

★ Answer and mark clearly the questions in the provided answer sheets.
Write down your name and student's ID on the each answer sheet you used.
* **Note:** No points will be given if no arguments are provided for an answer.
Good Luck! *~~ Yuling* ☺

1. (50 points) Find

$$(a) \lim_{x \rightarrow 0} (1 + 2x)^{1/x} \qquad (b) \int x\sqrt{2x+1} \, dx$$

$$(c) \int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx \qquad (d) \int \frac{1}{\sqrt{x}(\sqrt{x}+1)} \, dx \qquad (e) \int_{1/3}^{1/2} \frac{e^{1/x}}{x^2} \, dx$$

2. (10 points) Find $f(x)$ if $f'(x) = xe^{4-x^2}$ and the point $(-2, 1)$ is on the curve $y = f(x)$.

3. (10 points) Solve the given separable differential equation

$$\frac{dy}{dx} = \frac{2 - y^2}{xy}.$$

4. (10 points) Solve the given initial value problem:

$$\frac{dx}{dt} = \frac{\sin(\sqrt{t})}{\sqrt{t}}; \quad x(0) = -1.$$

5. (10 points) Find the average value of $f(x) = e^{-x}(4 - e^{2x})$ over the interval $-1 \leq x \leq 1$.

6. (10 points) Two functions $f(x)$ and $g(x)$ are continuous on the interval $-3 \leq x \leq 2$ and satisfy

$$\int_{-3}^2 f(x) \, dx = 5, \quad \int_{-3}^2 g(x) \, dx = -2, \quad \int_{-3}^1 f(x) \, dx = 0, \quad \int_{-3}^1 g(x) \, dx = 4,$$

calculate

$$\int_1^2 [3f(x) + 2g(x)] \, dx \quad \text{and} \quad \int_4^4 g(x) \, dx$$