榮譽第一

國立東華大學 應用數學系＿＿學年度第＿＿學期　考試科目：Cal AI　Quiz 1012 □期中□期末考試試卷

學號＿＿＿ 姓名 Yes 系所＿＿＿ 班別＿＿＿ 任課教師：Yes 共＿＿張

#1. $f(x) = \frac{x+3}{x-1} = \frac{\text{polynomial}}{\text{polynomial}}$ ∴ is defined as long as $x-1 \neq 0$

hence Domain of f is $\forall x \in R$, but $x \neq 1$

Note $f(x) = \frac{x-1+4}{x-1} = 1 + \frac{4}{x-1}$, and $\frac{4}{x-1} \in (0,\infty)$ for $x>1$, $\in(-\infty,0)$ $x<1$

$\Rightarrow$ Range of f is $(-\infty,1)\cup(1,\infty)$

#2. $f(x) = x^2 - x + 2 = (x-2)(x+1) = 0 \Rightarrow x = 2, -1$

i.e. $f(2) = f(-1) = 0$　i.e. Two different x w/ the same value of f

$\Rightarrow$ f is NOT 1-1

#3. Let $y = f(x) = \sqrt{x^2-9}$, $x \geq 3$ (hence $y \geq 0$)

$\Rightarrow x = \sqrt{y^2+9}$.

$\Rightarrow f^{-1}(x) = \sqrt{x^2+9}$, $x \geq 0$.

#4. (a) $\lim\limits_{x\to-2^-} \frac{x+2}{|x+2|} = \lim\limits_{x\to-2^-} \frac{x+2}{-(x+2)} = -1$　∵ $x \to -2^-$, $x<-2$, ∴ $|x+2| = -(x+2)$

(b) $\lim\limits_{x\to0^+}\left(x - \frac{1}{x}\right) = \lim\limits_{x\to0^+}\left(-\frac{1}{x}\right) = -\infty$. ∵ $\lim\limits_{x\to0^+} x = 0$, $\lim\limits_{x\to0^+}\frac{1}{x} = \infty$

(c) $\lim\limits_{x\to0} \frac{[1/(x-2)]-1}{x} = \lim\limits_{x\to0} \frac{1-(x-2)}{x(x-2)} = \lim\limits_{x\to0}\frac{3-x}{x(x-2)}$ D.N.E. ∵ $\lim\limits_{x\to0} 3-x = 3 \neq 0$

但 $\lim\limits_{x\to0} x(x-2) = 0$

(d) $\lim\limits_{x\to2} \frac{x-2}{x^2-4x+4} = \lim\limits_{x\to2}\frac{x-2}{(x-2)^2} = \lim\limits_{x\to2}\frac{1}{x-2}$ D.N.E ∵ $\lim\limits_{x\to2^+}\frac{1}{x-2} = \infty$

但 $\lim\limits_{x\to2^-}\frac{1}{x-2} = -\infty$

(e)

$$\lim_{\Delta x\to0}\frac{(x+\Delta x)^3 - (x+\Delta x) - (x^3 - x)}{\Delta x} = \lim_{\Delta x\to0}\frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x - \Delta x - x^3 + x}{\Delta x}$$

$$= \lim_{\Delta x\to0}\frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \Delta x}{\Delta x}$$

$$= \lim_{\Delta x\to0}\left[3x^2 + 3x\Delta x + (\Delta x)^2 - 1\right]$$

$$= 3x^2 - 1$$

#5. $h(x) = f(g(x)) \stackrel{\Delta}{=} \frac{1}{\sqrt{g(x)}} \stackrel{\Delta}{=} \frac{1}{\sqrt{x-1}}$ ∵ $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x-1$, $x>1$

∵ $\forall a>1$ $\lim\limits_{x\to a} h(x) = \lim\limits_{x\to a}\frac{1}{\sqrt{x-1}} \stackrel{\Delta}{=} \frac{1}{\sqrt{a-1}} = h(a)$ ∵ $\sqrt{a-1} \neq 0 \Rightarrow h$ is conti. on $(1,\infty)$

#6. To be conti. at $x=-1$: $\lim\limits_{x\to-1^+} f(x) = \lim\limits_{x\to-1^+}(ax+b) = -a+b$

$= \lim\limits_{x\to-1^-} f(x) = \lim\limits_{x\to-1^-} 2 = 2$

To be conti. at $x=3$: $\lim\limits_{x\to3^+} f(x) = \lim\limits_{x\to3^+} -2 = -2$

$= \lim\limits_{x\to3^-} f(x) = \lim\limits_{x\to3^-}(ax+b) = 3a+b$

$\Rightarrow \begin{cases} -a+b=2 \\ 3a+b=-2 \end{cases}$

$\Rightarrow a=-1, b=1$

For $x<-1$, $f(x)=2$, $-1<x<3$, $f(x) = -x+1$, $x>3$, $f(x) = -2$ are all conti. ft's. Hence $a=-1$, $b=1 \Rightarrow f(x)$ is conti. on R