榮譽第一

Sol. to Midterm Cal. BM 2017 11 29

國立東華大學 應用數學系 ____學年度第___學期 考試科目：_____ □期中□期末考試試卷

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#1. see sol. to Quiz 1. #2. see sol. to Quiz 2

#3.

(a) $\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$ "1^∞"

$= \lim_{x\to 0} \exp\left(\frac{1}{x}\ln(e^x+x)\right)$

$= \exp\left(\lim_{x\to 0}\frac{\ln(e^x+x)}{x}\right)$ ∵ exp. is cont'l.

$= \exp(2) = \boxed{e^2}$

∵ $\lim_{x\to 0}\frac{\ln(e^x+x)}{x}$ "$\frac{0}{0}$"

$= \lim_{x\to 0}\frac{\frac{e^x+1}{e^x+x}}{1} = \lim_{x\to 0}\frac{e^x+1}{e^x+x} = 2$

(b) $\lim_{x\to 0}\frac{\sin(2x)}{\sin(5x)}$ "$\frac{0}{0}$"

$= \lim_{x\to 0}\frac{2\cos(2x)}{5\cos(5x)} = \boxed{\frac{2}{5}}$

(c) $\lim_{x\to\infty} x^4 e^{-5x} = \lim_{x\to\infty}\frac{x^4}{e^{5x}}$ "$\frac{\infty}{\infty}$"

$= \lim_{x\to\infty}\frac{4x^3}{5e^{5x}}$ "$\frac{\infty}{\infty}$"

$= \lim_{x\to\infty}\frac{12\cdot x^2}{25e^{5x}}$ "$\frac{\infty}{\infty}$"

$= \lim_{x\to\infty}\frac{24\cdot x}{125e^{5x}}$ "$\frac{\infty}{\infty}$"

$= \lim_{x\to\infty}\frac{24}{625e^{5x}} = \boxed{0}$

#4. $x+y-1 = \ln(x^2+y^2)$

$\xrightarrow{\frac{d}{dx}}$ $1+\frac{dy}{dx} = \frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(1-\frac{2y}{x^2+y^2}\right) = \frac{2x}{x^2+y^2}-1$

$\Rightarrow \frac{dy}{dx} = \left(\frac{2x}{x^2+y^2}-1\right)\Big/\left(1-\frac{2y}{x^2+y^2}\right)$

at $(1,0) = (2-1)/1 = 1$

⇒ Tangent line at $(1,0)$

$y-0 = 1\cdot(x-1)$ i.e. $\boxed{y=x-1}$

#5. (a) $f(t) = 3t^5-5t^3$ cont'l. on $[-2,0]$

$\Rightarrow f'(t) = 15t^4-15t^2 = 15t^2(t^2-1)$

$= 0 \Rightarrow t = 0, \pm 1$: critical #s

but. $1 \notin [-2,0]$

$\Rightarrow f(0)=0$ $f(-1)=2$ (largest) $f(-2)=-56$ (smallest)

∴ $f(-1)=2$, abs. max

$f(-2)=-56$: abs. min'. of f on $[-2,0]$.

(b) $h(t) = (e^{-t}+e^t)^5$ on $[-1,1]$

$\Rightarrow h'(t) = 5(e^{-t}+e^t)^4(e^t-e^{-t}) = 0$

$\Leftrightarrow e^t = e^{-t} \Leftrightarrow t = 0$: critical #

$\Rightarrow h(0) = 2^5 = 32$: abs. min.

$h(-1) = (e+e^{-1})^5$ $(> 2^5)$

$h(1) = (e+e^{-1})^5$: abs. max

#6.

(a) $ye^{2x-x^3} = 5x+y^2\ln(x^2+1)$

$= 5x+4y^2\ln(x^2+1)$

$\xrightarrow{\frac{d}{dx}}$ $ye^{2x-x^3}\cdot(2-3x^2)+e^{2x-x^3}\cdot\frac{dy}{dx}$

$= 5+8y\ln(x^2+1)\cdot\frac{dy}{dx}+4y^2\frac{2x}{x^2+1}$

$\Rightarrow \frac{dy}{dx} = \frac{5+\frac{8y^2x}{x^2+1}-ye^{2x-x^3}(2-3x^2)}{e^{2x-x^3}-8y\ln(x^2+1)}$

(b) $f(x) = x^x 5^{x^2}$

$\ln f(x) = x\ln x + x^2\ln 5$

$\Rightarrow \frac{d}{dx}\ln f(x) = \frac{1}{f(x)}f'(x)$

$= (\ln x+1)+2x\ln 5$

$\Rightarrow f'(x) = x^x 5^{x^2}(\ln x+1+2x\ln 5)$

Yes 2/2

#6

(c) $\ln y = \ln(4x^2+e^{3x})^5 + \ln(e^{-6x}) - \ln(1+\cos(x^2)+x^3)^{\frac{4}{5}}$

$= 5\ln(4x^2+e^{3x}) - 6x - \frac{4}{5}\ln(1+\cos(x^2)+x^3)$

$\xrightarrow{\frac{d}{dx}} \frac{1}{y}\frac{dy}{dx} = 5\cdot\frac{8x+3e^{3x}}{4x^2+e^{3x}} - 6 - \frac{4}{5}\cdot\frac{(-\sin(x^2)2x+3x^2)}{1+\cos(x^2)+x^3}$

$= \frac{40x+15e^{3x}}{4x^2+e^{3x}} - 6 - \frac{12x^2-8x\sin(x^2)}{5(1+\cos(x^2)+x^3)}$

$\triangleq A. \Rightarrow \frac{dy}{dx} = y\cdot A.$

(d) $f(x) = \frac{e^{-x^3}+2x}{\log_8 x} = \frac{(e^{-x^3}+2x)\ln 8}{\ln x}$

$\Rightarrow f'(x) = \frac{\ln x\,(\ln 8\,(3x^2e^{-x^3}+2) - (e^{-x^3}+2x)\ln 8\cdot\frac{1}{x})}{(\ln x)^2}$

$= \frac{3x^2e^{-x^3}+2}{\log_8 x} - \frac{(e^{-x^3}+2x)\ln 8}{x(\ln x)^2}$

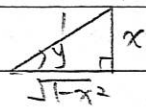
(e) $f(x)$ = inverse ft. of $\sin x$

$\Leftrightarrow \sin(f(x)) = x$

$\xrightarrow{\frac{d}{dx}} \cos(f(x))\,f'(x) = 1$

$\Rightarrow f'(x) = \frac{1}{\cos(f(x))} = \frac{1}{\sqrt{1-x^2}}$ for $-1<x<1$

∵ let $y = f(x)$ $\sin(y) = x$

$\Rightarrow$ 

$\Rightarrow \cos(y) = \sqrt{1-x^2}$

i.e. $\cos(f(x)) = \sqrt{1-x^2}$

Note $f(x) = \sin^{-1}(x) \overset{or}{=} \arcsin(x)$

$\therefore \frac{d}{dx}\sin^{-1}(x) = \frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$