Diving by \( \frac{dI}{dt} \), \( E = -M_{21} \frac{dI}{dt} \)

we could get the definition of \( M \)

\[
M = \frac{\mu_0}{4\pi} \int \int_S \frac{d\vec{s} \cdot d\vec{a}}{r}
\]

where \( d\vec{s} \) & \( d\vec{a} \) are two element of length

& \( r \) is the distance between them.

This is known as Newton's formula. \( M_{12} = \frac{\mu_0}{4\pi} \int \int_S \frac{d\vec{s} \cdot d\vec{a}}{r} \)

\( M_{12} = M_{21} \)

*Final Exam. 6/17 (E) 10:00 AM.*

\( \Phi = M.I \).

\[ \begin{align*}
\text{Flux} & \quad \text{mutual inductance} \\
\Rightarrow & \quad \text{Energy in magnetic field} \Rightarrow \text{Energy + Density of Energy}
\end{align*} \]

To obtain the energy of an electrostatic field, we calculate the work done by \( E \) in moving charge increment \( dq \).

Then the increment of \( dq \) can be described as \( dq = I dt \).

\[ W = \frac{1}{2} L I^2 (t) \]

1. If \( dW = I d\Phi \) is equal to the change in magnetic energy.
2. If there are \( n \) turns circuit, then the work done against the induced emf is given by \( dW = \sum_{i=1}^{n} I_i d\Phi_i \)

Magnetic Energy in terms of field vectors.

For simplicity, we assume that each circuit consist of a single loop, then the flux \( \Phi \)

\[ \Phi_i = \int \int_A \vec{n} \cdot d\vec{a} = \int \int_A \vec{n} \cdot d\vec{a} = \oint_{\text{closed loop}} \vec{A} \cdot d\vec{l} \]

\( \Rightarrow \) input the magnetic energy.
For n turns, the function of $dW = \frac{1}{2} \mathbf{I} \cdot \mathbf{Z}$ can be replaced as $W = \frac{1}{2} \mathbf{I} \cdot \mathbf{Z}$. \(\Rightarrow\) Magnetic energy

\[
W = \frac{1}{2} \mathbf{I} \cdot \mathbf{Z} = \frac{1}{2} \int \mathbf{I} \cdot \mathbf{A} \cdot d\mathbf{l}
\]

* The second step: we can change $\mathbf{J} \cdot d\mathbf{l} = (\mathbf{J} \cdot d\mathbf{a}) \cdot d\mathbf{l} = \frac{1}{2} \oint \mathbf{A} \cdot d\mathbf{a}$

\[
\mathbf{J} \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{a} \Rightarrow \oint \mathbf{A} \cdot d\mathbf{a} = \frac{1}{2} \int \mathbf{J} \cdot d\mathbf{a}
\]

* Mathematical method of $\mathbf{B} = \mathbf{v} \times \mathbf{A}$, $\mathbf{v} \times \mathbf{B} = \mathbf{H} \times \mathbf{J}$.

Recalculated $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}) = \mathbf{v} \cdot (\mathbf{v} \times \mathbf{A}) - \mathbf{A} \cdot (\mathbf{v} \times \mathbf{B})$

\[
\Rightarrow \mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}) = \mathbf{B} \cdot \mathbf{B} - \mathbf{A} \cdot \mathbf{H} \times \mathbf{J}
\]

\[
\Rightarrow \mathbf{B} \cdot \mathbf{B} - \mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}) = \mathbf{A} \cdot \mathbf{H} \times \mathbf{J}
\]

\[\text{V.I.P.}\]

$W = \frac{1}{2} \int \mathbf{J} \cdot d\mathbf{a}$, we obtain the expression of

\[
W = \frac{1}{2\mu_0} \int \mathbf{B} \cdot \mathbf{B} \, d\mathbf{v} - \frac{1}{2\mu_0} \int \mathbf{v} \cdot (\mathbf{v} \times \mathbf{B}) \, d\mathbf{v}
\]

\[
= \frac{1}{2\mu_0} \int \mathbf{B} \cdot \mathbf{B} \, d\mathbf{v} - \frac{1}{2\mu_0} \oint \mathbf{A} \times \mathbf{B} \cdot \mathbf{n} \, d\mathbf{a}
\]

Explain.

The integrations on the right are to be taken over the entire (volume) occupied by the current with the

Because $\mathbf{B}$ falls off at fast as $\frac{1}{r^2}$

\[
\mathbf{A} \rightarrow \frac{1}{r}
\]

Surface $\mathbf{d} \mathbf{a} \sim r^2$

Then the second term of surface integral vanishes.

\[
W = \frac{1}{2\mu_0} \int \mathbf{B} \cdot \mathbf{B} \, d\mathbf{v}, \text{ when } \mathbf{B} = \mu_0 \mathbf{H}
\]

\[
= \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} \, d\mathbf{v}, \text{ we may define the}
\]

* Energy density in magnetic field by $W = \frac{1}{2} \mathbf{H} \cdot \mathbf{B}$

\[
W = \frac{B^2}{2\mu_0}
\]
§ 7.13 Example. A long co-axial cubles carries current $I$.

The outer current $I$ →

inner current $I$ →

Find the magnetic energy stored in a section of length $l$.

* According to Ampere’s law, $B = \frac{\mu_0 I}{2\pi r}$.

* The magnetic energy per unit volume

$$ U = \frac{1}{2\mu_0} B^2 = \frac{\mu_0 I^2}{8\pi^2 l^2}.$$

* The unit volume of cylinder $U = \int_{\text{volume}} dV$, $dV = 2\pi r dp$.

So the magnetic energy is $U = \int_{l} dU = \int_{a}^{b} \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r dp dl$

$$ = \frac{\mu_0 I^2}{4\pi} \int_{a}^{b} \frac{1}{r} dr = \frac{\mu_0 I^2}{4\pi} \ln \frac{b}{a}, \text{ represented with } |. | .$$

$$ U = \frac{1}{2} |. I|^2 \quad |. \frac{\mu_0 I}{2\pi} \ln \frac{b}{a}|$$

HW: 06/10 return

P 7.18 P 7.24

P 7.20 P 7.30

§ 7.3 Maxwell Equation.

2.31. Electrodynamics equation. Before Maxwell

A. Gauss’s Law $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$\nabla \cdot B = 0$

Faraday’s Law $\nabla \times E = -\frac{\partial B}{\partial t}$

Ampere’s Law $\nabla \times B = \mu_0 j$

These equations represent the state of EM theory over a century.

The old eggs with the old rule that divergency of
curl is always zero

1. $\nabla \cdot (\nabla \times E) = \nabla \cdot (-\frac{\partial B}{\partial t}) = -\frac{\partial}{\partial t} (\nabla \cdot B) = 0$

2. $\nabla \cdot (\nabla \times B) = (\nabla \cdot j_0) \mu_0 = 0 ?$

Zero only at static steady state; Non-zero at dynamics state.

GEE-JUMP
Maxwell equation & Electromagnetic wave in vacuum.

1. Gauss's Law \( \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \)

2. Absence of magnetic monopole \( \nabla \cdot \mathbf{B} = 0 \)

3. Ampere's Law \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \)

4. Faraday's Law \( \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \)

5. The continuity eq. \( \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \)

\( \frac{94}{(b)} \) \( \frac{6/17}{(c)} \) Final Exam of EM.

\( \frac{\varepsilon_0}{\rho} \)

\( \frac{7.3.1}{\text{Ampere's Law}} \):
(1) Ampere's Law: \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \)
\( \nabla \cdot (\nabla \times \mathbf{B}) = \mu_0 \nabla \cdot \mathbf{J} = 0 \)

at steady, valid for dynamics?

(2) Continuity equation: \( \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0 \). Charge increment \( \Delta \rho \).

\( \frac{7.3.2}{\text{How Maxwell fixed Ampere's Law for Electrodynamics?}} \)

1. Continuity equation & Ampere's Law, they were all valid even in time varying situation & realize Ampere's Law was to consist with the continuity equation.

2. \( \nabla \cdot (\nabla \times \mathbf{B}) = 0 \), \( \nabla \cdot \mathbf{J} = 0 \).

This indeed true of \( \rho \) does not change with time.

But it is not true when \( \rho \) is changing with time.

\[ \frac{\Delta \rho}{\Delta t} = \text{constant} \]

\[ \frac{\rho}{t} = \text{constant} \]

\[ \frac{\Delta \rho + \rho}{\Delta t} + \text{constant} \]

\[ \text{Accel} \]

\[ \text{Speed} \]
Using Gauss's law, we can rewrite the continuity eq. as
\[ \nabla \cdot J = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t} \left[ \nabla \cdot E \right] \]
Replace \( \nabla \cdot \left[ \nabla + \frac{\partial}{\partial t} \right] E \cdot J \rightarrow 0 \), \( \nabla \cdot J_{\text{Maxwell}} = 0 \).

So the divergence of curl \( B \) can be rewritten as
\[ \nabla \cdot (\nabla \times B) = \nabla \cdot (\mu_0 J_{\text{Maxwell}}) = \nabla \cdot (\mu_0 \nabla + \frac{\partial}{\partial t} \mu_0 E) \rightarrow 0 \]

Maxwell said that if Ampere's law is modified by the addition of a new term the time derivative term.
\[ \nabla \times B = \mu_0 J + \frac{\partial}{\partial t} (\mu_0 \sigma_0 E) \]
Mag. Ele.

11) Is valid for steady-state phenomena is also compatible with the equation of continuity of time-dependent fields (dynamics).

2) The term \( \partial E / \partial t \) has the dimensions of current density
\[ \nabla \cdot (\varepsilon_0 E + \rho) = \dot{\rho}_E \rightarrow \nabla \cdot D = \dot{\rho}_E \]

Have a look in Gauss's law for Electric & Magnetic fields:
(1) \( \nabla \cdot E = \frac{\rho}{\varepsilon_0} \) + (2) \( \nabla \cdot B = 0 \) (\( \mu_0 \mu_m = 0 \))

Using the free space (\( \varepsilon_0 \) or \( \mu_0 = 0 \))
\[ \nabla \cdot E = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \cdot B = 0, \quad \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \]

Replace \( E \) by \( B \) \( \rightarrow \nabla \cdot B = 0, \quad \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \)

Replace \( B \) by \( -\mu_0 \mu_m \)
\[ \nabla \cdot E = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t} \]

There are something missing from \( \nabla \cdot B = 0 \) & \( \nabla \times E = -\frac{\partial B}{\partial t} \)

if we had \( \varepsilon_0, \mu_0 \) & \( m \). (In non-free space)
We have
1) \( \nabla \cdot E = \frac{\rho}{\varepsilon_0} \)
2) \( \nabla \times E = -\mu_0 \mu_m - \frac{\partial B}{\partial t} \)
3) \( \nabla \cdot B = \mu_0 \mu_m \)
4) \( \nabla \times B = \mu_0 \mu_0 \frac{\partial E}{\partial t} \)

\( \varepsilon_m \): magnetic charge density.
\( J_m \): magnetic current density.
"Maxwell's equation beg for the existence of magnetic charge" 7.35, 7.36