榮譽第一

國立東華大學
應用數學系______學年度第___學期　考試科目：Cal BM. Quiz 1 □期中□期末考試試卷

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#1. $\because \sqrt{x-9}$ is defined and $\neq 0$ if $x>9$

$\Rightarrow f(x)=\frac{x}{\sqrt{x-9}}$ has domain $x>9$

Now. $x\in R$ $\sqrt{x-9}>0$ if $x>9$

$\Rightarrow f(x)=\frac{x}{\sqrt{x-9}}=\sqrt{x-9}+\frac{9}{\sqrt{x-9}}=(\sqrt[4]{x-9}-\frac{3}{\sqrt[4]{x-9}})^2+6$

$\geq 6=f(18)$, $\forall x>9 \Rightarrow f$'s range is $[6,\infty)$

#2 $f(x)=x^4 \Rightarrow f(1)=1=f(-1)$ $\therefore$ Not 1-1

#3. $f(x)=\sqrt{x^2-4}$, $x\geq 2$

Let $y=\sqrt{x^2-4}$, $x\geq 2$, hence $y\geq 0$

$\Rightarrow x=\sqrt{y^2+4}$ $\because x\geq 2$

$\Rightarrow f^{-1}(x)=\sqrt{x^2+4}$, $x\geq 0$.

#4 (a) $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}=\lim_{x\to -1}\frac{(2x-3)\cancel{(x+1)}}{\cancel{x+1}}$ $\because x\neq -1$

$=\lim_{x\to -1}(2x-3)\overset{\because \text{polynomial}}{=}-2-3=-5.$

(b) $\lim_{\Delta t\to 0}\frac{(t+\Delta t)^2-4(t+\Delta t)+2-(t^2-4t+2)}{\Delta t}$

$=\lim_{\Delta t\to 0}\frac{2t\Delta t+(\Delta t)^2-4\Delta t}{\Delta t}$

$=\lim_{\Delta t\to 0}(2t+\Delta t-4)=2t-4.$

(c) $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$

$=\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$=\lim_{x\to 3}\frac{x+1-2^2}{(x-3)(\sqrt{x+1}+2)}$

$=\lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}=\frac{1}{4}$

(d) $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

$=\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$=\lim_{x\to 0}\frac{\cancel{x}+\cancel{2}-\sqrt{2}^2}{\cancel{x}(\sqrt{x+2}+\sqrt{2})}=\frac{1}{2\sqrt{2}}$

(e) $\lim_{x\to 0^-}\frac{x+1}{x}$ $\begin{matrix}\to 1\neq 0, >0\\ \to 0 \quad x<0\end{matrix}$

$=-\infty.$

#5. $f(x)=\frac{x-1}{x^2-4x+3}$ is a rational fc.

hence conti. whenever it is defined

i.e. $x^2-4x+3=(x-3)(x-1)\neq 0$

$\Rightarrow$ For all $x\neq 1$ or 3

$f(x)=\frac{x-1}{x^2-4x+3}$ is conti.

$\because 1,3\in[0,4]$. $\therefore f$ has two

disconti. at $x=1,3$ on $[0,4]$

Note $f(x)=\frac{x-1}{(x-1)(x-3)}$, $x\neq 1,3$

$=\frac{1}{x-3}$

hence $x=1$ is a removable disconti

of f. by re-defining $f(1)=-\frac{1}{2}$. but

$x=3$ is not removable $\because \lim_{x\to 3}f(x)$ D.N.E.

#6. To be conti. at $x=-1$

$\lim_{x\to -1^+}f(x)=\lim_{x\to -1^+}(ax+b)=-a+b$ $\Rightarrow -a+b=2$ —①

$\lim_{x\to -1^-}f(x)=\lim_{x\to -1^-}2=2$

conti. at $x=3$

$\lim_{x\to 3^+}f(x)=\lim_{x\to 3^+}-2=-2$ $\Rightarrow 3a+b=-2$ —②

$\lim_{x\to 3^-}f(x)=\lim_{x\to 3^-}(ax+b)=3a+b$

①② $\Rightarrow a=-1,\ b=1.$

For $x<-1$, $f(x)=2$, $-1<x<3$, $f(x)=-x+1$

and $x>3$, $f(x)=-2$ are all polynomials

hence conti.

Therefore $a=-1,\ b=1 \Rightarrow f(x)$ is conti

on R.