A comparison between Electratic & Magnetostatics. It should be compared between two static fields.

1. Both $\mathbf{B}$ and $\mathbf{E}$ are defined in terms of forces.
   \[ \mathbf{F}_E = q \mathbf{E}, \quad \mathbf{F}_B = q (\mathbf{v} \times \mathbf{B}) \]
   $\mathbf{E}$: polar vector, $\mathbf{v}$: axial vector

2. Where $\mathbf{E}$ is defined via $\mathbf{F} = q \mathbf{E}$ on a stationary charge and is consequently a polar vector.

3. $\mathbf{B}$ is defined via $\mathbf{F} = q (\mathbf{v} \times \mathbf{B})$. The cross-product of two polar vectors is an axial vector.

4. Gauss' law appear different for these two static fields.
   \[ \mathbf{V} \cdot \mathbf{E} = \rho, \quad \mathbf{V} \cdot \mathbf{B} = 0 \]
   \[ \int \mathbf{V} \cdot d\mathbf{r} = \int \rho d\mathbf{r} = Q/\varepsilon. \]

5. The B-field is divergence-less at all points & its solenoid. It is lines form closed loop.

6. $\mathbf{E}$ is curl-free ($\nabla \times \mathbf{E} = 0$) solenoid $\mathbf{E}$-field is easy to make.
* J.J. Thomson 1897.

The measurement of \( e/m \)

Using Lorentz force \( \mathbf{F} = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \)

1. If the sum of electron's force is zero then \( q \mathbf{E} = q (\mathbf{v} \times \mathbf{B}) \sim |E/B| = v \)

2. The axial in \( x \) and \( y \) is

\[
x = vt, \quad y = \frac{1}{2}at^2 = \frac{1}{2} \frac{E}{m} t^2 = \frac{1}{2} \frac{eE}{m} t^2
\]

\[
y = \frac{eE}{2mv_0} x^2
\]

\[
\frac{e}{m} \approx \frac{2yE}{B^2 x^2} \approx 1.7 \times 10^{-9} \text{ C/kg}
\]

Millikan \( e \sim 1.602 \times 10^{-19} \text{ C} = 9.1 \times 10^{-31} \text{ kg} \)

* Example 5.2 Cycloid motion p. 205

If \( \mathbf{B} \) is points in \( x \)-direction \( \Rightarrow \)

\[
\mathbf{E} = \mathbf{v} \times \mathbf{B} = q \left[ \begin{array}{c} \hat{x} \ \hat{y} \ \hat{z} \\
0 \ \ 0 \ \ 0
\end{array} \right] = qBz (\hat{y} \hat{y} - \hat{z} \hat{z})
\]

\[
\mathbf{E} = \mathbf{F} = qE \hat{z}
\]

\[
\mathbf{F} = \mathbf{m} \ddot{a} = m (\dot{y} \hat{y} + \dot{z} \hat{z}) = qBz \hat{y} + (qE - qB\dot{y}) \hat{z}
\]

So the equation of \( \mathbf{F} = \mathbf{m} \mathbf{a} \) can compare each other
\[ y = m\ddot{y} = qB\ddot{z} \quad \ddot{y} = \frac{qB}{m} \ddot{z} \]

Define \( \frac{qB}{m} = \omega \) then \( \ddot{y} = \omega \ddot{z} \)

\[ \ddot{z} = \omega (\frac{E}{B} - \ddot{y}) \]

0. If \( \ddot{y} = \omega \ddot{z} \quad \ddot{y} = \omega \ddot{z} \)

\[ \ddot{z} = \omega (\frac{E}{B} - \ddot{y}) \Rightarrow \ddot{y} = \omega^2 (\frac{E}{B} - \ddot{y}) \]

2. Let \( \ddot{y} = s \) then the equation can be rewritten as

\[ s = \omega^2 (\frac{E}{B} - s) \]

Get the solution of \( s \)

\[ s = A \cos \omega t + B \sin \omega t + \frac{E}{B} = \ddot{y} \]

\[ \therefore y(t) = \int \ddot{y}(t) \, dt = \frac{A}{\omega} \sin \omega t + \frac{B}{\omega} \cos \omega t + \frac{E}{B} t + C \]

Insert to \( \ddot{z} = \omega (\frac{E}{B} - \ddot{y}) \)

\[ \ddot{z} = \omega (\frac{E}{B} - A \cos \omega t - B \sin \omega t - \frac{E}{B}) \]

\[ \ddot{z} = -\omega (A \cos \omega t + B \sin \omega t) \]

So the \( \ddot{z} = -A \sin \omega t + B \cos \omega t + C' \)

\[ z = \frac{A}{\omega^2} \cos \omega t + \frac{B}{\omega} \sin \omega t + C'' \]

Check the initial condition:

\[ t = 0 \Rightarrow \dot{y}(0) = 0 \quad \dot{z}(0) = 0 \]

\[ y(t) = \frac{E}{NB} (\omega t - \sin \omega t) = R (\omega t - \sin \omega t) \]

\[ z(t) = \frac{E}{NB} (1 - \cos \omega t) = R (1 - \cos \omega t) \]

\[ R = \text{radius} \]
5.1.3 Current vs charge.

A. Electric current $\leftrightarrow$ charge at rest
\[ I \quad \downarrow \quad Q \]

Definition: Electric current is the flow of electric charge.

B. The magnitude of an electric charge at a point is defined as the time deviation of electric charge.

\[ I(t) = \dot{Q}(t) \] Formly this is written as
\[ I(t) = \frac{dQ}{dt} \iff Q(t) = \int_{t_0}^{t} I(t) \, dt + Q. \]

C. Current density

- It is a measure of the density of electrical current. It is defined as a vector whose magnitude is $I/A$.
- The total charges pass through the area $A$ in a time, then the $dQ = \varrho (\nabla \cdot \vec{V}) \vec{A} \cdot \hat{n}$

\[ n = \text{charge particle per unit volume}. \]

If $A \parallel \vec{V}$, $I = \frac{dQ}{dt} = n \varrho A \vec{V}$

\[ J = \frac{I}{A} = n \varrho \vec{V} \iff \vec{J} = \varrho \vec{V} \]
Current density \( J = \frac{e \nu}{2} \)  

5.1.3 Electric Current

1. Charge density \( \lambda = \frac{Q}{l} \)

2. Current \( I = \frac{Q}{t} \)

3. \( \frac{I}{\lambda} = \frac{Q}{\lambda l} = \nu \Rightarrow I = \lambda \nu \)

For Example of Ampere's experiment then non-uniform

\[ I = \lambda \nu = \lambda_+ \nu_+ + \lambda_- \nu_- \]

\[ F_{\text{mag}} = \int \vec{q} \vec{v} \times \vec{B} \text{ d}q \]

\[ \Rightarrow \int (\vec{v} \times \vec{B}) \text{ d}q = \int (\vec{v} \times \vec{B}) \cdot \text{d}l \]

\[ = \int (\lambda \vec{v}) \times \vec{B} \text{ d}l = \int (\vec{I} \times \vec{B}) \text{ d}l \]

* For example of closed path

\[ \text{for current is independent on position} \]

\[ \vec{F}_{\text{mag}} = I \int \vec{B} \times \text{d}l \]

* For \( \vec{B} \) independent on the position

\[ \text{if } \vec{B} \text{ is uniform} \]

\[ \vec{F}_{\text{mag}} = I \left[ \int \text{d}l \right] \times \vec{B} \quad \text{for closed path} \]

= 0
Example 5.4

(a) A current $I$ is uniformly distributed over a wire of circuit cross section which radius $a$.

Then we define the current density $J = I/A \Rightarrow I = \int J \, da \quad I = I/\pi a^2$

(b) Find the total current $I$ if $J = ks$

$I = \int ks \, da = \int_0^a ks \cdot 2\pi rs \, ds = 2\pi k \int s^2 \, ds = 2\pi k \frac{1}{3} a^3$

PS. If the current crossing a surface $S$ can be written as

$I = \int J \, da = \oint E \cdot d\ell = \int \vec{v} \cdot J \, d\ell$

flux $\Rightarrow$ current $\Rightarrow -\frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{S} = -\frac{Q}{\varepsilon}$

$\vec{v} \cdot \vec{E} = \frac{Q}{\varepsilon}$

$\Rightarrow \int \vec{v} \cdot E \, d\ell = \frac{Q}{E}$. The final form of continuity equation is described as $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$. 