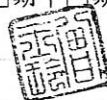


榮譽第一

國立東華大學
應用數學系

學年度第 學期

考試科目: Cal BM. Quiz 1 期中 期末考試試卷



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|--|---|
| <p>#1. $\sqrt{x-9}$ is defined and $\neq 0$ if $x > 9$. $\Rightarrow f(x) = \frac{x}{\sqrt{x-9}}$ has domain $x > 9$</p> | <p>(a) $\lim_{x \rightarrow 0^-} \frac{x+1}{x} \rightarrow 1 \neq 0, > 0$ $\rightarrow 0 \quad x < 0$ $= -\infty$</p> |
| <p>Now. $x \in \mathbb{R} \quad \sqrt{x-9} > 0, \forall x > 9$ $\Rightarrow f(x) = \frac{x}{\sqrt{x-9}} = \sqrt{x-9} + \frac{9}{\sqrt{x-9}} = (\sqrt{x-9} - \frac{3}{\sqrt{x-9}})^2 + 6$ $\Rightarrow 6 = f(18), \forall x > 9 \Rightarrow f \text{ has range } [6, \infty)$</p> | <p>#5. $f(x) = \frac{x-1}{x^2-4x+3}$ is a rational ft. hence conti. whenever it is defined</p> |
| <p>#2. $f(x) = x^c \Rightarrow f(1) = 1 = f(-1) \quad \therefore$ Not 1-1</p> | <p>i.e. $x^2-4x+3 = (x-3)(x-1) \neq 0$ \Rightarrow For all $x \neq 1$ or 3</p> |
| <p>#3. $f(x) = \sqrt{x^2-4}, x \geq 2$ Let $y = \sqrt{x^2-4}, x \geq 2$, hence $y \geq 0$ $\Rightarrow x = \sqrt{y^2+4} \quad \forall x \geq 2$ $\Rightarrow f^{-1}(x) = \sqrt{x^2+4}, x \geq 0$</p> | <p>$f(x) = \frac{x-1}{x^2-4x+3}$ is conti. $\therefore 1, 3 \in [0, 4] \quad \therefore f$ has two disconti. at $x=1, 3$ on $[0, 4]$</p> |
| <p>#4 (a) $\lim_{x \rightarrow -1} \frac{2x^2-x-3}{x+1} \xrightarrow{-1} \lim_{x \rightarrow -1} \frac{(2x-3)(x+1)}{x+1}$ \therefore polynomial $\quad \therefore x \neq -1$ $= \lim_{x \rightarrow -1} (2x-3) = -2-3 = -5$</p> | <p>Note $f(x) = \frac{x-1}{(x-1)(x-3)}, x \neq 1, 3$ $= \frac{1}{x-3}$</p> |
| <p>(b) $\lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - 4(x+\Delta x) + 2 - (x^2 - 4x + 2)}{\Delta x}$ $= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2 - 4\Delta x}{\Delta x}$ $= \lim_{\Delta x \rightarrow 0} (2x + \Delta x - 4) = 2x - 4$</p> | <p>hence $x=1$ is a removable disconti. of f. by re-defining $f(1) = -\frac{1}{2}$. but $x=3$ is not removable $\because \lim_{x \rightarrow 3} f(x)$ D.N.E.</p> |
| <p>(c) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3} \xrightarrow{3} \frac{0}{0}$ $= \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$ $= \lim_{x \rightarrow 3} \frac{x+1-2}{(x-3)(\sqrt{x+1} + 2)}$ $= \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1} + 2} = \frac{1}{4}$</p> | <p>#6. To be conti. at $x=-1$ $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (ax+b) = -a+b \Rightarrow -a+b = 2$</p> |
| <p>(d) $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} \xrightarrow{0} \frac{0}{0}$ $= \lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} \cdot \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}}$ $= \lim_{x \rightarrow 0} \frac{x+2-\sqrt{2}^2}{x(\sqrt{x+2} + \sqrt{2})} = \frac{1}{2\sqrt{2}}$</p> | <p>$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} 2 = 2$ conti. at $x=3$</p> |
| <p>$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} -2 = -2 \Rightarrow 3a+b = -2$ $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (ax+b) = 3a+b$</p> | <p>$\Rightarrow a = -1, b = 1$</p> |
| <p>For $x < -1, f(x) = 2, -1 < x < 3, f(x) = -x+1$ and $x > 3, f(x) = -2$ are all polynomials hence conti.</p> | <p>Therefore $a = -1, b = 1 \Rightarrow f(x)$ is conti. on \mathbb{R}.</p> |