2014. 10. 08. Quiz 1.

1. (a). $\lim_{x\to 1^-} \frac{x-\sqrt{x}}{x-1} = \lim_{x\to 1^-} \frac{x-\sqrt{x}}{x-1} \quad \frac{x+\sqrt{x}}{x+\sqrt{x}}$

$= \lim_{x\to 1^-} \frac{x^2-x}{(x-1)(x+\sqrt{x})} = \lim_{x\to 1^-} \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+\sqrt{x})}$

$= \lim_{x\to 1^-} \frac{x}{(x+\sqrt{x})} = \frac{1}{1+1} = \frac{1}{2}$

(b) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{\cancel{\sqrt{x}-3}}{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}} = \frac{1}{\lim_{x\to 9} \sqrt{x}+3}$

$= \frac{1}{\sqrt{\lim_{x\to 9} x}+3} = \frac{1}{6}$

(c) $\lim_{x\to +\infty} \frac{x^2+x-5}{1-2x-x^3} = \lim_{x\to +\infty} \frac{\frac{1}{x}+\frac{1}{x^2}-\frac{5}{x^3}}{\frac{1}{x^3}-2\frac{1}{x^2}-1} = 0$

$\lim_{x\to -\infty} \frac{\frac{1}{x}+\frac{1}{x^2}-\frac{5}{x^3}}{\frac{1}{x^3}-2\frac{1}{x^2}-1} = 0$

(d) $\lim_{x\to 0^+} \sqrt{x(1+\frac{1}{x^2})} = \lim_{x\to 0^+} \sqrt{x+\frac{1}{x}} = +\infty$

2. (a). $f(x) = 5x^3-3x+\sqrt{x}$ is not continuous for $x<0$ since square roots of negative number do not exist in the real numbers

(b) $f(x) = \frac{x^2-2x+1}{x^2-x-2} = \frac{(x-1)^2}{(x-2)(x+1)}$

is not continuous at $x=-1$ or $x=2$, where the denominator is 0

3.

$$f(x) = \begin{cases} \dfrac{x^2-1}{x+1}, & \text{if } x < -1 \\ Ax^2+x-3, & \text{if } x \geq -1 \end{cases}$$

The $f(x)$ is continuous everywhere except possibly at $x=-1$ since $\dfrac{x^2-1}{x+1}$ is a rational function and Ax^2+x-3 is a polynomial.

Since $f(-1) = A-4$, in order for $f(x)$ to be continuous at $x=-1$, A must be chosen so that

$$\lim_{x \to -1} f(x) = A-4$$

As x approaches -1 from the right.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (Ax^2+x-3) = A-4$$

and as x approaches -1 from the left.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(\frac{x^2-1}{x+1}\right) = \lim_{x \to 1^-} \frac{(x+1)(x-1)}{x+1} = -2$$

$\lim\limits_{x \to -1} f(x)$ exists whenever $A-4=-2 \Rightarrow A=2$

Furthermore, for $A=2$.

$\lim\limits_{x \to -1} f(x) = -2$. $f(-1) = 2-4 = -2$.

Thus, $f(x)$ is continuous at $x=-1$ only when $A=2$

4. Let $f(x) = \sqrt[3]{x} - (x^2+2x-1)$

$f(x)$ is continuous at all x and

$f(0) = 1$, $f(1) = 1 - (1+2-1) = -1$.

By the Intermediate value property, there is at least one number $0 \le c \le 1$ such that $f(c) = 0$, and $x = c$ is a solution.

5. For $f(x) = 2 - 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(2-3(x+h)^2) - (2-3x^2)}{h}$$

$$= \lim_{h \to 0} (-6x - 3h) = -6x, \text{ for all } x.$$

At the point $c=1$, the slop of the tangent line is $m = f'(1) = -6$. The point $(1, -1)$ is on the tangent line so by the point-slop formula the equation of the tangent line is

$y - (-1) = -6(x-1)$ or $y = -6x + 5$