⋆ Answer and mark clearly the questions in the provided answer sheets. Write down your name and student's ID on the each answer sheet you used.
∗ **Note:** No points will be given if no arguments are provided for an answer.
For your information:

- $\int \sin u\, du = -\cos u + C$ and $\int \cos u\, du = \sin u + C$
- $\int \sec^2 u\, du = \tan u + C$ and $\int \sec u \tan u\, du = \sec u + C$
- $\sin^2 u + \cos^2 u = 1$ and $\tan^2 u + 1 = \sec^2 u$

Good Luck! ∼∼ *Yuling* ⌣̈

1. (10 points) (Multiple choice) Which of the following integrals are improper integrals?

$$(a)\ \int_{-2}^{5} x^{-2}\, dx \qquad (b)\ \int_0^2 \frac{x^3+x+7}{e^{2x}(x-2)}\, dx \qquad (c)\ \int_1^e x\ln x\, dx$$

$$(d)\ \int_{-\infty}^{5} 2xe^{-x^2}\, dx \qquad (e)\ \int_{-1}^{4} \frac{1}{\sqrt{|x|}(x-1)}\, dx$$

2. (7 points) Find the maximum and minimum values of the function $f(x,y) = xy^2$ subject to the constraint $x^2+y^2=1$

3. (49 points) Find

$$(a)\ \int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx \qquad (b)\ \int_0^1 \frac{e^x-e^{-x}}{e^x+e^{-x}}\, dx \qquad (c)\ \int_1^e (\ln x)^3\, dx \qquad (d)\ \int_0^{\pi} (\sin x + \cos x)^2\, dx$$

$$(e)\ \int \frac{1}{(x^2-4)^2}\, dx \qquad (f)\ \int \frac{1}{\sqrt{4x^2-4}}\, dx \qquad (g)\ \int \cos(x)\, 2^x\, dx$$

4. (14 points) Use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence

$$(a)\ \int_1^{\infty} \frac{(\sin(x))^2}{x^2}\, dx \qquad (b)\ \int_1^{\infty} \frac{1}{\sqrt{x+\sqrt{x}}}\, dx$$

5. (7 points) Find $f'(x)$ if $f(x) = \int_{\cos^2(2x)}^{x^4} e^{-t^2}(t+\ln t)\, dt$

6. The gamma function is defined as the improper integral

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}\, e^{-x}\, dx,$$

which is convergent for any $\alpha > 0$. Giving that $\Gamma(1) = 1$, $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$, for all $\alpha > 0$, hence $\Gamma(n+1) = n!$ for positive integer n, also that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$,

(a) (7 points) show that

$$\int_{-\infty}^\infty e^{-\frac{x^2}{2}}\, dx = \sqrt{2\pi},$$

(b) (7 points) find

$$\int_{-\infty}^\infty e^{-\left(\frac{x^2+4x}{8}\right)}\, dx$$