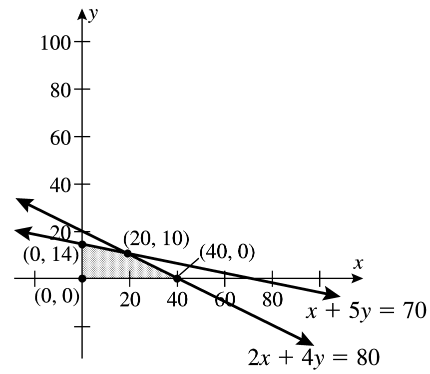
40. Let x = number of dozens of cans of food A, and
y = number of dozens of cans of food B made.

The objective is to maximize the profit, P, from selling the cans of food. P is given by:

$P = 3x + 10y$

The limited supply of food supplements constrain the problem.

$$\begin{cases} 2x + 4y \le 80 & \text{supplement 1 available} \quad (1) \\ x + 5y \le 70 & \text{supplement 2 available} \quad (2) \\ x \ge 0 & (3) \\ y \ge 0 & (4) \end{cases}$$



The constraints are graphed and the set of feasible points is shaded. The corner points are (0, 0), (40, 0), (0, 14), and (20, 10). The point (20, 10) is found by solving the system of equations $\begin{cases} 2x + 4y = 80 & (1) \\ x + 5y = 70 & (2) \end{cases}$. When (1) is subtracted from two times (2), we get $6y = 60$ or $y = 10$. Back-substituting into (2) gives $x = 20$.

The objective function P is evaluated at the corner points as shown in the table.

Corner Point (x, y)	Value of Objective Function $P = 3x + 10y$
(0, 0)	$P = 3(0) + 10(0) = 0$
(40, 0)	$P = 3(40) + 10(0) = 120$
(0, 14)	$P = 3(0) + 10(14) = 140$
(20, 10)	$P = 3(20) + 10(10) = 160$

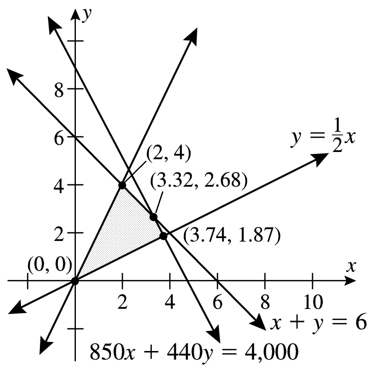
The maximum profit is \$160. It is attained when 20 dozen cans of food A are made and 10 dozen cans of food B are made.

Chapter 3 Project

1. Objective function: $c = 31.4x + 114.74y$

2. Constraints:

$$\begin{cases} x + y \le 6 & \text{total quantity constraint} \quad (1) \\ x \ge \frac{1}{2}y & \text{proportion peanuts} \quad (2) \\ y \ge \frac{1}{2}x & \text{proportion raisins} \quad (3) \\ 850x + 440y \le 4000 & \text{calorie limit constraint} \quad (4) \\ x \ge 0 & \text{non-negativity constraint} \quad (5) \\ y \ge 0 & \text{non-negativity constraint} \quad (6) \end{cases}$$

3. 

The corner points are: (0, 0), (3.74, 1.87), (3.32, 2.68), and (2, 4).

The point (3.32, 2.68) is the solution to the system $\begin{cases} x + \quad y = \quad 6 & (1) \\ 850x + 440y = 4000 & (2) \end{cases}$.

Substituting $6 - x$ for y in (2), gives $410x = 1360$ or $x = 3.32$. Back-substitution into (1) gives $y = 2.68$.

The point (3.74, 1.87) is the solution to the system $\begin{cases} 850x + 440y = 4000 \\ \qquad\quad y = 1/2x \end{cases}$

4. The objective function c is evaluated at the corner points as shown in the table.

Corner Point (x, y)	Value of Objective Function $c = 31.4x + 114.74y$
(0, 0)	$c = 31.4(0) + 114.74(0) = 0$
(3.74, 1.87)	$c = 31.4(3.74) + 114.74(1.87) = 331.9998$
(3.32, 2.68)	$c = 31.4(3.32) + 114.74(2.68) = 411.7512$
(2, 4)	$c = 31.4(2) + 114.74(4) = 521.76$

Using 2 cups of peanuts and 4 cups of raisins maximizes the amount of carbohydrates in the mix at 521.76 grams.

5. The total mix contains 6 cups, so there are $521.76 \div 6 = 86.96$ grams of carbohydrates per cup.

Since each cup of raisins contain 440 calories and each cup of peanuts have 850 calories, the calorie count per cup is $(4 \cdot 440 + 2 \cdot 850) \div 6 = 576.67$ calories per cup of mix.

6. To find the mix that will maximize the protein we use the same corner points, but we change the objective function to $p = 34.57x + 4.67y$. The objective function p is evaluated at each corner points as shown in the table.

Corner Point (x, y)	Value of Objective Function $p = 34.57x + 4.67y$
(0, 0)	$p = 34.57(0) + 4.67(0) = 0$
(3.74, 1.87)	$p = 34.57(3.74) + 4.67(1.87) = 138.025$
(3.32, 2.68)	$p = 34.57(3.32) + 4.67(2.68) = 127.288$
(2, 4)	$p = 34.57(2) + 4.67(4) = 87.82$

Using 3.74 cups of peanuts and 1.87 cups of raisins maximizes the amount of protein in the mix at 138.03 grams.

In a cup of this protein rich mix, there are $138.025 \div (3.74 + 1.87) = 24.6$ grams of protein.

The new mix has $(3.74 \cdot 850 + 1.87 \cdot 440) \div 5.61 = 713.3$ calories per cup.

7. To determine the recipe needed to minimize the fat in the mix, construct a new objective function and evaluate it at the corner points.

Objective function: $f = 72.5x + 0.67y$

Corner Point (x, y)	Value of Objective Function $f = 72.5x + 0.67y$
(0, 0)	$f = 72.5(0) + 0.67(0) = 0$
(3.74, 1.87)	$f = 72.5(3.74) + 0.67(1.87) = 272.403$
(3.32, 2.68)	$f = 72.5(3.32) + 0.67(2.68) = 242.496$
(2, 4)	$f = 72.5(2) + 0.67(4) = 147.68$

Since you intend to eat at least 3 cups of mix, we cannot use the corner point (0, 0). So you would minimize the fat content by making a mix that consists of 2 cups of peanuts and 4 cups of raisins.

8. This newest mix has 521.76 grams of carbohydrates. See problem 4.

9. This low–fat mix has 87.82 grams of protein. See problem 6.

Mathematical Questions from Professional Exams

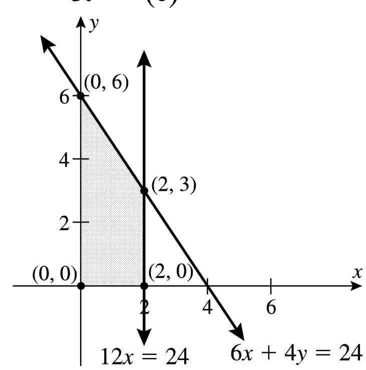
1. (b) **2.** (a) **3.** (c)

4. Linear Programming problem:

Maximize $P = 2X + 1Y$

subject to the constraints

$$\begin{cases} 6x + 4y \le 24 & \text{machine 1 constraint} \\ 12x \le 24 & \text{machine 2 constraint} \\ x \ge 0 & \text{non-negativity constraint} \\ y \ge 0 & \text{non-negativity constraint} \end{cases}$$



The constraints are graphed, and the set of feasible points is shaded.

Corner points: (0, 0), (2, 0), (0, 6), and (2, 3).

Corner Point (x, y)	Value of Objective Function $P = 2x + y$
(0, 0)	$P = 2(0) + 0 = 0$
(2, 0)	$P = 2(2) + 0 = 4$
(0, 6)	$P = 2(0) + 6 = 6$
(2, 3)	$P = 2(2) + 3 = 7$

Maximum profit is \$7, and is attained when **(c)** 2 units of product X and 3 units of product Y are produced.

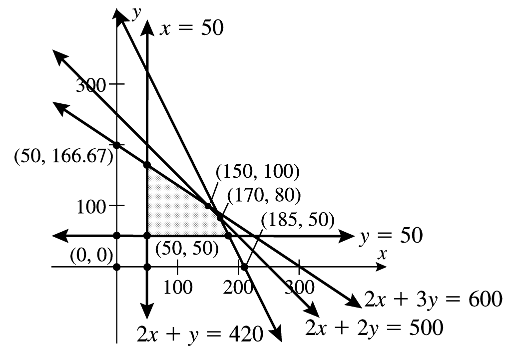
5. Profit is the difference between revenue and cost, so profit for product $Q = 20 - 12 = 8$, and profit for product $P = 17 - 13 = 4$. The objective function is **(d).**

6. Linear Programming problem:

Maximize $P = 4X + 2Y$

subject to the constraints

$$\begin{cases} 2x + \ \ y \le 420 & \text{machining constraint} \\ 2x + 2y \le 500 & \text{assembling constraint} \\ 2x + 3y \le 600 & \text{finishing constraint} \\ x \ge 50 & \\ y \ge 50 & \end{cases}$$



The constraints are graphed, and the set of feasible points is shaded.

Corner points: (50, 50), (185, 50), (170, 80), (150, 100) and (50, 166.67).

Corner Point (x, y)	Value of Objective Function $P = 4x + 2y$
(50, 50)	$P = 4(50) + 2(50) = 300$
(185, 50)	$P = 4(185) + 2(50) = 840$
(170, 80)	$P = 4(170) + 2(80) = 840$
(150, 100)	$P = 4(150) + 2(100) = 800$
(50, 166.67)	$P = 4(50) + 2(166.67) = 533.34$

(c) producing 170 of product X and 80 of product Y maximizes profit.

7. **(c)**

8. Evaluate the objective function at the corner points.

Corner Point (x, y)	Value of Objective Function $P = 3x + 4y$
(0, 20)	$P = 3(0) + 4(20) = 80$
(30, 0)	$P = 3(30) + 4(0) = 90$
(20, 10)	$P = 3(20) + 4(10) = 100$

(b) producing 20 units of product A and 10 units of product B maximizes profit.

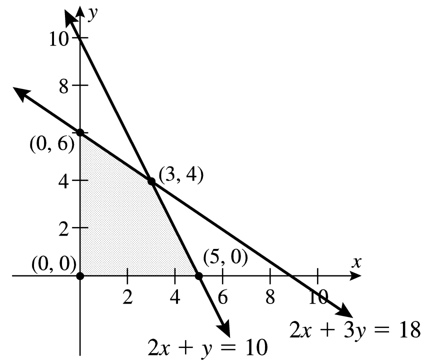
9. **(b)**

10. Linear Programming problem:

Maximize $P = 4x + 2y$

subject to the constraints

$$\begin{cases} 2x + 3y \le 18 & \text{machine 1 constraint} \\ 2x + y \le 10 & \text{machine 2 constraint} \\ x \ge 0 & \text{non-negativity constraint} \\ y \ge 0 & \text{non-negativity constraint} \end{cases}$$



The constraints are graphed, and the set of feasible points is shaded.
Corner points: (0, 0), (5, 0), (0, 6), and (3, 4).

Corner Point (x, y)	Value of Objective Function $P = 4x + 2y$
(0, 0)	$P = 4(0) + 2(0) = 0$
(5, 0)	$P = 4(5) + 2(0) = 20$
(0, 6)	$P = 4(0) + 2(6) = 12$
(3, 4)	$P = 4(3) + 2(4) = 20$

(a) maximum profit is $20

11. **(b)**

12. **(e)**

13. **(d)**

14. **(b)**