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榮譽第一
國立東華大學
應用數學系 ______學年度第____學期　考試科目：________________ □期中□期末考試試卷

學號 410932040 姓名 朱珈[illegible]　系所 企管　班別 一　任課教師：__________ 共____張

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1. $f(x) = \frac{x^4}{x^3+1} = x^4(x^3+1)^{-1}$

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$f'(x) = 4x^3(x^3+1)^{-1} + x^4(-1)(x^3+1)^{-2}(3x^2)$

$= \frac{x^6+4x^3}{(x^3+1)^2}$

$= \frac{x^3(x^3+4)}{(x^3+1)^2}$

$\Rightarrow x = 0$ or $\sqrt[3]{-4}$

$\Rightarrow (0,0)$, $(\sqrt[3]{-4}, 2.117)$ #

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4. (a) $g(x) = x\sqrt{x+3} = x(x+3)^{\frac{1}{2}}$

$g'(x) = (x+3)^{\frac{1}{2}} + x(\frac{1}{2})(x+3)^{-\frac{1}{2}}(1)$

$= \frac{(x+3)+\frac{1}{2}x}{(x+3)^{\frac{1}{2}}} = \frac{3x+6}{2(x+3)^{\frac{1}{2}}} \Rightarrow x=-2$

$g''(x) = \frac{6(x+3)^{\frac{1}{2}} - (3x+6)(x+3)^{-\frac{1}{2}}}{4(x+3)}$

$= \frac{3x+12}{4(x+3)^{\frac{3}{2}}}$

$\Rightarrow$ No inflection point　why?　max:

$g''(-2) = \frac{3}{2} > 0 \Rightarrow (-2,-2)$ is relative mini. #

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2. (a) $f(t) = (t^2-9)\sqrt{t+2} = (t^2-9)(t+2)^{\frac{1}{2}}$, $(-1,-8)$

$f'(t) = 2t(t+2)^{\frac{1}{2}} + (t^2-9)(\frac{1}{2})(t+2)^{-\frac{1}{2}}(1)$

$= \frac{2t^2+4t+\frac{1}{2}t^2-\frac{9}{2}}{(t+2)^{\frac{1}{2}}}$

$= \frac{5t^2+8t-9}{2\sqrt{t+2}}$

$f'(-1) = m = -6 \Rightarrow y-(-8) = -6(x-(-1))$

$y = -6x-14$ #

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(b) $f(x) = \frac{4}{1+x^2} = 4(1+x^2)^{-1}$

$f'(x) = -4(1+x^2)^{-2}(2x)$

$= -\frac{8x}{(1+x^2)^2} \Rightarrow x=0$

$f''(x) = -8(1+x^2)^{-2} + (-8x)(-2)(1+x^2)^{-3}(2x)$

$= \frac{-8-8x^2+32x^2}{(1+x^2)^3} = \frac{24x^2-8}{(1+x^2)^3} \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$f''(0) = -8 < 0 \Rightarrow (0,4)$ is relative max.

$f(\frac{1}{\sqrt{3}}) = 3$　$f(-\frac{1}{\sqrt{3}}) = 3 \Rightarrow (\frac{1}{\sqrt{3}},3)$ and $(-\frac{1}{\sqrt{3}},3)$ are points of inflection of $f(x)$ #

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(b) $f(x) = \frac{x+1}{\sqrt{2x-3}} = (x+1)(2x-3)^{-\frac{1}{2}}$, $(2,3)$

$f'(x) = (2x-3)^{-\frac{1}{2}} + (x+1)(-\frac{1}{2})(2x-3)^{-\frac{3}{2}}(2)$

$= \frac{(2x-3)+(-x-1)}{(2x-3)^{\frac{3}{2}}} = \frac{x-4}{(2x-3)^{\frac{3}{2}}}$

$f'(2) = m = -2 \Rightarrow y-3 = -2(x-2)$

$y = -2x+7$ #

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5. $f(x) = -x^2+2x-1 \Rightarrow x=1$

$f''(x) = -2x+2 \Rightarrow x=1$

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(c) $2x - y - x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$, $(-2,-1)$

$\frac{dy}{dx} = \frac{2x-y}{x+2y} \Rightarrow m = \frac{3}{4} \Rightarrow y+1 = \frac{3}{4}(x+2)$

$y = \frac{3}{4}x+\frac{1}{2}$ #

(a) $f'(x) < 0$ for $x<1$, $f(x)$ is increasing on $(-\infty,1)$

$f'(x) > 0$ for $x>1$, $f(x)$ is decreasing on $(1,\infty)$ #

(b) $f''(1) = 0 \Rightarrow f$ is concave downward #

(c) x-values of relative max. $= x=1$ #　check.

No inflection points #

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3. (a) $y = \sqrt{\tan(9x)}$

$\frac{dy}{dx} = \frac{1}{2}(\tan(9x))^{-\frac{1}{2}} \cdot \sec^2(9x)(9)$

$= \frac{9\sec^2(9x)}{2\sqrt{\tan(9x)}}$ #

03

6. $f(x) = \frac{4}{3}x\sqrt{3-x} = \frac{4}{3}x(3-x)^{\frac{1}{2}}$　$[0,3]$

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(b) $f(x) = \sqrt{4-x} = (4-x)^{\frac{1}{2}}$

$f'(x) = \frac{4}{3}(3-x)^{\frac{1}{2}} + (\frac{4}{3}x)(\frac{1}{2})(3-x)^{-\frac{1}{2}}(-1)$

$= \frac{12-4x-2x}{3(3-x)^{\frac{1}{2}}} = \frac{12-6x}{3(3-x)^{\frac{1}{2}}} \Rightarrow x=2$

$f'(x) = \frac{1}{2}(4-x)^{-\frac{1}{2}}(-1) = -\frac{1}{2}(4-x)^{-\frac{1}{2}}$

$f''(x) = \frac{1}{4}(4-x)^{-\frac{3}{2}}(-1) = -\frac{1}{4}(4-x)^{-\frac{3}{2}}$　it's undefined #

$f(0)=0$　$f(2)=\frac{8}{3}$　$f(3)=0$

$f'''(x) = \frac{3}{8}(4-x)^{-\frac{5}{2}}(-1) = -\frac{3}{8}(4-x)^{-\frac{5}{2}} \Rightarrow f'''(5) = -\frac{3}{8\sqrt{(-1)^5}}$

$\Rightarrow (2,\frac{8}{3})$ is absolute max. #　min = ______

$f'''(-5) = -\frac{3}{8} \cdot 9^{-\frac{5}{2}} = -\frac{1}{648}$

4(a) $g(x) = x\sqrt{x+3} = x(x+3)^{\frac{1}{2}}$ (note that $x \geq -3$)

└ end point $(-3,0)$

① $g'(x) = (x+3)^{\frac{1}{2}} + \frac{1}{2}x(x+3)^{\boxed{-\frac{1}{2}}}$

$= [(x+3) + \frac{1}{2}x](x+3)^{\boxed{-\frac{1}{2}}}$

$= (\frac{3}{2}x+3)(x+3)^{-\frac{1}{2}}$

⇒ Critical point : $(-2,-2)$ ($f'(-2) = 0$)

* Singular point : $(-3, 0)$ ($f'(-3)$ doesn't exist)

② $g''(x) = \frac{3}{2}(x+3)^{-\frac{1}{2}} + \boxed{(-\frac{1}{2})}(\frac{3}{2}x+3)(x+3)^{-\frac{3}{2}}$

$= [\frac{3}{2}(x+3) - (\frac{3}{4}x+\frac{3}{2})](x+3)^{-\frac{3}{2}}$

$= (\frac{3}{4}x+3)(x+3)^{-\frac{3}{2}}$

⇒ $g''(x) = 0$ iff $x = -4$,

but * $g(x)$ is not defined at $x = -4$

⇒ No inflection point.

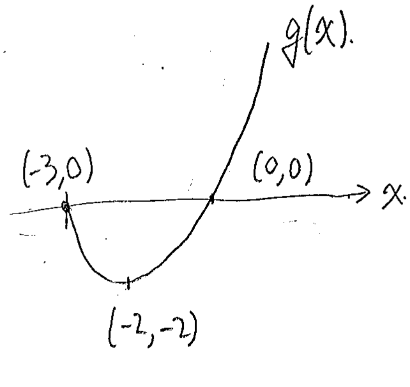
③ Check relative extrema :

* ∵ $g''(-2) > 0$

$(-3,0)$ is end point with $g'(x) < 0$ for $x \in (-3,-2)$

∴ relative minimum : $g(-2) = -2$

relative maximum : $g(-3) = 0$



5, $f'(x) = -x^2+2x-1$

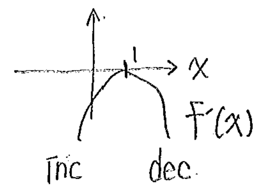
$\Rightarrow f''(x) = -2x+2 \quad \Rightarrow \begin{cases} f''(x) > 0 & \text{if } x \in (-\infty, 1) \\ f''(x) < 0 & x \in (1, \infty) \end{cases}$

(a) From $f''(x)$, we know $f'(x)$ is

Increasing on $(-\infty, 1)$

decreasing on $(1, \infty)$

(or $f'(x) = -(x-1)^2$

$\Rightarrow$ 

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(b) From $f''(x)$, we know $f(x)$ is

concave up on $(-\infty, 1)$

concave down on $(1, \infty)$

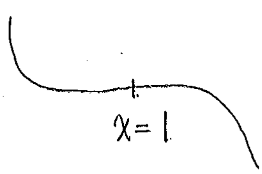
(c) $f''(x) = 0$ iff $x = 1$.

$\Rightarrow$ ① f has inflection point at $x=1$

② However, f is decreasing on $\mathbb{R} \setminus \{1\}$ $\left(\because f'(x) < 0 \ \forall x \neq 1 \right)$

Thus, there is no relative extrema.

< Sketch >



< Note >

You may not write that

$f(x) = -\frac{1}{3}x^3 + x^2 - x$

(Check that the following has the same first derivative

$g(x) = -\frac{1}{3}x^3 + x^2 - x + 1$

$h(x) = -\frac{1}{3}x^3 + x^2 - x + 5$)

6. $f(x) = \frac{4}{3}x\sqrt{3-x} = \frac{4}{3}x(3-x)^{\frac{1}{2}}$

$\Rightarrow f'(x) = \frac{4}{3}(3-x)^{\frac{1}{2}} + \left(\frac{1}{2}\right)\cdot\boxed{(-1)}\cdot\frac{4}{3}x(3-x)^{-\frac{1}{2}}$

$= \left[\frac{4}{3}(3-x) - \frac{2}{3}x\right](3-x)^{-\frac{1}{2}}$

$= (-2x+4)(3-x)^{-\frac{1}{2}}$

Check Critical point : $(2, \frac{8}{3})$

* End point : $(0,0)$, $(3,0)$

$\Rightarrow$ absolute max : $(2, \frac{8}{3})$

absolute min : $(0,0)$, $(3,0)$

Sketch :