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# Fee versus royalty licensing in a Cournot duopoly model

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## Abstract

This paper finds that royalty licensing can be superior to fixed-fee licensing for the patent-holding firm when the cost-reducing innovation is non-drastic. The reason for this result is that the patent-holding firm enjoys a cost advantage over the licensee under royalty licensing while the two firms compete on equal footing under fixed-fee licensing. © 1998 Elsevier Science S.A.

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## 1. Introduction

Despite the fact that licensing by means of a royalty is more prevalent than licensing by means of a fixed fee,<sup>1</sup> the theoretical literature has overwhelmingly found that licensing by means of a fixed fee is superior to licensing by means of a royalty for both the patent holder and consumers (e.g., Kamien and Tauman, 1986). The model that has been mostly studied in the literature is the licensing of a cost-reducing innovation to existing firms with inferior production technologies by a patent holder which is itself a non-producer.

The present paper studies and compares licensing by means of a fixed fee and licensing by means of a royalty in a homogeneous-good Cournot duopoly where one of the firms has a cost-reducing innovation. The key difference between the present model and models in the existing literature is that here the patent holder is also a producer in the industry while it is an outsider to the industry in existing models. An outside patent holder is only interested in the total licensing revenue while a patent-holding firm is interested in its total income (licensing revenue plus profit). In contrast to the finding in the literature that fixed-fee licensing is generally better than royalty licensing for the patent holder, it is found here that licensing by means of a royalty is superior to licensing by means of a fixed fee from the viewpoint of the patent-holding firm when the innovation is non-drastic. In the case

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<sup>1</sup>According to Rostoker (1984), royalty alone was used 39% of the time, fixed fee alone 13%, and royalty plus fixed fee 46%, among the firms surveyed.

of a drastic innovation, the patent-holding firm becomes a monopoly and licensing does not occur. Similar to the finding in the literature, it is found that licensing by means of a fixed fee is at least as good as licensing by means of a royalty for consumers.

An earlier work is that of Arrow (1962), who studies licensing to a perfectly competitive industry and to a monopoly using a royalty. Kamien and Schwartz (1982) extend Arrow's model to an oligopolistic industry. Kamien and Tauman (1984) examine licensing to a perfectly competitive industry by means of both a fee and a royalty. Kamien and Tauman (1986) compare licensing by means of a fee and licensing by means of a royalty in an homogeneous-good oligopoly with an outside patent holder. Marjit (1990) studies licensing by a Cournot duopolist to its competitor by means of a fee licensing. Kamien (1992) contains an excellent survey of the patent licensing literature.

## 2. Licensing in an homogeneous Cournot duopoly

We consider a Cournot duopoly producing an homogeneous product. The (inverse) market demand function is given by  $p = a - Q$ , where  $p$  denotes price and  $Q$  represents industry output. With the old technology, both firms produce at constant unit production cost  $c$  ( $0 < c < a$ ). The cost-reducing innovation by firm 1 creates a new technology that lowers its unit cost by the amount of  $\varepsilon$ .

A licensing game consists of three stages. In the first stage, the patent-holding duopolist acts as a Stackelberg leader in setting a fixed licensing fee or a royalty rate. In the second stage, the other firm (the would-be licensee) acts as a Stackelberg follower in deciding whether to accept the offer from the patent holder. In the last stage, the two firms engage in a noncooperative competition in quantities. The patent-holding firm sets its fixed licensing fee or royalty rate to maximize its total income which is the sum of the profit from its own production and the licensing revenue.

### 2.1. Cournot equilibrium

We start our analysis by considering the Cournot duopoly where firm 1 has an unspecified constant unit production cost of  $c_1$  and firm 2 has an unspecified constant unit production cost of  $c_2$ . Results of this model will serve as a reference for deriving results for the alternative licensing models studied later. Throughout the paper, subscripts 1 and 2 denote firms 1 and 2, respectively.

Firm 1's profit function is represented by  $\Pi_1 = (a - q_1 - q_2 - c_1)q_1$ . Choosing  $q_1$  to maximize  $\Pi_1$  yields firm 1's quantity-reaction function given as  $q_1 = (a - c_1 - q_2)/2$ . Maximizing firm 2's profit function  $\Pi_2 = (a - q_1 - q_2 - c_2)q_2$  yields firm 2's quantity-reaction function as  $q_2 = (a - c_2 - q_1)/2$ . The intersection of these reaction functions gives the firms' Cournot equilibrium quantities

$$q_1^* = \frac{a - 2c_1 + c_2}{3} \text{ and } q_2^* = \frac{a - 2c_2 + c_1}{3}. \quad (1)$$

The firms' equilibrium profits are

$$\Pi_1^* = \frac{(a - 2c_1 + c_2)^2}{9} \text{ and } \Pi_2^* = \frac{(a - 2c_2 + c_1)^2}{9}. \quad (2)$$

We now go back to consider the model posited at the beginning of Section 2. We first consider the Cournot equilibrium when firm 1 cannot license its innovation to firm 2. In this case, firm 1 will have in use the new technology while firm 2 will have the old technology. Thus, firm 1's unit production cost is  $c - \varepsilon$  and firm 2's is  $c$ .

We will need to consider two separate cases: non-drastic and drastic innovations, depending on the magnitude of the innovation. A *drastic* innovation is one where the innovating firm will become a monopoly if licensing does not occur. In other words, a drastic innovation is one where the monopoly price with the new technology is equal to or less than the unit production cost of the old technology (so that the firm using the old technology is driven out of the market). It is easy to verify that the monopoly price with the new technology is less than or equal to  $c$  if  $\varepsilon \geq a - c$ . Hence, a  $\varepsilon$  that is greater than or equal to  $a - c$  gives a drastic innovation.

### 2.1.1. Non-drastic innovation ( $\varepsilon < a - c$ )

In this case, both firms will produce a positive level of output when licensing does not occur. Substituting  $c_1 = c - \varepsilon$  and  $c_2 = c$  into Eqs. (1) and (2) gives the firms' Cournot equilibrium quantities (the superscript NL denotes 'no licensing')

$$q_1^{\text{NL}} = \frac{a - c + 2\varepsilon}{3} \text{ and } q_2^{\text{NL}} = \frac{a - c - \varepsilon}{3}, \quad (3)$$

and their equilibrium profits

$$\Pi_1^{\text{NL}} = \frac{(a - c + 2\varepsilon)^2}{9} \text{ and } \Pi_2^{\text{NL}} = \frac{(a - c - \varepsilon)^2}{9}. \quad (4)$$

### 2.1.2. Drastic innovation ( $\varepsilon \geq a - c$ )

By Eq. (1), if  $\varepsilon \geq a - c$  then firm 2 will drop out of the market, making firm 1 a monopoly. Solving the monopoly problem yields the firms' quantities

$$q_1^{\text{NL}} = \frac{a - c + \varepsilon}{2} \text{ and } q_2^{\text{NL}} = 0, \quad (5)$$

and their profits

$$\Pi_1^{\text{NL}} = \frac{(a - c + \varepsilon)^2}{4} \text{ and } \Pi_2^{\text{NL}} = 0. \quad (6)$$

## 2.2. Licensing by a fixed fee

We consider next licensing by means of a fixed fee only. Under the fixed-fee licensing method, firm 1 licenses its cost-reducing technology to firm 2 at a fixed fee  $F$  which is invariant of the quantity firm 2 will produce using the new technology. The maximum license fee firm 1 can charge firm 2 is what will make firm 2 indifferent between licensing and not licensing the new technology. In the case that licensing occurs, both firms will produce at constant unit cost  $c - \varepsilon$ .

The third stage equilibrium is that given in Section 2.1 if licensing does not occur in stage two of the game. To find the third stage equilibrium when licensing occurs in the second stage of the game,

substituting  $c_1 = c_2 = c - \varepsilon$  into Eqs. (1) and (2) yields the firms' equilibrium quantities (the superscript F denotes 'fee licensing')

$$q_1^F = q_2^F = \frac{a - c + \varepsilon}{3}, \quad (7)$$

and their equilibrium profits

$$\Pi_1^F = \Pi_2^F = \frac{(a - c + \varepsilon)^2}{9}. \quad (8)$$

With a non-drastic innovation ( $\varepsilon < a - c$ ), Eqs. (4) and (8) imply that the maximum license fee firm 1 can charge is

$$F = \Pi_2^F - \Pi_2^{\text{NL}} = \frac{(a - c + \varepsilon)^2}{9} - \frac{(a - c - \varepsilon)^2}{9} = \frac{4(a - c)\varepsilon}{9}. \quad (9)$$

From (8) and (9), firm 1's total income (profit plus licensing fee) under fixed-fee licensing is

$$\Pi_1^F + F = \frac{(a - c + \varepsilon)^2}{9} + \frac{4(a - c)\varepsilon}{9}. \quad (10)$$

Comparing (4) and (10) we obtain that  $\Pi_1^F + F > \Pi_1^{\text{NL}}$  if and only if  $\varepsilon < 2(a - c)/3$ . Hence, under fixed-fee licensing, firm 1 will license its innovation if  $\varepsilon < 2(a - c)/3$  and it will not if  $2(a - c)/3 \leq \varepsilon < a - c$ .

With a drastic innovation ( $\varepsilon \geq a - c$ ), Eqs. (6) and (8) imply that the maximum license fee firm 1 can charge equals  $F = \Pi_2^F - \Pi_2^{\text{NL}} = (a - c + \varepsilon)^2/9$ . Firm 1's total income is

$$\Pi_1^F + F = \frac{2(a - c + \varepsilon)^2}{9}. \quad (11)$$

From (6) and (11), we obtain that  $\Pi_1^F + F < \Pi_1^{\text{NL}}$ . Hence, under the fixed-fee licensing method firm 1 will not license its new technology and will become a monopoly when the innovation is drastic.

Summarizing the above results, we have the following proposition.

**Proposition 1.** *Under fixed-fee licensing, firm 1 will license its innovation to firm 2 if and only if  $\varepsilon < 2(a - c)/3$ . In particular, firm 1 will become a monopoly when the innovation is drastic.*

### 2.3. Licensing by a royalty

In this subsection, we consider licensing by means of a royalty only. Under a royalty licensing method, firm 1 licenses its new technology to firm 2 at a fixed royalty rate  $r$  and the amount of royalty firm 2 pays will depend on the quantity firm 2 will produce using the new technology. In this case, firm 1's unit production cost is  $c - \varepsilon$ , firm 2's unit production cost is  $c - \varepsilon + r$  if it licenses from firm 1 and  $c$  if it does not license. Note that the maximum royalty rate firm 1 can charge obviously cannot exceed  $\varepsilon$  (i.e.,  $0 \leq r \leq \varepsilon$ ).

The third stage equilibrium is that given in Section 2.1 if licensing does not occur in stage two of

the game. To find the third stage equilibrium when licensing occurs in the second stage of the game, substituting  $c_1 = c - \varepsilon$  and  $c_2 = c - \varepsilon + r$  into Eqs. (1) and (2) yields the firms' equilibrium quantities (the superscript R denotes 'royalty licensing')

$$q_1^R = \frac{a - c + \varepsilon + r}{3} \text{ and } q_2^R = \frac{a - c + \varepsilon - 2r}{3}, \quad (12)$$

and their equilibrium profits

$$\Pi_1^R = \frac{(a - c + \varepsilon + r)^2}{9} \text{ and } \Pi_2^R = \frac{(a - c + \varepsilon - 2r)^2}{9}. \quad (13)$$

From (12) and (13), firm 1's total income is

$$\Pi_1^R + rq_2^R = \frac{(a - c + \varepsilon + r)^2}{9} + \frac{r(a - c + \varepsilon - 2r)}{3}. \quad (14)$$

Choosing  $r$  to maximize firm 1's total income, we obtain that if the innovation is non-drastic (i.e.,  $\varepsilon < a - c$ ) then the optimal  $r = \varepsilon$  and if the innovation is drastic (i.e.,  $\varepsilon \geq a - c$ ) then the optimal  $r = (a - c + \varepsilon)/2$ . With this result, we are ready to examine firm 1's licensing decision by means of a royalty.

If the innovation is non-drastic, substituting  $r = \varepsilon$  into (12)–(14) gives the firms' equilibrium quantities

$$q_1^R = \frac{a - c + 2\varepsilon}{3} \text{ and } q_2^R = \frac{a - c - \varepsilon}{3}, \quad (15)$$

their profits

$$\Pi_1^R = \frac{(a - c + 2\varepsilon)^2}{9} \text{ and } \Pi_2^R = \frac{(a - c - \varepsilon)^2}{9}, \quad (16)$$

and firm 1's total income is

$$\Pi_1^R + rq_2^R = \frac{(a - c + 2\varepsilon)^2}{9} + \frac{\varepsilon(a - c - \varepsilon)}{3}. \quad (17)$$

Comparing (4) and (17), we see that licensing is better than not licensing for firm 1 under a royalty licensing method when the innovation is non-drastic.

If the innovation is drastic, substituting  $r = (a - c + \varepsilon)/2$  into (12) and (13) yields the monopoly outcome as given by Eqs. (5) and (6). Hence, licensing by a royalty is the same as not licensing (firm 2 produces 0 with or without licensing). We will assume that firm 1 will not license its innovation in this case.

Summarizing the results above, we have the following proposition.

**Proposition 2.** *Under royalty licensing, firm 1 will license its innovation to firm 2 if the innovation is non-drastic. In the case of a drastic innovation, firm 1 will become a monopoly.*

#### 2.4. Comparison: fee versus royalty licensing

We evaluate next the superiority of a fixed fee licensing versus a royalty licensing. There are three cases to consider depending on whether licensing occurs under both licensing methods. The first two cases involve a non-drastic innovation and the last case corresponds to a drastic innovation.

Case (1):  $\varepsilon < 2(a - c)/3$ . In this case, firm 1 licenses its innovation to firm 2 under either licensing method. By (10) and (17), the difference between firm 1's total income under fee licensing and that under royalty licensing is

$$\begin{aligned} (\Pi_1^F + F) - (\Pi_1^R + rq_2^R) &= \left[ \frac{(a - c + \varepsilon)^2}{9} + \frac{4(a - c)\varepsilon}{9} \right] - \left[ \frac{(a - c + 2\varepsilon)^2}{9} + \frac{\varepsilon(a - c - \varepsilon)}{3} \right] \\ &= -\frac{(a - c)\varepsilon}{9} < 0. \end{aligned}$$

Hence, for firm 1, licensing by means of a royalty is superior to licensing by means of a fee in this case. From (7) and (15) we have  $q_1^F + q_2^F > q_1^R + q_2^R$ . This implies that licensing by means of a fee is better than licensing by means of a royalty for consumers.

Case (2):  $2(a - c)/3 \leq \varepsilon < a - c$ . In this case, firm 1 licenses its innovation under royalty licensing but does not license under fee licensing. Hence, licensing by means of a royalty must be superior to licensing by means of a fee for firm 1 since firm 1 could choose not to license its innovation under a royalty licensing method. Comparing (3) and (15) we have  $q_1^{NL} + q_2^{NL} = q_1^R + q_2^R$ . Hence, licensing by means of a fee is the same as licensing by means of a royalty for consumers.

Case (3):  $\varepsilon \geq a - c$ . In this case, firm 1 becomes a monopoly and licensing will not occur under either licensing method. Hence, the two licensing methods yield the same outcome for both firms and consumers. In summary, we have the following proposition.

**Proposition 3.** *With either a non-drastic or a drastic innovation, licensing by means of a royalty is at least as good as licensing by means of a fee for the patent-holding firm (firm 1), and licensing by means of a fee is at least as good as licensing by means of a royalty for consumers.*

This proposition is in contrast to the result in the literature which purports that licensing by means of a fixed fee is at least as good as licensing by means of a royalty for both the non-producing patent holder and consumers (e.g., Kamien and Tauman, 1986). Proposition 3 shows that a patent-holding firm always prefers a royalty licensing to a fee licensing as long as its innovation is non-drastic, and it is indifferent between the two licensing methods in the case of a drastic innovation.

The reason that licensing by a royalty can be better than licensing by a fee for the patent-holding firm is the following. The patent holder enjoys a cost advantage under royalty licensing while the two firms compete on equal footing (equal unit variable cost) under fee licensing. Hence, the patent-holding firm reaps the reward of licensing while still enjoys the benefit of its cost advantage under royalty licensing. In an homogeneous-good duopoly model, this advantage only disappears when licensing does not occur under either licensing method.

### 3. Extensions

In this section, we briefly discuss two extensions of the basic duopoly model with linear demand studied in the preceding section: one is concerned with an arbitrary number of firms in the industry, the other is with a general industry demand function. In summary, the basic result from the previous section that royalty licensing may be superior to fee licensing for the patent-holding firm continues to hold in these two extensions. (Proofs for these extensions are available from the author upon request.)

Consider first licensing by a patent-holding firm when there is more than one other firm in the industry. Three licensing policies have typically been studied in the literature: license auctioning, fee licensing, and royalty licensing. Here we find two results regarding these three alternative licensing policies. First, an auction is the same or better than fee licensing for the patent-holding firm. Second, royalty licensing is the same or better than either an auction or fee licensing for the patent-holding firm if the innovation is large or the number of firms in the industry is few, and the opposite happens if otherwise. The first result is consistent with findings in the literature and for the same reason. A licensee is willing to bid more in an auction than the fee it is willing to pay under fee licensing because it competes with one fewer licensee if it does not purchase a license under fee licensing as compared to an auction in which the total number of licensed firms remains unchanged if it does not get a license. The second result is different from existing results in the literature which purport that an auction is the best licensing policy for the patent holder (Kamien, 1992). The reason for the second result above is the following. The patent-holding firm enjoys a cost advantage over all of its competitors under royalty licensing while it competes on an equal footing (i.e., equal unit variable cost) with all licensees under an auction or fee licensing. This advantage of royalty licensing will only be overwhelmed by the other licensing methods when the innovation is small or when the number of firms in the industry is large. In these situations, the efficiency of an auction or fee licensing (in that all licensees produce at the same low level unit variable cost as the patent-holding firm) overrides the cost advantage of royalty licensing by generating a substantially larger amount of licensing revenue than that under royalty licensing.

Now consider licensing by the patent-holding firm in a duopoly with a general demand function. Consideration of a general demand in a licensing model was first given by Kamien et al. (1992). By adopting the same assumption as theirs that the price elasticity of demand is increasing in price, we are able to show that royalty licensing is at least as good as fee licensing for the patent-holding firm. The reason for this is the same as that for the linear duopoly model: the cost advantage enjoyed by the patent-holding firm is never overwhelmed by the efficiency gain for the duopoly industry under fee licensing.

### 4. Conclusion

This paper has studied and compared licensing by means of a fixed fee and licensing by means of a royalty in a simple Cournot duopoly model where one of the firms has a cost-reducing innovation. The innovation of this paper is to treat the patent holder as also a producer in the product market, as opposed to as an independent research unit in the existing literature. In contrast to the findings in the literature, this paper has found that licensing by means of a royalty may be superior to licensing by

means of a fixed fee from the viewpoint of the patent-holding firm.<sup>2</sup> This conclusion is found to hold when there is an arbitrary number of firms in the industry or when a general demand function is considered.

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<sup>2</sup>That a royalty licensing may be superior to a fixed fee licensing is also found by Muto (1993). However, Muto considers a differentiated-goods duopoly with Bertrand competition where the innovator is an independent outsider to the industry.