商用微積分期中考　　　　　　　　　　　　　　　　教師：曾玉玲 (Yes)
日期：　　　姓名：Yes.　　　學號：　　　系別：

1. (8 points) Show that if a function f is differentiable at $x = c$, then it is continuous at $x = c$.

$f(x) = \frac{f(x)-f(c)}{x-c} \cdot (x-c) + f(c)$

$\Rightarrow \lim_{x\to c} f(x) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c} \cdot \lim_{x\to c}(x-c) + f(c)$

$= f'(c)\cdot 0 + f(c)$, ∵ f is diff. at $x=c$

$= f(c)$

i.e. f is conti. at $x=c$.

2. (24 points) Find the indicated limit or show it does not exist. If the limiting value is infinite, indicate whether it is ∞ or $-\infty$.

(a) $\lim_{x\to 0}(e^{-5x}+3x)^{1/x}$

$= e^{\lim_{x\to0}\frac{\ln(e^{-5x}+3x)}{x}}$

$= e^{L} = e^{-2}$

w/

$L = \lim_{x\to0} \frac{\ln(e^{-5x}+3x)}{x}$ $\frac{\to \text{"}0\text{"}}{\to 0}$

$= \lim_{x\to0} \frac{\frac{-5e^{-5x}+3}{e^{-5x}+3x}}{1}$

$= \lim_{x\to0} \frac{-5e^{-5x}+3}{e^{-5x}+3x}$ $\frac{\to -5+3 = -2}{\to 1}$

$= -2$

(b) $\lim_{x\to 0}\frac{\sin(\sqrt{2}x)}{\sin(8x)}$, $\frac{\to \text{"}0\text{"}}{\to 0}$

$= \lim_{x\to0} \frac{\sqrt{2}\cos(\sqrt{2}x)}{8\cos(8x)}$

$= \frac{\sqrt{2}}{8}\cdot\frac{\cos(0)}{\cos(0)}$

$= \frac{\sqrt{2}}{8}$

(c) $\lim_{t\to\infty} t^5 e^{-3t}$

$= \lim_{t\to\infty} \frac{t^5}{e^{3t}}$ $\frac{\to \text{"}\infty\text{"}}{\to \infty}$

$= \lim_{t\to\infty} \frac{5\cdot t^4}{3e^{3t}}$ $\frac{\to \text{"}\infty\text{"}}{\to \infty}$

$= \cdots$

$= \lim_{t\to\infty} \frac{5\cdot4\cdot3\cdot2\cdot1}{3^5\cdot e^{3t}}$ $\frac{\to 5!}{\to \infty}$

$= 0$

3. (8 points) Find the equation of the tangent line to the curve of $\frac{d}{dx}\boxed{2x+xy-2=\ln(x^3+y^2)}$ at the point $(1,0)$.

$\Rightarrow 2 + y + x\cdot\frac{dy}{dx} = \frac{1}{x^3+y^2}\left(3x^2 + 2y\cdot\frac{dy}{dx}\right)$

代 $x=1$, $y=0$

$2 + \frac{dy}{dx} = 3 \Rightarrow \frac{dy}{dx} = 1$.

∴ Tangent line is $y-0 = 1\cdot(x-1)$

4. (8 points) Find the equation of the tangent line to the graph of $f(x) = x - \ln(\sqrt{x})$ at the point where $x = 1$.

$= x - \frac{1}{2}\ln x$

$\Rightarrow f'(x) = 1 - \frac{1}{2x}$.　代 $x=1$　$f'(1) = \frac{1}{2}$.

$f(1) = 1 - \ln\sqrt{1} = 1$.

∴ Tangent line is $y-1 = \frac{1}{2}\cdot(x-1)$

5. (16 points) Find the absolute maximum and absolute minimum (if any) of

(a) $f(x) = \ln(4x - x^2)$ for $1 \le x \le 3$.

(b) $h(t) = (e^{-t} + e^{t})^3$ for $-1 \le t \le 3$.

(a) $f'(x) = \dfrac{4-2x}{4x-x^2} = 0 \Rightarrow x = 2.$

$f(1) = \ln(3)$. abs min.

$f(2) = \ln(4)$. abs. max.

$f(3) = \ln(3)$. abs. min.

(b) $h'(t) = 3(e^{-t}+e^{t})^2(e^{t}-e^{-t}) = 0 \Rightarrow t=0$

$h(-1) = (e + e^{-1})^3$

$h(0) = (e^0 + e^0)^3 = 8$ = abs. min

$h(3) = (e^{-3}+e^{3})^3$ = abs. max.

7. (8 points) Find the second derivative,that is $f''(x)$, of $f(x) = e^{x^2+1} + 2e^{-3x}$.

$f'(x) = 2x e^{x^2+1} - 6e^{-3x}$

$\Rightarrow f''(x) = 2e^{x^2+1} + 2x\, e^{x^2+1} \cdot (2x) - 6e^{-3x} \cdot (-3)$

$= (4x^2+2)e^{x^2+1} + 18e^{-3x}$

6. (40 points) Find the derivative $\frac{dy}{dx}$ or $f'(x)$ where

(a) $e^{6x-x^3}\log_2 y = 8y\,\sin(2x) + y\,\ln((3x^2+1)^2)$ $= \frac{1}{\ln 2}\ln y$

(b) $f(x) = x^x\, 9^{x^3}$

(c) $f(x) = 7\left[\tan\,(\ln\,\sqrt{2x}\,)\right]^3$

(d) $y = \dfrac{(4x^2e^{-5x})^4(2x^5-6x+2)^3}{[1+\cos(4x^2-2x)+x^9]^{3.2}}$

(e) $f(x) = \arcsin(x) =$ the inverse function of $\sin(x)$

(a) $\frac{d}{dx}(a) \Rightarrow e^{6x-x^3}\cdot(6-3x^2)\cdot\log_2 y + e^{6x-x^3}\cdot\frac{1}{\ln 2}\,\frac{1}{y}\cdot\frac{dy}{dx}$

$= 8\sin(2x)\cdot\frac{dy}{dx} + 16y\cos(2x) + 2\ln(3x^2+1)\cdot\frac{dy}{dx} + 2y\cdot\frac{6x}{3x^2+1}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{e^{6x-x^3}\cdot(6-3x^2)\log_2 y - 16y\cos(2x) - 2y\frac{6x}{3x^2+1}}{8\sin(2x) + 2\ln(3x^2+1) - e^{6x-x^3}\frac{1}{\ln 2}\frac{1}{y}}$

(b) $y = f(x)$, $\ln y = \ln x^x\cdot 9^{x^3} = x\ln x + x^3\ln 9$

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \ln x + x\cdot\frac{1}{x} + 3x^2\cdot\ln 9 = \ln x + 1 + 3x^2\ln 9$

$\Rightarrow f'(x) = \frac{dy}{dx} = y\cdot(\ln x + 1 + 3x^2\ln 9) = x^x\cdot 9^{x^3}(\ln x + 1 + 3x^2\ln 9)$

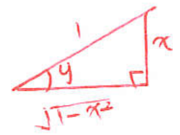
(c) $f'(x) = 3\cdot 7\left[\tan(\ln\sqrt{2x})\right]^2\cdot\sec^2(\ln\sqrt{2x})\cdot\frac{1}{\sqrt{2x}}\cdot\frac{1}{2}(2x)^{-\frac{1}{2}}\cdot 2$

(d) $\ln y = 4\ln(4x^2) + 4(-5x) + 3\ln(2x^5-6x+2) - 3.2\ln[1+\cos(4x^2-2x)+x^9]$

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = 4\cdot\frac{8x}{4x^2} - 20 + 3\cdot\frac{10x-6}{2x^5-6x+2} - 3.2\cdot\left(\frac{-\sin(4x^2-2x)(8x-2)+9x^8}{1+\cos(4x^2-2x)+x^9}\right) = A$

$\frac{dy}{dx} = y\cdot A.$

(e) $y = f(x) = \arcsin(x) = \sin^{-1}(x)$ i.e. $y = \sin^{-1}(x)$



$\Rightarrow \sin y = x$

$\Rightarrow \cos y\cdot\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$

$= \frac{1}{\sqrt{1-x^2}}$ $-1 < x < 1$