榮譽第一

國立東華大學 應用數學系 ____學年度第____學期 考試科目：112 ☑期中☐期末考試試卷

學號 410932024 姓名 馮祐倫 系所 企管一 班別____ 任課教師：____ 共____張

8 (1) $f(x)=\frac{f(x)-f(c)}{x-c}(x-c)+f(c)$

$\lim_{x\to c} f(x)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}\cdot\lim_{x\to c}(x-c)+f(c)$

$=f'(c)\times 0+f(c)$

$=f(c)$

$\because f$ is diff at $x=c$

i.e $f$ is conti at $x=c$

8 (2) $2+y+x\frac{dy}{dx}=\frac{3x^2+2y}{x^3+y^2}\frac{dy}{dx}$，代入 $(1,0)$

$2+\frac{dy}{dx}=\frac{3}{1}\frac{dy}{dx}$

$\frac{dy}{dx}=1=$ slope

$\Rightarrow y-0=1\cdot(x-1)$ #

8 (3) $f'(x)=1-\frac{1}{2x}$，代入 $x=1$

$f'(1)=\frac{1}{2}=$ slope

$f(1)=1$，代入 $(1,1)$

$\Rightarrow y-1=\frac{1}{2}(x-1)$ #

8 (4) a. $f'(x)=\frac{4-2x}{4x-x^2}\overset{set}{=}0$

$x=2$.

$f(1)=\ln 3 \Rightarrow$ abs.min

$f(2)=\ln 4 \Rightarrow$ abs.max

$f(3)=\ln 3 \Rightarrow$ abs.min

8 (4) b. $h'(t)=3(e^{-t}+e^{t})^2(-e^{-t}+e^{t})\overset{set}{=}0$

$t=0$

$h(-1)=(e^{1}+e^{-1})^3$

$h(0)=(e^{0}+e^{0})^3=8 \Rightarrow$ abs.min

$h(3)=(e^{-3}+e^{3})^3 \Rightarrow$ abs.max

8 (5) a. $f'(x)=6\times 3\cdot\tan^2(\ln\sqrt{3x})\cdot(\sec^2\ln\sqrt{3x})$

$\times(\frac{1}{\sqrt{3x}})\times\frac{1}{2}\times(3x)^{-\frac{1}{2}}\times 3$

$f'(x)=18(\tan^2(\ln\sqrt{3x}))(\sec^2\ln\sqrt{3x})\times\frac{1}{2x}$ #

(5) b. $\ln f(x)=\ln x^{x}+\ln 6^{x^3}$ 8

$=x\ln x+x^3\ln 6$

$\frac{1}{y}\frac{dy}{dx}=1\ln x+x\frac{1}{x}+3x^2\ln 6+x^3\times 0$

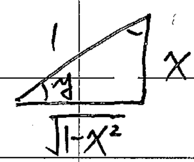
$\frac{dy}{dx}=y(\ln x+1+3x^2\ln 6)$

$=x^{x}6^{x^3}(\ln x+1+3x^2\ln 6)$ #

(5) c. $f(x)=\sin^{-1}(x)$ 8

$\Rightarrow \sin y=x$

$\Rightarrow \cos y\frac{dy}{dx}=1$



$\Rightarrow \frac{dy}{dx}=\frac{1}{\cos y}$

$=\frac{1}{\sqrt{1-x^2}}$ #

(5) d. $(2-3x^2)e^{2x-x^3}\log_5 y+e^{2x-x^3}\frac{1}{(\ln 5)y}\frac{dy}{dx}$ 8

$=2\sin(x^2)\frac{dy}{dx}+2y\cdot\cos(x^2)\cdot 2x$

$+2\ln(3x^2+1)\frac{dy}{dx}+2y\frac{6x}{3x^2+1}$

$\frac{dy}{dx}=\frac{(2-3x^2)e^{2x-x^3}\log_5 y-4xy\cos(x^2)-\frac{12xy}{3x^2+1}}{2\sin(x^2)+2\ln(3x^2+1)-e^{2x-x^3}\frac{1}{\ln 5\, y}}$ #

(5) e. $\ln y=4\ln(4x^3e^{-2x})+3\ln(2x^5-8x+2)$ 8

$-7.4\ln[1+\cos(5x^2-10x)+x^{5.2}]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\Big[4\times(\frac{12x^2}{4x^3})+4\times(-2)+3\cdot\frac{10x^4-8}{2x^5-8x+2}$

$-7.4\frac{-\sin(5x^2-10x)(10x-10)+5.2x^{4.2}}{1+\cos(5x^2-10x)+x^{5.2}}\Big]\overset{set}{=}A$

$\Rightarrow \frac{dy}{dx}=yA$ #

80

8. (6) $f'(x) = 2xe^{x^2-1} + 30e^{6x} + \frac{2x}{x^2+2}$

$f''(x) = 2e^{x^2-1} + 2x \times (2x) e^{x^2-1} + 180e^{6x} + \frac{2(x^2+2) - 2x(2x)}{(x^2+2)^2}$

$= (4x^2+2)e^{x^2-1} + 180e^{6x} + \frac{-2x^2+4}{(x^2+2)^2}$ #

8 (7) a. $= e^{\lim_{x\to 0} \frac{\ln(e^{-3x}+5x)}{x}} = e^{L} = \boxed{e^2}$

$\lim_{x\to 0} \frac{\ln(e^{-3x}+5x)}{x} \rightarrow$ "$\frac{0}{0}$"

$\Rightarrow \lim_{x\to 0} \frac{-3e^{-3x}+5}{e^{-3x}+5x}$ $\rightarrow -3+5=2$, $\rightarrow 1$

$=2$ #

8. (7) b. $\lim_{x\to 0} \frac{\sin(\sqrt{5}x)}{\sin(3x)} \rightarrow$ "$\frac{0}{0}$"

$\Rightarrow \lim_{x\to 0} \frac{\sqrt{5}\cos(\sqrt{5}x)}{3\cos(3x)}$ $\rightarrow \sqrt{5}\times 1$, $\rightarrow 3\times 1$

$= \frac{\sqrt{5}}{3}$ #

8. (7) c. $\lim_{t\to\infty} \frac{t^3}{e^{5t}} \rightarrow$ "$\frac{\infty}{\infty}$"

$\Rightarrow \lim_{t\to\infty} \frac{3t^2}{5e^{5t}} \rightarrow$ "$\frac{\infty}{\infty}$"

$\Rightarrow \lim_{t\to\infty} \frac{6t}{5^2 \cdot e^{5t}} \rightarrow$ "$\frac{\infty}{\infty}$"

$\Rightarrow \lim_{t\to\infty} \frac{6}{5^3 \cdot e^{5t}}$ $\rightarrow 6$, $\rightarrow \infty$

$= 0$ #

104

**榮譽第一**

國立東華大學 應用數學系 109 學年度第 1 學期 考試科目：微積分(商用) ☑期中□期末考試試卷

學號 410932011 姓名 李光翔 系所 企管一 班別 ____ 任課教師：曾玉玲 共 ___ 張

8

1. 證 diff ⇒ conti

if f is diff at $x=c$

then. $f'(c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$ exist

令 $h=x-c$

$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

$\lim_{x\to c}(f(x)-f(c))=\lim_{x\to c}\left(\frac{f(x)-f(c)}{x-c}\times(x-c)\right)$

$=\lim_{x\to c}f'(c)\times 0$

$=0$.

又. $\lim_{x\to c}(f(x)-f(c))=\lim_{x\to c}f(x)-\lim_{x\to c}f(c)$

$=\lim_{x\to c}f(x)-f(c)=0$

故 $\lim_{x\to c}f(x)=f(c)$. #

8

2.

eq= $y-y_0=f'(x)(x-x_0)$

求 $f'(x)$ i.e. $\frac{dy}{dx}$.

$\frac{d}{dx}(2x+xy-2)=\frac{d}{dx}\ln(x^3+y^2)$

$2+(1\cdot y+x\cdot\frac{dy}{dx})=\frac{1}{x^3+y^2}\cdot(3x^2+2y\frac{dy}{dx})$

$2+y+x\cdot\frac{dy}{dx}=\frac{3x^2+2y\frac{dy}{dx}}{x^3+y^2}$

$2x^3+yx^3+x^4\frac{dy}{dx}+2y^2+y^3+xy^2\frac{dy}{dx}=3x^2+2y\frac{dy}{dx}$

$2x^3+yx^3+2y^2+y^3-3x^2=-x^4\frac{dy}{dx}-xy^2\frac{dy}{dx}+2y\frac{dy}{dx}$

$2x^3+yx^3+2y^2+y^3-3x^2=(2y-x^4-xy^2)\frac{dy}{dx}$.

$\frac{2x^3+yx^3+2y^2+y^3-3x^2}{2y-x^4-xy^2}=\frac{dy}{dx}=$

當 $x=1$ $y=0$. $\Rightarrow\frac{dy}{dx}=\frac{-1}{-1}=1$.

$\Rightarrow$ eq= $y=(x-1)$ #

8

3.

$f(x)=x-\ln(x^{\frac{1}{2}})$

$f(x)=x-\frac{1}{2}\ln x$

$f'(x)=1-\frac{1}{2}\frac{1}{x}=1-\frac{1}{2x}$

當 $x=1$ 時 $f(1)=1-\ln\sqrt{1}=1-\ln 1=1$.

eq: $y-1=(1-\frac{1}{2\cdot 1})(x-1)$

$\Rightarrow y-1=\frac{1}{2}(x-1)$ #

8

4.(a).

$f(x)=\ln(4x-x^2)$. $\forall\ 1\le x\le 3$.

$f'(x)=\frac{1}{4x-x^2}\cdot(4-2x)=\frac{4-2x}{4x-x^2}=\frac{2(2-x)}{x(4-x)}$

$f'(x)=0$. 當 $x=2$.

$f'(x)$ D.N.E 當 $x=0$ or $x=4$.

但 $x=0$、$x=4$ 皆不在 Domain 中，⇒ 不合.

critical number: $x=2$.

$f(x)$ ↗ | ↘

$f'(x)$ + 2 −

abs max: $(2, f(2))=(2,\boxed{\ln 4})$ #

端點 $f(1)=\ln(4-1)=\ln 3$

$f(3)=\ln(12-9)=\ln 3$.

abs min: $(1, f(1))=(1,\boxed{\ln 3})$ #

abs min: $(3, f(3))=(3,\boxed{\ln 3})$ #

4.(b) ? $h'(t)=3(e^{-t}+e^{t})^2\cdot(-e^{-t}+e^{t})=0$

$\Rightarrow t=0$. $h(0)=8$ abs. min. $h(1)=(e^{-1}+e)^3$ $h(3)=(e^{-3}+e^3)^3$ abs. max

8

5.(a).

$f(x)=6[\tan(\ln(\sqrt{3x}))]^3$

$f'(x)=6\cdot 3[\tan(\ln(\sqrt{3x}))]^2\cdot\sec^2(\ln(\sqrt{3x}))\cdot\frac{1}{\sqrt{3x}}\cdot\frac{1}{2}(3x)^{-\frac{1}{2}}$

$\times 3$. #

8

5.(b).

$f(x)=x^x\cdot 6^{x^3}$

$\ln f(x)=\ln(x^x\cdot 6^{x^3})=\ln x^x+\ln 6^{(x^3)}$

$=x\ln x+x^3\ln 6$.

$\frac{d}{dx}\ln f(x)=(1\cdot\ln x+x\cdot\frac{1}{x})+\ln 6\cdot 3x^2$

$=\ln x+1+\ln 6\cdot 3x^2$

48

$f'(x) = (x^x \cdot 6^{x^3}) \times (\ln x + 1 + \ln 6 \cdot 3x^2)$ #

8

5.(c).

$f(x) = \sin^{-1}(x)$.

令 $y = \sin^{-1}(x)$.

$\sin(\sin^{-1}(x)) = x$

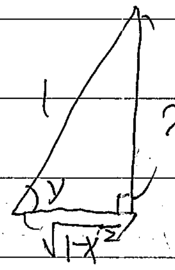
$\sin(y) = x$.

$\frac{d}{dx}\sin(y) = 1$.

$\cos(y) \cdot \frac{dy}{dx} = 1$.

$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-x^2}}$ #

* $\sin y = x$.



8

5.(d).

$\frac{d}{dx}(e^{2x-x^3} \cdot \log_5 y) \rightarrow$ A式

$= \frac{d}{dx}(2y\sin(x^2) + 2y\ln(3x^2+1)) \rightarrow$ B式

A式 $= e^{2x-x^3} \cdot (2-3x^2) \cdot \log_5 y + e^{2x-x^3} \cdot \frac{1}{y} \cdot \frac{dy}{dx} \cdot \frac{1}{\ln 5}$

B式 $= 2\frac{dy}{dx} \cdot \sin(x^2) + 2y\cos(x^2) \cdot (2x) + 2 \cdot \frac{dy}{dx}\ln(3x^2+1)$

$+ 2y \cdot \frac{1}{(3x^2+1)} \cdot (6x)$.

综合A.B得 $\frac{e^{2x-x^3}}{y\ln 5}\frac{dy}{dx} - 2\sin(x^2)\frac{dy}{dx} - 2\ln(3x^2+1)\frac{dy}{dx}$

$= -e^{2x-x^3} \cdot (2-3x^2) \cdot \log_5 y + 2y(2x)\cos(x^2) + 2y\frac{(6x)}{(3x^2+1)}$

$\Rightarrow \left[\frac{e^{2x-x^3}}{y\ln 5} - 2\sin(x^2) - 2\ln(3x^2+1)\right]\frac{dy}{dx}$

$= -e^{2x-x^3}(2-3x^2) \cdot \log_5 y + 4xy\cos(x^2) + \frac{12xy}{(3x^2+1)}$

$\frac{dy}{dx} = \dfrac{-e^{2x-x^3}(2-3x^2) \cdot \log_5 y + 4xy\cos(x^2) + \frac{12xy}{3x^2+1}}{\frac{e^{2x-x^3}}{y\ln 5} - 2\sin(x^2) - 2\ln(3x^2+1)}$ #

6、 $f(x) = e^{x^2-1} + 5e^{6x} + \ln(x^2+2)$. 8

$f'(x) = 2xe^{x^2-1} + 30e^{6x} + 2x(x^2+2)^{-1}$.

$f''(x) = \frac{d}{dx}f'(x) = 2e^{x^2-1} + 4x^2e^{x^2-1} + 180e^{6x}$

$+ \frac{2}{(x^2+2)} - \frac{2x}{(x^2+2)^2}$ #

7.(a). 8

$\lim_{x\to 0}(e^{-3x}+5x)^{\frac{1}{x}} = \lim_{x\to 0} e^{\ln(e^{-3x}+5x)^{\frac{1}{x}}}$

$= e^{(\lim_{x\to 0}\ln(e^{-3x}+5x)^{\frac{1}{x}})}$

$\lim_{x\to 0}\ln(e^{-3x}+5x)^{\frac{1}{x}} = \lim_{x\to 0}\frac{1}{x}\ln(e^{-3x}+5x)$

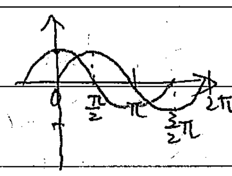
$= \lim_{x\to 0}\frac{\ln(e^{-3x}+5x) \to 0}{(x) \to 0}$ "$\frac{0}{0}$"

$= \lim_{x\to 0}\frac{\frac{1}{e^{-3x}+5x} \cdot (e^{-3x} \cdot (-3) + 5)}{1} = \lim_{x\to 0}\frac{e^{-3x} \cdot (-3) + 5}{e^{-3x}+5x}$

$= \frac{(-3)+5}{1+0} = \boxed{2}$

$\lim_{x\to 0}(e^{-3x}+5x)^{\frac{1}{x}} = e^2$ #

7.(b). 8



$\lim_{x\to 0}\frac{\sin(\sqrt{5}x) \to 0}{\sin(3x) \to 0}$ "$\frac{0}{0}$".

$= \lim_{x\to 0}\frac{\cos(\sqrt{5}x) \cdot \sqrt{5}}{\cos(3x) \cdot 3} = \frac{\sqrt{5}}{3}$ #

7.(c). 8

$\lim_{t\to\infty} t^3e^{-5t} = \lim_{t\to\infty}\frac{(t^3) \to \infty}{(e^{5t}) \to \infty}$ "$\frac{\infty}{\infty}$".

$= \lim_{t\to\infty}\frac{(3t^2) \to \infty}{(e^{5t}(5)) \to \infty}$ "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty}\frac{(6t) \to \infty}{(25e^{5t}) \to \infty}$ "$\frac{\infty}{\infty}$"

$= \lim_{t\to\infty}\frac{(6) \to 6}{(125e^{5t}) \to \infty} = 0$ #

8 5.(e.)

令 $g(x) = (4x^3e^{-2x})^4(2x^5-8x+2)^3$

$h(x) = [1+\cos(5x^2-10x) + x^{5.2}]^{7.4}$

$y' = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{(h(x))^2}$

$g'(x) = 4(4x^3e^{-2x})^3 \cdot (12x^2e^{-2x} + 4x^3 \cdot e^{-2x} \cdot (-2)) \cdot (2x^5-8x+2)^3 + (4x^3e^{-2x})^4 \cdot 3(2x^5-8x+2)^2 \cdot (10x^4-8)$.

$h'(x) = 7.4[1+\cos(5x^2-10x) + x^{5.2}]^{6.4} \cdot (-\sin(5x^2-10x) \cdot (10x-10) + 5.2x^{4.2})$.

$y' = \dfrac{\overbrace{4(4x^3e^{-2x})^3 \cdot (12x^2e^{-2x} + 4x^3 \cdot e^{-2x} \cdot (-2)) \cdot (2x^5-8x+2)^3 + (4x^3e^{-2x})^4 \cdot 3(2x^5-8x+2)^2 \cdot (10x^4-8)}^{g'(x)} \times \overbrace{[1+\cos(5x^2-10x) + x^{5.2}]^{7.4}}^{h(x)}}{[1+\cos(5x^2-10x) + x^{5.2}]^{7.4\times 2}}$

$- \dfrac{\overbrace{(4x^3e^{-2x})^4(2x^5-8x+2)^3}^{g(x)} \times \overbrace{7.4[1+\cos(5x^2-10x) + x^{5.2}]^{6.4} \cdot (-\sin(5x^2-10x) \cdot (10x-10) + 5.2x^{4.2})}^{h'(x)}}{[1+\cos(5x^2-10x) + x^{5.2}]^{7.4\times 2}}$ #