Solution of quiz 2

1

$$f(x)=\sqrt[3]{\frac{x}{x+2}}=\left(\frac{x}{x+2}\right)^{1/3}$$

$$f'(x)=\frac{1}{3}\left(\frac{x}{x+2}\right)^{-2/3}\cdot\frac{(x+2)(1)(x)(1)}{(x+2)^2}$$

$$=\frac{(x+2)^{2/3}}{3x^{2/3}}\cdot\frac{2}{(x+2)^2}$$

$$=\frac{2}{3x^{2/3}(x+2)^{4/3}}$$

$m=f'(-1)=\frac{2}{3}$ and $f(-1)=-1$, so the equation of the tangent line at $(-1,-1)$ is

$y+1=\frac{2}{3}(x+1)$, or $y=\frac{2}{3}x-\frac{1}{3}$.

2

$$h(x)=\sqrt{5x^2+g(x)}=\left[5x^2+g(x)\right]^{1/2}$$

Using the general power rule,

$$h'(x)=\frac{1}{2}\left[5x^2+g(x)\right]^{-1/2}\left[10x+g'(x)\right]$$

$$=\frac{10x+g'(x)}{2\sqrt{5x^2+g(x)}}$$

Substituting $x=0$, $g(0)=4$ and $g'(0)=2$,

$$h'(0)=\frac{0+2}{2\sqrt{0+4}}=\frac{2}{4}=\frac{1}{2}$$

3

$$x^2+xy+y^2=3$$

$$2x+x\frac{dy}{dx}+y\cdot 1+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-2x-y}{x+2y}$$

(a) $\frac{-2x-y}{x+2y}=0$ when $-2x-y=0$, or

$y=-2x$.

Substituting in the original equation,

$$x^2-2x^2+4x^2=3$$

$$3x^2=3$$

$$x=\pm 1$$

When $x=-1$, $y=-2(-1)=2$, and when $x=1$, $y=-2(1)=-2$. So, there are horizontal tangents at $(-1, 2)$ and $(1, -2)$.

(b) $x+2y=0$ when $x=-2y$.

Substituting in the original equation,

$$4y^2-2y^2+y^2=3$$

$$3y^2=3$$

$$y=\pm 1$$

When $y=-1$, $x=-2(-1)=2$, and when $y=1$, $x=-2(1)=-2$. So, there are vertical tangents at $(-2, 1)$ and

$(2, -1)$.

4

$$x^2+3y^2=5$$

$$2x+6y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{3y}$$

$$\frac{d^2y}{dx^2}=\frac{(3y)(-1)-(-x)\left(3\frac{dy}{dx}\right)}{(3y)^2}$$

$$=\frac{-3y+3x\frac{dy}{dx}}{9y^2}$$

$$=\frac{-3y+3x\left(\frac{-x}{3y}\right)}{9y^2}$$

$$=\frac{-3y-\frac{x^2}{y}}{9y^2}\cdot\frac{y}{y}$$

$$=\frac{-3y^2-x^2}{9y^3}$$

$$=\frac{-(x^2+3y^2)}{9y^3}$$

$$=-\frac{5}{9y^3}$$

5 (a)

$h(t)=\frac{t^2}{t^2+t-2}=\frac{t^2}{(t+2)(t-1)}$ defined for

$t\neq -2, 1$

$$h'(t)=\frac{(t^2+t-2)(2t)-(t^2)(2t+1)}{(t^2+t-2)^2}$$

$$=\frac{t(t-4)}{(t^2+t-2)^2}$$

$h'(t)=0$ when $t=0, 4$

When $-2<t<0$, $h'(t)>0$ so h increasing

$0<t<1$, $h'(t)<0$ so h decreasing

$1<t<4$, $h'(t)<0$ so h decreasing

$t>4$, $h'(t)>0$ so h increasing

When $t=0$, $h(0)=0$ and the point $(0, 0)$ is a relative maximum.

When $t=4$, $h(4)=\frac{8}{9}$ and the point $\left(4, \frac{8}{9}\right)$ is a relative minimum.

5 (b)

$$f'(x)=\frac{(x+1)^2(4-3x)^3}{(x^2+1)^2}$$

$f'(x)=0$ when $x=-1, \frac{4}{3}$

When $x<-1$, $f'(x)>0$ so f increasing

$-1<x<\frac{4}{3}$, $f'(x)>0$ so f increasing

$x>\frac{4}{3}$, $f'(x)<0$ so f decreasing

When $x=-1$, f does not have a relative extremum, and when $x=\frac{4}{3}$, f has a relative maximum.

5

(c) $f(x) = \frac{(x-2)^3}{x^2}$

$$f'(x) = \frac{x^2[3(x-2)^2(1)] - (x-2)^3(2x)}{x^4}$$

$$= \frac{x(x-2)^2[3x - 2(x-2)]}{x^4}$$

$$= \frac{(x-2)^2(x+4)}{x^3}$$

$f'(x) = 0$ when $x = -4, 2$

$$f''(x) = \frac{1}{x^6}(x^3[(x-2)^2(1) + (x+4)(2)(x-2)] - [(x-2)^2(x+4)(3x^2)])$$

$$= \frac{x^2(x-2)(x[(x-2) + 2(x+4)] + 3(x-2)(x+4))}{x^6}$$

$$= \frac{24(x-2)}{x^4}$$

$f''(-4) = -\frac{9}{16} < 0$ and $f(-4) = -13.5$. So, $(-4, -13.5)$ is a relative maximum. $f''(2) = 0$, so the test fails.

6

$$f(x) = x(2x+1)^2$$

$$= x(4x^2 + 4x + 1)$$

$$= 4x^3 + 4x^2 + x$$

$f'(x) = 12x^2 + 8x + 1$

$f''(x) = 24x + 8 = 8(3x+1)$

$f''(x) = 0$ when $x = -\frac{1}{3}$

When

$x < -\frac{1}{3}$, $f''(x) < 0$ so f is concave down

$x > -\frac{1}{3}$, $f''(x) > 0$ so f is concave up

Since the concavity changes at the critical

7 (a) $f(t) = 3t^5 - 5t^3,\ -2 \le t \le 0$

$f'(t) = 15t^4 - 15t^2 = 15t^2(t+1)(t-1)$

$f'(t) = 0$ when $t = -1$, $t = 0$ and $t = 1$, of which $t = -1$ and $t = 0$ are in the interval.

$f(-1) = 2$, $f(0) = 0$, $f(-2) = -56$

So, $f(-1) = 2$ is the absolute maximum and $f(-2) = -56$ is the absolute minimum.

7 (b) $f(x) = \dfrac{1}{x},\ x > 0$

$f'(x) = -\dfrac{1}{x^2}$

$f'(x)$ is never zero and $f'(x)$ is undefined when $x = 0$, which is not in the domain. Also, there are no endpoints. So, there is no absolute maximum or absolute minimum.