 - t(x_B - x_A)(2 - x_A - x_B)$$

$$p_B^* = 0$$

In words, firm $A$ charges the highest price such that no consumer wants to buy from $B$, given the partition $C_A$.

For completeness, we now describe what happens when $x_A + x_B < 1$ and $t(x_B - x_A) > an - \beta n^2 > t(x_B - x_A)(2 + x_A + x_B)/3$. Expression (10) can be used to show that all consumers patronize store $B$ and that the equilibrium prices are given by

$$p_A^* = 0$$

$$p_B^* = an - \beta n^2 - t(x_B - x_A)(x_B + x_A)$$

Note also that this result has a contestable market flavor in that the potential competition exercised by firm $B$ (resp. $A$) leads firm $A$ (resp. $B$) to charge a price much lower than the monopoly price.

Since the conditions identified above are mutually exclusive, for any location pair such that (6) holds, there exists a unique subgame perfect Nash equilibrium in the price-network game. We may then summarize these findings as follows:

**Proposition 1.** Vanity yields higher market prices while weak conformity generates lower prices. Furthermore, under weak conformity and horizontal differentiation, the firm with the locational advantage may capture the whole market.

We now proceed by studying how the market splitting changes with the population size. To this end, assume that both firms are active so that (13) is verified. Evaluating the marginal consumer (5) at the price equilibrium (11) and (12), we have

$$\hat{x}^* = \frac{t(x_B - x_A)(2 + x_A + x_B) - 3(an - \beta n^2)}{6[t(x_B - x_A) - (an - \beta n^2)]}$$

When the two stores are symmetrically located ($x_A + x_B = 1$), $\hat{x}^* = 1/2$ and is therefore independent of $n$ regardless of the sign of $\alpha$. This shows that symmetric models prevent the emergence of the snowball effects described below. On the contrary, $\hat{x}^* > 1/2$ if and only if $x_A + x_B > 1$, i.e., when firm $A$ is the store with the locational advantage.

Furthermore, when $\alpha > 0$, differentiating (17) with respect to $n$ shows that $\hat{x}^*$
moves rightward if and only if $x_A + x_B > 1$ since $n \leq \alpha/2\beta$. In other words, the market share of the store with the locational advantage increases with the population size. However, the growth of firm A’s market share slows down as $n$ rises as long as $\beta > 0$.

As $n$ keeps rising, three cases may arise, as depicted in Fig. 1 in which the externality $E(n)$ is represented for three parameter configurations such that $\alpha/2\beta$ is constant. In the first one, (14) holds for all $n \leq \alpha/2\beta$, that is

$$E(\alpha/2\beta) = \frac{\alpha^2}{4\beta} < \frac{t}{3}(x_B - x_A)(4 - x_A - x_B)$$

which implies that (6) is satisfied for all admissible values of $n$. The market equilibrium involves two active firms. As $n$ rises, firm A increases its share but never monopolizes the market. This is because products are very differentiated and/or because the externality is strongly concave, thus showing that network effects become marginally weak (see the bottom parabola in Fig. 1).

In the second case represented by the intermediate parabola in Fig. 1, (14) is violated before $n$ reaches the threshold $\alpha/2\beta$ whereas (6) holds for all admissible $n$, that is

$$\frac{t}{3}(x_B - x_A)(4 - x_A - x_B) < \frac{\alpha^2}{4\beta} < t(x_B - x_A)$$

Since the externality function is not too concave, there is first market sharing but a growing population magnifies the locational advantage of firm A to the point where this firm eventually captures the entire market. The price equilibrium is first given by (11) and (12) and then by (15) and (16).

Finally, when

$$t(x_B - x_A) < \frac{\alpha^2}{4\beta}$$

Fig. 1. The externality function.
we observe the sequence described in the second case. However, (6) is violated before the threshold $\alpha / 2\beta$ is reached (see the top parabola of Fig. 1). The corresponding market solution is then as described in the next section.

Note that, under vanity ($\alpha < 0$), it is readily verified that the market share of the disadvantaged firm rises with the population size but never exceeds $1/2$. Hence, we have shown:

**Proposition 2.** Under weak conformity, the market share of the firm with the locational advantage expands as the population grows. This firm may even secure the total market when the concavity parameter of the externality function is small. On the contrary, under vanity, the market share of the firm with the locational advantage shrinks when the population increases and the market outcome approaches equal sharing.

We now consider the impact of a variation of $n$ on equilibrium profits. Consider, first, the case of vanity. Since both prices and demands increase with $n$,

$$
\frac{d \Pi_i^*}{dn} > 0 \quad i = A, B
$$

which means that an increase in population is always profitable to each firm when consumer behavior is characterized by vanity (assuming that all consumers keep buying). Assume now that there is weak conformity. The results are ambiguous because the equilibrium prices fall with $n$. For simplicity, we restrict ourselves to the case of symmetric locations. Demands increase linearly with $n$ whereas prices decrease so that the impact on profits is a priori unspecified. Since $E(n)$ is increasing for $n \leq \alpha / 2\beta$, it is readily verified that

$$
\frac{d \Pi_i^*}{dn} > 0 \quad \text{for} \quad 2\alpha n - 3\beta n^2 < t(x_B - x_A) \quad i = A, B
$$

while

$$
\frac{d \Pi_i^*}{dn} < 0 \quad \text{for} \quad t(x_B - x_A) < 2\alpha n - 3\beta n^2 \quad i = A, B
$$

Hence, under weak conformity, the demand effect and the price effect just balance at the two positive roots (if they exist) of the equation $-3\beta n^2 + 2\alpha n - t(x_B - x_A) = 0$. When $\alpha^2 / 3\beta < t(x_B - x_A)$, the roots are not real and increasing the population size benefits both firms as long as (6) holds.

When $\alpha^2 / 3\beta > t(x_B - x_A)$, the two roots are positive. The small one is always less than $\alpha / 2\beta$, however, the large one is admissible if and only if $\alpha^2 / 4\beta < t(x_B - x_A)$, which is equivalent to (6) when $n$ takes its highest admissible value.

---

*This is obvious for firm $B$. In order to show that $n_i$ also increases with $n$, we use the fact that $x_A + x_B < 4$, which must hold for the two firms to be active.*
Thus, as soon as \( n \) exceeds the first root, both firms are hurt by a population increase, but a rise in population is again profitable to each firm when \( n \) exceeds the second root. Clearly, the relationship between profits and the population size is not monotone, even when \( \beta = 0 \).

### 3.2. Robustness with respect to the externality function

In order to gain more insights on the nature of the assumptions that drive the results above, we consider the general case in which \( E(n) \) where \( E \) is strictly increasing and concave \((E' > 0, E'' \leq 0)\). We assume that there is a subgame perfect Nash equilibrium which is unique and interior, a situation that corresponds to the case considered above. For any given price pair \((p_A, p_B)\), the consumer equilibrium \(0 < \hat{x} < 1\) is therefore a solution to

\[
G(p_A, p_B, \hat{x}) = E(\hat{x}n) - E[(1 - \hat{x})n] - 2\hat{x}(x_B - x_A) + t(x_B^2 - x_A^2) + p_B - p_A = 0
\]

which defines implicitly the function \( \hat{x}(p_A, p_B) \). From the first order conditions for profit maximization, we obtain

\[
p_A = -\frac{\hat{x}}{d\hat{x}/dp_A} \quad \text{and} \quad p_B = \frac{1 - \hat{x}}{d\hat{x}/dp_B}
\]

From (18), it follows that

\[
\frac{d\hat{x}}{dp_i} = -\frac{\partial G/\partial p_i}{\partial G/\partial \hat{x}} \quad i = A, B
\]

Hence, (19) and (20) imply that

\[
p_B - p_A = (2\hat{x} - 1)\frac{\partial G}{\partial \hat{x}} = (2\hat{x} - 1)[nE'(\hat{x}n) + nE'(1 - \hat{x})n - 2t(x_B - x_A)]
\]

The marginal consumer at the equilibrium prices, \( \hat{x}^* \), is thus implicitly defined by

\[
H(\hat{x}^*) = E(\hat{x}^*n) - E[(1 - \hat{x}^*)n] - 2\hat{x}^*(x_B - x_A) + t(x_B^2 - x_A^2) + (2\hat{x}^* - 1)[nE'(\hat{x}^*n) + nE'(1 - \hat{x}^*)n - 2t(x_B - x_A)] = 0
\]

Clearly, we have

\[\text{We are indebted to an anonymous referee for suggesting the analysis performed in this subsection.}\]
First, we study the sign of the numerator of (21). That is,
\[
\frac{\partial H}{\partial n} = \frac{\partial H}{\partial \dot{x}^*} \dot{x}^* E'(\dot{x}^* n) - (1 - \dot{x}^*) E'((1 - \dot{x}^*) n)
\]
\[+ (2\dot{x}^* - 1)[\dot{x}^* n E''(\dot{x}^* n) + E'(\dot{x}^* n) + (1 - \dot{x}^*) n E''((1 - \dot{x}^*) n)]
\]
\[+ E'((1 - \dot{x}^*) n) \]  
(22)
Assuming that A is the large firm ($\dot{x}^* > 1/2$), it is readily verified that $\partial H/\partial n > 0$ if $yE'(yn)$ is strictly increasing in $y$. This is equivalent to assuming that the absolute value of the elasticity of the marginal externality ($E'(yn)$) with respect to $y$ is smaller than one. Alternately, this amounts to saying that the concavity of the externality function measured by the relative risk aversion coefficient is smaller than one.

However, this condition on the concavity of the externality function may be too restrictive. For example, it does not hold in our quadratic case, although we have $\partial H/\partial n > 0$ for all admissible $n$. The inspection of (22) shows that a necessary and sufficient condition for $\partial H/\partial n > 0$ to be satisfied when $\dot{x}^* > 1/2$ is that $xE'(xn) - (1 - x)E'((1 - x)n)$ be strictly increasing in $x$. Indeed, since the bracketed term in (22) is the derivative of the first term with respect to $\dot{x}^*$ and since the first term equals zero at $\dot{x}^* = 1/2$, (22) is positive for $\dot{x}^* > 1/2$ if and only if the first term is strictly increasing in $\dot{x}^*$. This latter condition holds for all admissible $n$ in the quadratic case.

Consider now the sign of the denominator of (21). That is,
\[
\frac{\partial H}{\partial \dot{x}^*} = 3[nE'(\dot{x}^* n) + nE'((1 - \dot{x}^*) n) - 2t(x_B - x_A)]
\]
\[+ (2\dot{x}^* - 1)n^2 [E''(\dot{x}^* n) - E''((1 - \dot{x}^*) n)]
\]
For $\dot{x}^* > 1/2$, this expression is negative when the following two conditions hold:
\[2t(x_B - x_A) > nE'(\dot{x}^* n) + nE'((1 - \dot{x}^*) n) \]  
(23)
and
\[E'' \leq 0 \]  
(24)
The first inequality means that the differentiation effect, expressed by $2t(x_B - x_A)$, dominates the network effect, $nE'(\dot{x}^* n) + nE'((1 - \dot{x}^*) n)$. Since the derivative of the latter term with respect to $\dot{x}^*$ is
\[n[E''(\dot{x}^* n) - E''((1 - \dot{x}^*) n)] \leq 0 \]
when $E'' \leq 0$. Accordingly, (23) may be replaced by
\[t(x_B - x_A) > nE'(n/2) \]  
(25)
which is equivalent to the condition (6) that corresponds to the case of weak conformity or vanity. In consequence, the denominator of (21) is negative if $E'' > 0$ and $t(x_B - x_A) > nE'(n/2)$.

To sum-up: the share of the large firm ($\hat{x}$) increases in population size when (i) the absolute value of the elasticity of the marginal externality is smaller than one and (ii) $E'' > 0$ and $t(x_B - x_A) > nE'(n/2)$.

As shown by the foregoing developments, $\hat{x}$ also increases in population size under the following alternative conditions: (i) $xE'(xn) - (1 - x)E'((1 - x)n)$ is strictly decreasing in $x$ and (ii) $E'' = 0$ and $t(x_B - x_A) < nE'(n/2)$. However, the latter inequality is likely to mean that one is the case of multiple consumer equilibria as discussed in the section below.

Yet, it should be clear that the concavity of the externality function is not a sufficient condition for the share of the large firm to grow with the population size. For example, this share decreases when $xE'(xn) - (1 - x)E'((1 - x)n)$ is strictly decreasing in $x$ and (ii) $E'' = 0$ and $t(x_B - x_A) > nE'(n/2)$.

4. Market equilibrium under strong conformity

In this section, we return to the case of a quadratic externality function as in Section 3.1, and assume that $a$, $b$, $n$, $t$ and the firms’ locations are such that (7) holds, thus implying that the bandwagon effect dominates the product differentiation effect. As in the foregoing section, we still assume that $n > a/2b$. Here, the partition $C$ is still an equilibrium partition but $\hat{x}$ as given by (5) is now increasing (resp. decreasing) in $p_A$ (resp. $p_B$). As will be seen below, there exist other equilibrium consumer partitions so that demands are defined by correspondences. The branch corresponding to $\hat{x}$ is increasing in own price so that the associated profit is increasing too.

The existence of multiple consumer partition equilibria suggests that we may expect several equilibria in the full game. To determine these equilibria when $x_A + x_B \geq 1$, consider the price space depicted in Fig. 2. It follows from Section 3 that the price pairs leading to the consumer partition where firm $A$ gets the whole market, i.e., $C_A$, are given by (9). Let $d_1$ be the straight line obtained when the equality holds in (9) and let $d_2$ be the straight line associated with the equality in (10). Finally, define $I$ (resp. $II$) as the price domain strictly above (resp. strictly below) the line $d_1$ (resp. $d_2$), while the domain $III$ lies in between $d_1$ and $d_2$ and

---

1. Of course, condition (i) is always satisfied when the concavity of the externality function, as expressed by the relative risk aversion coefficient, is larger than one.
2. Recall that a subgame perfect Nash equilibrium is here a pair of prices and a mapping from the set of prices pairs into the set of equilibrium consumer partitions $\{C, C_A, C_B\}$.
3. The case $x_A + x_B < 1$ can be similarly tackled.
Fig. 2. Partition of the price space.

includes these lines. Within III, all three consumer partitions, $\hat{C}$, $C_A$ and $C_B$, are equilibria of the consumer subgame; in domain I, $C_B$ is the only equilibrium partition while, in domain II, $C_A$ is the only equilibrium.

We first eliminate the domains in which no price equilibrium in pure strategies may arise. In domain I, no equilibrium exists because firm A has a zero market share and will always find it profitable to capture a positive demand by decreasing its price up to the corresponding value on $d_1$. The same holds, mutatis mutandis, for domain II.

Hence, we are left with domain III, which is such that any pair of prices belonging to this domain induces a subgame that has three equilibria, that is $\hat{C}$, $C_A$ and $C_B$. However, not all these price pairs are part of the equilibria of the full game. In order to determine these equilibria, we partition III into (at most) four domains as represented in Fig. 3. Since $x_A + x_B \geq 1$, it could be that domains $A$ and $B$ are empty. However, in order to be general, we consider the case where the four domains $A, B, C$, and $D$ are nonempty.

Fig. 3. Partition of domain III.
In domain A, no firm can be sure to capture the whole market by undercutting its rival because prices $p_A$ and $p_B$ are too low to allow a deviation into domain I (resp. II) for firm B (resp. firm A). Accordingly, \textit{any price pair belonging to A together with any of the three equilibrium consumer partitions $(\bar{C}, C_A, C_B)$ is an equilibrium outcome.} Indeed, take the strategies of consumers to be such that: $C = C_A$ for all $(p_A, p_B)$ in domain III with $p_B \neq p_B^*$ and $C = C_B$ for all $(p_A, p_B^*)$ in domain III with $p_A \neq p_A^*$. Then, none of the firms has an incentive to unilaterally deviate, whatever the consumer partition associated with $(p_A^*, p_B^*)$.

Consider now domain D. There is no equilibrium of the game involving the partition $C_A$ or $C_B$ because the firm with no consumers can always reduce its price and obtain a positive demand at a positive price. More precisely, for any pair of prices $(p_1, p_2)$ in domain D and partition $C_A$ (resp. $C_B$), firm B (resp. firm A) can always decrease its price and reach domain I (resp. domain II), which constitutes a profitable deviation.

It remains to consider a pair of prices $(p_A, p_B)$ in domain D together with partition $C$. Take the strategies of consumers to be such that: $C = C_B$ for all $(p_A, p_B)$ in domain III with $p_A \neq p_A^*$, and $C = C_A$ for all $(p_A, p_B^*)$ in domain III with $p_B \neq p_B^*$. Clearly a unilateral increase in price can never be profitable, given these consumers strategies. On the other hand, a decrease in price of, say, firm A such that $(p_A, p_B^*)$ belongs to domain III is also unprofitable given the consumers strategies. The only candidate to a profitable deviation for firm A is thus a decrease in price such that the new price pair belongs to domain II since in this case firm A may be able to offset the decrease in price by the increase in demand (recall that in domain II, $D_A = n$). Thus, for a pair $(p_A, p_B)$ in domain D together with partition $\bar{C}$ to be an equilibrium outcome it must be that a decrease in $p_A$ proves to be non-profitable. This condition writes as:

$$II_A = p_A \hat{x}(p_A, p_B) \equiv p_B - [an - \beta n^2 - t(x_B - x_A)(x_B + x_A)]$$

(26)

where the RHS of this inequality is the supremum on the profit firm A can earn in domain II. In the same spirit, we have to ensure that a decrease in the price of firm B leading to domain I is not profitable either. This writes as:

$$II_B = p_B[1 - \hat{x}(p_A, p_B) \equiv p_A - [an - \beta n^2 - t(x_B - x_A)(2 - x_A - x_B)]$$

(27)

where the RHS is the supremum on the profit firm B can earn in domain I. Consequently, $(p_A, p_B) \in D$ and $\bar{C}$ is an equilibrium outcome if and only if (26) and (27) hold.

Finally, we come to domain B (resp. C). The partition $C_A$ (resp. $C_B$) can never be part of the equilibrium because firm B (resp. A) can always capture a positive market share by a sufficient price drop. On the other hand, partitions $C_B$ and $\bar{C}$ (resp. $C_A$ and $\bar{C}$) can belong to an equilibrium of the whole game.

Consider a price pair $(p_A, p_B)$ in domain B together with partition $C_B$, and choose the consumers’ strategies such that $C = C_B$ for all $(p_A, p_B)$ in domain III.
and $C = C_A$ for all $(p_A, p_B)$ in domain III. Given such strategies there is no profitable deviation for any of the firms. Thus any pair of prices in domain $B$ together with partition $C_B$ is an equilibrium. By the same kind of reasoning, so is any pair of prices in domain $C$ and partition $C_A$. Consider, finally, a pair of prices $(p_A, p_B)$ in domain $B$ together with partition $C$. In the same spirit as the analysis developed for domain $D$, consumers strategies can be constructed such that the only candidate to a possible profitable deviation is a decrease in firm $B$’s price that leads the price pair into domain $I$. For such a deviation not to be profitable, condition (27) has to be met. Similarly, a pair of prices $(p_A, p_B)$ in domain $C$ together with partition $C$ is an equilibrium outcome if and only if condition (26) is met.

To sum-up, we have shown:

**Proposition 3.** For any location pair such that (7) holds, there exist multiple subgame perfect Nash equilibria in the price-network game.

Let us denote by $\mathcal{E}$ the following set of equilibria:

(i) $0 \leq p_A^* = \alpha n - \beta n^2 - t(x_B - x_A)(2 - x_A - x_B), \quad p_B^* = 0$ and the partition $C^* = C_A$;

(ii) $p_A^* = 0, 0 \leq p_B^* = \alpha n - \beta n^2 - t(x_B - x_A)(x_B + x_A)$ and $C^* = C_B$; and

(iii) $p_A^* = p_B^* = 0$ together with $C^* = C, C_A$ or $C_B$.

Clearly, the last equilibrium corresponds to the Bertrand equilibrium in which the consumers either split between the two stores or patronize any single store. Since there are three equilibrium consumer partitions, no firm can take advantage over its competitor so that the only equilibrium prices are $p_A^* = p_B^* = 0$. In the other equilibria, one store serves the whole market so that the corresponding firm is able to charge a positive price which does not exceed the limit price identified in Section 3.

However, as seen above, there exist other equilibria. For example, in the case of symmetric locations ($x_A + x_B = 1$) and $\beta = 0$, the shaded area represented in Fig. 4 describes the set of prices that can be associated with partition $C$ and form an equilibrium outcome. In particular, when $p_j^* = \alpha n - t(x_B - x_A)$, firm $j$’s best reply is given by $p_j^* = 4[\alpha(n - t(x_B - x_A))/3$ for $i, j = A, B$ and $i \neq j$, as shown by (26) and (27) in which $x_A + x_B = 1$ and $x = 1/2$. The remaining equilibria outcomes are: all prices in domain $A$ together with either partition $C_A$ or $C_B$, all prices in domain $B$ together with partition $C_B$ and all prices in domain $C$ together with partition $C_A$.

The foregoing argument together with (26) and (27) implies that the equilibrium prices are bounded above. This can be seen in Fig. 3 for the case of symmetric locations.
Our point is that the equilibria that do not belong to $\mathcal{E}$ do not satisfy the following axiom, which we use as a refinement.\footnote{An alternative axiom which leads to the same set of outcomes (the same equilibrium payoffs and the same equilibrium strategies for the active firms) is as follows: the probability that a consumer patronizes a firm does not increase with the price of this firm. In other words this alternative axiom means that each firm conjectures a non-increasing demand function for its product. We are indebted to J.F. Mertens for having suggested these axioms.}

**Axiom of Invariance.** If the price pair $(p_A, p_B)$ induces an equilibrium consumer partition $(N^*_A, N^*_B)$, then $(p_A + \Delta, p_B + \Delta)$ induces the same partition $(N^*_A, N^*_B)$ regardless of the value of $\Delta$ such that (i) $p_A + \Delta$ and $p_B + \Delta$ are nonnegative and (ii) all consumers want to buy.

The justification for this axiom is as follows. Let the two firms charge $p_A$ and $p_B$. Then, the subgame induced by these prices is identical to the subgame induced by the price pair $p_A + \Delta$ and $p_B + \Delta$ since each consumer’s payoff function is the same in both subgames (up to the constant $\Delta$). The axiom then requires that consumers use the same strategy in all identical subgames.

We prove in Appendix A the following two statements: (i) any equilibrium which does not belong to $\mathcal{E}$ violates the axiom of invariance while (ii) any equilibrium in $\mathcal{E}$ satisfies this axiom. As a consequence, the following result holds.

**Proposition 4.** The set of subgame perfect equilibrium outcomes under the invariance axiom is given by $\mathcal{E}$. For the firms, the equilibria are such that at least one firm sets a price equal to zero while the other firm charges a price that does not exceed its limit price. When a firm charges a positive equilibrium price, it supplies the whole market.
Accordingly, even when the restriction given by the invariance axiom is imposed, there is multiplicity of equilibria. However all the equilibrium prices can be viewed as fairly enough. Indeed, the highest price a firm can charge has the nature of a limit price and is identical to the equilibrium price (15): it corresponds to the highest price compatible with keeping the whole market, the other firm setting its lowest possible price. At the other extreme, the Bertrand outcome is another equilibrium where either one firm captures the whole market or both firms share the market.

More surprising, perhaps, is the result that, in equilibrium, the firm, say A, supplying the entire market may set any price \( p_A \) below its limit price, again when its rival sets a zero price. This is because any positive deviation from \( p_A \) can be associated with the partition \( C_B \). Such a partition is crucial for sustaining \( p_A \) as an equilibrium price. By contrast, an equilibrium in which \( p_A \) equals firm A’s limit price seems to be more robust. This is because any deviation from this price is unprofitable for A regardless of the consumer partition associated with the corresponding subgame.\(^{11}\)

When strongly positive network effects prevail but not by much, the set \( \mathcal{E} \) reduces to the prices sustaining the firm with the locational advantage as the single active firm. This is in accord with what we have seen in the case of weak conformity when the network effects are ‘strong enough’. As the intensity of the externality keeps rising, the set \( \mathcal{E} \) expands and encompasses market configurations where the disadvantaged firm gets the entire market. This is due to the fact that the network effects are now so strong that the locational advantage of a firm is no longer sufficient to exclude the other firm at all equilibria. However the locational advantage does not completely vanish. It manifests itself in a higher limit price for the corresponding firm.

5. Concluding remarks

We have shown that the introduction of network effects into the spatial model of product differentiation may have some significant impact on the market outcome. The main effect is that vanity relaxes price competition while weak conformity intensifies competition. In the latter case, fiercer competition may even lead to the exit of a firm (in contrast to what is observed in standard models of horizontal differentiation).

On the other hand, the nature of results changes in the case of strong conformity. First, there is multiplicity of price equilibria generated by the

\(^{11}\)One might therefore wish for a more selective refinement than the axiom of invariance, but it is not clear to us what it should be.
existence of several equilibrium consumer partitions. Second, under the invariance axiom, almost all equilibria involve a single network.

At this point, it would seem natural to investigate the location choice of firms. First, we would like to stress the fact that, in some cases, locations are given by extraneous considerations, so that the study of price competition is the relevant issue. Second, when firms are free to choose their locations, we may argue as follows. In the case of vanity, it is straightforward to show that firms locate on opposite sides of the market, as in the Hotelling model with quadratic transportation costs without externality. In the case of conformity, the analysis is more complex. Indeed, when firms are far enough, it is likely that we have weak conformity. However, when stores are sufficiently close, (7) must hold and we have strong conformity. In this region, at least one firm makes zero profits. If the set of location choices available to firms is large enough, this firm may always choose a location such that (13) holds, thus ensuring a positive profit to itself. Therefore, if a location equilibrium in pure strategies exists, it must be that firms are sufficiently far apart to fall in the weak conformity case. This suggests that differentiation is stronger, the higher the intensity of the externality, at least when the domain of locational opportunities is sufficiently large. In practice, the outcome will depend on the extent of the set of location choices, on the population size through the intensity of the externality as well as on the timing of the location game.

In the foregoing analysis, the conformity case has been restricted to the domain \( n < \frac{\alpha}{2\beta} \). When the population size starts increasing from low values, we have seen that the market share of the firm with the locational advantage rises and may reach one for some critical value of the population size. When \( n \) increases further, additional equilibria may also emerge in that the firm with the locational advantage may be out of business. For \( n > \frac{\alpha}{2\beta} \), the externality function starts decreasing with the population size because of the emergence of some congestion in the consumption of the product. For a sufficiently large increase in \( n \), one returns to the case covered by (6) in which the market is shared between the two firms. In addition, when \( n > \frac{\alpha}{\beta} \) the nature of the market outcome is the same as in the vanity case since \( E(n) \) is negative in (13).

As a final comment, we would like to discuss briefly the case of more than two firms. For simplicity, assume that firms are located equidistantly along a circle, as in Salop (1979). It is readily verified that vanity leads to higher profits once all consumers in the population in question buy the product. This induces a larger variety of brands, as often observed in luxury good industries where snob effects are likely to be present. As expected, the situation is more complex under conformity. This is because a price change by a firm affects directly the networks of its adjacent competitors, which in turn affects the networks of the subsequent firms, and so on. In other words, the demand of a firm is a function of the price system and competition is no longer localized. Nevertheless, Friedman and Grilo (1999) have shown that the symmetric equilibrium price is a monotone function of
the number of firms. However, depending on $\alpha$ and $\beta$, this function may be either decreasing (as usual) or increasing. Likewise, the number of firms at the free entry equilibrium is affected in a determined but not simple way by changes in $\alpha$ and $\beta$. In the special case where $\beta = 0$, these authors show that more conformity ($\alpha$ rises) leads to a smaller number of firms at the free entry equilibrium while the corresponding equilibrium price also decreases. This suggests that the conformity effect dominates the competition effect associated with the number of firms.

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Appendix 1

(i) Consider any price pair $e = (p_A, p_B)$ in the domain $III$ represented in Fig. 1 such that both prices are strictly positive. We show below that $e$ together with $\tilde{C}$, $C_A$ or $C_B$ fails to be an equilibrium outcome of the full game under the additional restriction on consumers’ strategies imposed by the invariance axiom. Let $e$ and $\tilde{C}$ be the first candidate. For this to be an equilibrium outcome, any unilateral increase in $p_A$ must be associated with $C_B$ because otherwise both price and demand of firm $A$ would increase. Then the invariance axiom implies that any price pair in the region above the straight-line with slope 1 and passing through $e$, denoted $d_e$, must be associated with the partition $C_B$. But then, firm $B$ can increase its profit by slightly decreasing its price (observe that such a move is possible since $p_B > 0$). Hence $e$ and $\tilde{C}$ cannot be an equilibrium outcome under the invariance axiom. We now come to $e$ and $C_A$. For this to be an equilibrium outcome, any unilateral deviation in $p_A$ must be associated with $C_B$. The invariance axiom implies that any price pair belonging to a neighborhood of the line $d_e$ must also be associated with $C_A$. In this case, firm $A$ can always increase its profits by slightly increasing its price so that $e$ and $C_A$ is not an equilibrium outcome. A similar argument applies to $e$ and $C_B$. Note that the possibility of a price decrease is crucial for the argument above. This observation helps us to understand that $\tilde{E}$ is the set of equilibria under the invariance axiom. (ii) Consider now a price pair $e' = (p_A, p_B)$ in $\tilde{E}$ such that $p_A = 0$ and $p_B > 0$. We show that $e'$ with $\tilde{C}$ or $C_A$ cannot be an equilibrium outcome. Let us start with $e'$ and $\tilde{C}$. It is then readily verified that the first part of the argument developed in (i) still applies. Similarly, the second part shows that $e'$ and $C_A$ is not an equilibrium outcome either. It

remains to study \( e' \) and \( C_B \). Denote by \( d_e \), the line with slope 1 passing through \( e' \). Clearly, \( e' \) and \( C_B \) can be sustained as an equilibrium outcome by associating \( C_B \) with all prices pairs on and above \( d_e \) and \( C_A \) with all price pairs below \( d_e \). Such assignment of consumer partitions is compatible with the invariance axiom. Hence we have shown that \( e' \) and \( C_B \) is an equilibrium outcome. A similar argument applies to any price pair \( e' = (p_A, p_B) \) in \( e' \) such that \( p_A > 0 \) and \( p_B = 0 \) to show that \( e' \) and \( C_A \) is an equilibrium outcome. Finally, we deal with the case where \( p_A = p_B = 0 \). It is easy to see that \( e' = (0,0) \) with either \( C_A \) or \( C_B \) is an equilibrium outcome respecting the invariance axiom when any price pair above (below) the diagonal is associated with \( C_B \) (\( C_A \)) while the partition on the diagonal is the same as the one associated with \( e' \).

References


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