§ 6.4.2 Ferromagnetism

The magnetic domains under applied magnetic fields.

\[ B = \mu_0 (H + M) \]

* Induced magnetization \( M \)

* Saturated applied fields \( H \)

\[ J = -m_1 \cdot B \] => calculate the stability

Problem 6.21

+ Paramagnetism

For statistical analysis, it may be shown that the probability of finding any one atom having energy

1) due to orientation in the field is proportional to the maxwell distribution at \( k_B T \)

2) paramagnetic material in nature.

\[ J = -m_1 \cdot B \] potential energy

\[ \text{probability } \sim e^{-\frac{J}{k_B T}} \]

\[ \text{where } \theta \text{ is the angle between } m_1 \text{ and } B, \text{ then } J = -m_1 B \cos \theta \]

The probability is \[ e^{-\frac{m_1 B \cos \theta}{k_B T}} \]

From this, Curie can calculate the average value of \( m_1 B \).

\[ \langle m_1 B \rangle = \frac{1}{3} \frac{m_1 B}{k_B T} \]

The magnetization of a paramagnetic with \( N \) atoms per unit volume is

\[ \vec{M} = 4N \frac{m_1 B}{3k_B T} \] \( \Rightarrow \) \[ \vec{M} = \chi_m \bar{H} \]

\[ \chi_m = \frac{4N m_1}{3k_B T} \] Curie Law

\( \chi \propto \frac{1}{T} \) paramagnetism

\( \chi \propto \frac{1}{T^2} \) antiferromagnetism

\( \chi \propto \frac{1}{T} \) ferromagnetism

Susceptibility \( \chi \)

Curie Weiss Law

\( \chi \propto \frac{1}{T} \) paramagnetism

\( \chi \propto \frac{1}{T^2} \) antiferromagnetism

\( \chi \propto \frac{1}{T} \) ferromagnetism
When electric current in material is proportional to the voltage,

\[ I \propto V \text{ called as Ohm's law} \]

\[ f = \text{force per unit charge} \quad I R = V \quad "\text{macroscopic view}" \]

* Into microscopic view

The current density defined as electric current per unit area

\[ \mathbf{J} = \frac{I}{A} \]

can be expressed in terms of the electron density.

\[ \mathbf{J} \propto I \propto V \propto f \]

static, steady, dynamics.

* Microscopic view

* The number of atoms per unit volume

\[ N = N_{A} \left( \frac{\text{atoms}}{\text{mole}} \right) \frac{V}{A} \left( \frac{\text{m}^{3}}{\text{atom}} \right) = \rho \left( \frac{\text{atom}}{\text{m}^{3}} \right) \]

* Ohm's law

\[ I = \frac{V}{R} = \text{current} \quad \mathbf{J} = \mathbf{E} \quad \mathbf{E} = \mathbf{V} \]

Then the current density is

\[ \mathbf{J} = \frac{V}{R} \frac{A}{A} = \frac{V}{R} \frac{A}{A} = \mathbf{E} \]

\[ \mathbf{J} = \mathbf{E} \cdot \mathbf{A} \]

* The current density is proportional to the \( E \times \frac{1}{\rho} \).

Then define a conductivity \( \sigma = \frac{1}{\rho} \) conductivity

\[ \rho \rightarrow 0, \quad \sigma \rightarrow \infty \implies \text{Superconductivity} \]

§ 2.1.1 Ohm's Law

* If force on charge will fast/slow velocity

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So the current density \( J \) is \( \propto -E \).

* From the Lorentz force \( F = q(\vec{E} + \vec{v} \times \vec{B}) \)

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Force per unit charge \( f = \frac{F}{q} = (\frac{tan}{2} + \vec{E}) \)

* Then the current density \( J = \sigma \vec{E} = qf \)

\[ \text{(7.1)} \]

* If we consider the electromagnetic force of electron/magnetic force,

then \( J = \sigma (\vec{E} + \vec{v} \times \vec{B}) \).

\[ \text{(7.2)} \]

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\[ \begin{align*}
\text{steady} & \Rightarrow \text{static state} \\
\frac{\Delta Q}{\Delta t} = I \\
I = \frac{Q}{t}
\end{align*} \]

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Ex 7.2 Two long cylinders (radius a & b) are separated by
material of conductivity \( \sigma \). If they are
maintained at a potential difference \( V \), what
current flows from one to the other in a length \( L \).

* The Electric-field between a and b cylinder

\[ \vec{E} = \frac{\lambda}{2 \pi \varepsilon_0 s} \mathbf{\hat{s}} \]

\[ \text{where } \lambda = \text{charge per unit length on the linear cylinder} \]

\[ I = \int s \, J \, da \]

The total current \( I = \int s \, J \, da = \int s \, E \, da \),

\[ da = 2\pi s \]  

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The potential between the cylinder is

\[ V = -\int \vec{E} \cdot d\vec{s} = -\int \frac{\lambda}{2 \pi \varepsilon_0 s} d\vec{s} \]

\[ V = \frac{\lambda}{2 \pi \varepsilon_0 s} \ln \frac{b}{a} \Rightarrow \lambda = \frac{2 \pi \varepsilon_0 V}{\ln \frac{b}{a}} \]

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* If the current is \( I = \frac{\sigma \lambda}{L} \)

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* So the current \( I = \frac{2 \pi \varepsilon_0 V}{\ln \frac{b}{a}} \). Then the resistance is \( R = \frac{\sigma}{2 \pi \varepsilon_0} \).

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IF a free charge toward the end of the conductor and establish
an electric field \( \vec{E} \) given by \( \vec{E} = \vec{v} \times \vec{B} \).

The end of the conductor is \( V_{ba} = \int_a^b \vec{E} \cdot d\vec{s} = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{s} \)
Then the difference of voltage is dependent on $\vec{V} \times \vec{B}$.

The $\text{emf}$ can be changed as $V_{\text{emf}} = \frac{1}{2} \int \vec{B} \cdot (\vec{V} \times \vec{B}) \cdot d\ell$ (average method).

If $\vec{V}, \vec{B}, d\ell$ are constants, then $\varepsilon = VB \ell$, called emf.

§ 2.1.2.

$V$ * Voltage = Applied force $F$ on charges per charge $\Rightarrow f$

$I$ * Current density $\Rightarrow J = \sigma F$, $\sigma$ = conductivity $= \frac{1}{\rho}$

$\text{emf}$ * Electromotive Force $\varepsilon = V = \int \vec{E} \cdot d\ell \Rightarrow \text{Electricity}$

$F = q \vec{E} + q \vec{v} \times \vec{B} \Rightarrow$ introduce a steady state $\Rightarrow \text{Electrodynamics}$

* Now we consider a conductor of length $L$ moving with velocity $\vec{v}$ perpendicular to a uniform field $\vec{B}$.

$\vec{F} = q \vec{v} \times \vec{B}$

The free charge establishes an electric field $q \vec{E}$. Static $\rightarrow$ Steady State.

Then $q \vec{E} \rightarrow q \vec{v} \times \vec{B}$ the potential difference between the ends of the conductor is $V = \int \vec{E} \cdot d\ell \Rightarrow \text{Electricity, Electrodynamic}$.

We obtain $\varepsilon = \frac{1}{2} \int \vec{B} \cdot (\vec{V} \times \vec{B}) \cdot d\ell$ (average $\Rightarrow$ Average Voltage per charge.

§ 2.1.3. Motional $\text{emf}$

$\Rightarrow \varepsilon = \frac{\Delta V}{\Delta t} \Rightarrow \frac{\Delta A}{\Delta t} \vec{B} ?$
If the \(B\) is point into the page, connecting a resistance \(R\), moving with velocity \(v\), let us define the flux of area through regions 1 and 2 as \(\Phi = BA\).

1. Then we know \(\Phi = \int \mathbf{V} \cdot \mathbf{B} \, dl = \int \mathbf{F}_{\text{mag}} \cdot \mathbf{d}l\).

2. Due to the changing of area, we introduce a idea of magnetic flux \(\Phi = BA\).

Considering the moving \(\mathbf{V}\):

1. In region 2 the loop is moving out. Then the flux is decreasing. 通量减少: \(\frac{d\Phi}{dt} = B h \frac{dx}{dt} = B h v\).

2. In region 1, the loop is moving in. Then the flux is increasing. 通量增加: \(\frac{d\Phi}{dt} = + Bh v\).

3. So the emf generate in loop is minus the rate of charge of flux through the loop in region 2. 通量变化率: \(E = -\frac{d\Phi}{dt}\).

**Conclusion:** \(E = -\frac{d\Phi}{dt}\).

**Problem 7.12**

A long solenoid of radius \(a\) is driven by an alternating current. So that the field inside is \(B(t) = B_0 \cos (\omega t) \mathbf{z}\). A circuit \(\Phi = BA\) changing area \(\mathbf{A}\) changing \(B\)-field \(\mathbf{B}\) dynamic. loop of wire of radius \(\frac{a}{2}\):

1. \(\Phi = \frac{\pi a^2}{4} B_0 \omega\).

So the \(E = -\frac{d\Phi}{dt} = -\frac{d}{dt} \left[ \frac{\pi a^2}{4} B_0 \cos \omega t \right] = -\frac{\pi a^2}{4} B_0 \omega \sin \omega t\).

then \(I = \frac{E}{R} = \frac{\pi a^2}{4} B_0 \sin \omega t\).

2. Loop area \(\frac{\pi a^2}{4}\).