Calculus BM
Instructor: Yu-Ling Tseng Midterm 20171129

⋆ Answer and mark clearly the questions in the provided answer sheets.
Write down your name and student's ID on the each answer sheet you used.
∗ **Note:** No points will be given if no arguments are provided for an answer.
For your information:

- $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
- $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ and $\frac{d}{dx}x^r = rx^{r-1}$, for all r
- $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\cos(x) = -\sin(x)$

Good Luck! $\sim\sim$ *Yuling* ‿

1. (8 points) Find the constants a and b such that the function $f(x)$ is continuous on the entire real number line, where

$$f(x) = \begin{cases} 2 & x \le -1 \\ ax + b & -1 < x < 3 \\ -2 & x \ge 3 \end{cases}$$

2. (8 points) You are given $f'(x) = -x^2 + 2x - 1$. Find the intervals on which (a) $f'(x)$ is increasing or decreasing, (b) the graph of f is concave upward or concave downward, and (c) find the x-values of the relative extrema and inflection points of f.

3. (24 points) Find the indicated limit or show it does not exist. If the limiting value is infinite, indicate whether it is ∞ or $-\infty$.

$$(a)\quad \lim_{x\to 0}(e^x + x)^{1/x} \qquad (b)\quad \lim_{x\to 0}\frac{\sin(2x)}{\sin(5x)}, \qquad (c)\quad \lim_{x\to\infty} x^4 e^{-5x}$$

4. (8 points) Find the equation of the tangent line to the curve of $x + y - 1 = \ln(x^2 + y^2)$ at the point $(1, 0)$.

5. Find the absolute maximum and absolute minimum (if any) of

   (a) (8 points) $f(t) = 3t^5 - 5t^3$ on the closed interval $-2 \le t \le 0$.
   (b) (8 points) $h(t) = (e^{-t} + e^t)^5$ for $-1 \le t \le 1$.

6. (40 points) Find the derivative $\frac{dy}{dx}$ or $f'(x)$ where

   (a) $y\,e^{2x-x^3} = 5x + y^2\,\ln((x^2+1)^4)$ (b) $f(x) = x^x\,5^{x^2}$

   (c) $y = \frac{(4x^2+e^{3x})^5 e^{-6x}}{(1+\cos(x^2)+x^3)^{4/5}}$ (d) $f(x) = \frac{e^{-x^3}+2x}{\log_8 x}$

   (e) $f(x) =$ the inverse function of $\sin(x)$