第二次分組報告題目

Group 1: Inventory Control A department store stocks three brands of toys: A, B, and C. Each unit of brand A occupies 1 square foot of shelf space, each unit of brand B occupies 2 square feet, and each unit of brand C occupies 3 square feet. The store has 120 square feet available for storage. Surveys show that the store should have on hand at least 12 units of brand A and at least 30 units of A and B combined. Each unit of brand A costs the store \$8, each unit of brand B \$6, and each unit of brand C \$10. Minimize the cost to the store.

Let x_1, x_2, and x_3 represent the number of units of toy A, toy B, and toy C, respectively, stocked by the store. The store wants to minimize costs and meet the demand for the toys. They want to

Minimize $C = 8x_1 + 6x_2 + 10x_3$

subject to the conditions

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &\le 120 \\
x_1 &\ge 12 \\
x_1 + x_2 &\ge 30 \\
x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 &\ge 0
\end{aligned}
$$

Since the minimum problem must have all constraints written as greater than or equal to inequalities, we rewrite the first constraint as

$$-x_1 - 2x_2 - 3x_3 \ge -120$$

The matrix (on the left) representing the system and its transpose (on the right) are:

$$
\begin{array}{c}
\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\
\left[\begin{array}{ccc|c}
-1 & -2 & -3 & -120 \\
1 & 0 & 0 & 12 \\
1 & 1 & 0 & 30 \\
8 & 6 & 10 & 0
\end{array}\right]
\end{array}
\qquad
\begin{array}{c}
\begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\
\left[\begin{array}{ccc|c}
-1 & 1 & 1 & 8 \\
-2 & 0 & 1 & 6 \\
-3 & 0 & 0 & 10 \\
-120 & 12 & 30 & 0
\end{array}\right]
\end{array}
$$

The dual of the problem is

Maximize $P = -120y_1 + 12y_2 + 30y_3$

subject to the conditions

$$
\begin{aligned}
-y_1 + y_2 + y_3 &\le 8 \\
-2y_1 + \quad y_3 &\le 6 \\
-3y_1 &\le 10 \\
y_1 \ge 0 \quad y_2 \ge 0 \quad y_3 &\ge 0
\end{aligned}
$$

We set up the initial simplex tableau and solve the maximum problem.

BV	P	y_1	y_2	y_3	s_1	s_2	s_3	RHS
s_1	0	−1	1	1	1	0	0	8
$\rightarrow s_2$	0	−2	0	[1]	0	1	0	6
s_3	0	−3	0	0	0	0	1	10
P	1	120	−12	−30	0	0	0	0

BV	P	y_1	y_2	y_3	s_1	s_2	s_3	RHS
$\rightarrow s_1$	0	−1	[1]	0	1	−1	0	2
y_3	0	−2	0	1	0	1	0	6
s_3	0	−3	0	0	0	0	1	10
P	1	60	−12	0	0	30	0	180

BV	P	y_1	y_2	y_3	s_1	s_2	s_3	RHS
y_2	0	−1	1	0	1	−1	0	2
y_3	0	−2	0	1	0	1	0	6
s_3	0	−3	0	0	0	0	1	10
P	1	72	0	0	12	18	0	204

The store can obtain a minimum cost of \$204 by stocking 12 units of toy A, 18 units of toy B, and no units of toy C.

Group 2: Production Schedule A company owns two mines. Mine A produces 1 ton of high-grade ore, 3 tons of medium-grade ore, and 5 tons of low-grade ore each day. Mine B produces 2 tons of each grade ore per day. The company needs at least 80 tons of high-grade ore, at least 160 tons of medium-grade ore, and at least 200 tons of low-grade ore. How many days should each mine be operated to minimize costs if it costs \$2000 per day to operate each mine?

Let x_1 and x_2 represent the number of days mine A and mine B, respectively are operated. The company wants to save costs of operation while meeting demand.

Minimize $C = 2000x_1 + 2000x_2$

subject to the conditions

$$
\begin{aligned}
x_1 + 2x_2 &\geq 80 \\
3x_1 + 2x_2 &\geq 160 \\
5x_1 + 2x_2 &\geq 200 \\
x_1 \geq 0 \quad x_2 &\geq 0
\end{aligned}
$$

The matrix (on the left) representing the system and its transpose (on the right) are:

$$
\begin{array}{cc|c} x_1 & x_2 & \\ 1 & 2 & 80 \\ 3 & 2 & 160 \\ 5 & 2 & 200 \\ \hline 2000 & 2000 & 0 \end{array}
\qquad
\begin{array}{ccc|c} y_1 & y_2 & y_3 & \\ 1 & 3 & 5 & 2000 \\ 2 & 2 & 2 & 2000 \\ \hline 80 & 160 & 200 & 0 \end{array}
$$

The dual of the problem is

Maximize $P = 80y_1 + 160y_2 + 200y_3$

subject to the conditions

$$
\begin{aligned}
y_1 + 3y_2 + 5y_3 &\leq 2000 \\
2y_1 + 2y_2 + 2y_3 &\leq 2000 \\
y_1 \geq 0 \quad y_2 \geq 0 \quad y_3 &\geq 0
\end{aligned}
$$

We set up the initial simplex tableau and solve the maximum problem.

BV	P	y_1	y_2	y_3	s_1	s_2	RHS
$\rightarrow s_1$	0	1	3	[5]	1	0	2000
s_2	0	2	2	2	0	1	2000
P	1	–80	–160	–200	0	0	0

$\longrightarrow$

BV	P	y_1	y_2	y_3	s_1	s_2	RHS
y_3	0	$\frac{1}{5}$	$\frac{3}{5}$	1	$\frac{1}{5}$	0	400
$\rightarrow s_2$	0	[$\frac{8}{5}$]	$\frac{4}{5}$	0	$-\frac{2}{5}$	1	1200
P	1	–40	–40	0	40	0	80000

$\longrightarrow$

BV	P	y_1	y_2	y_3	s_1	s_2	RHS
y_3	0	0	[$\frac{1}{2}$]	1	$\frac{1}{4}$	$-\frac{1}{8}$	250
$\rightarrow y_1$	0	1	$\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{5}{8}$	750
P	1	0	–20	0	40	0	110000

$\longrightarrow$

BV	P	y_1	y_2	y_3	s_1	s_2	RHS
y_2	0	0	1	2	$\frac{1}{2}$	$-\frac{1}{4}$	500
$\rightarrow y_1$	0	1	–1	0	$-\frac{1}{2}$	$\frac{3}{4}$	500
P	1	0	–20	0	40	0	120000

The solution to the minimum problem is minimal $C = 120{,}000$ when $x_1 = 40$ and $x_2 = 20$. The mining company can minimize its costs while meeting demand if it operates mine A for 40 days and mine B for 20 days. The minimum cost is $120,000.

Group 3: Advertising A local appliance store has decided on an advertising campaign utilizing newspaper and radio. Each dollar spent on newspaper advertising is expected to reach 50 people in the "Under $25,000" and 40 in the "Over $25,000" bracket. Each dollar spent on radio advertising is expected to reach 70 people in the "Under $25,000" and 20 people in the "Over $25,000" bracket. If the store wants to reach at least 100,000 people in the "Under $25,000" and at least 120,000 in the "Over $25,000" bracket, how should it proceed so that the cost of advertising is minimized?

We let x_1 represent the amount spent on newspaper advertising, and x_2 represent the amount spent on radio advertising. The store wants to minimize the cost of advertising, while reaching certain potential customers. They want to

Minimize $\quad C = x_1 + x_2$

subject to the conditions

$$50x_1 + 70x_2 \geq 100{,}000$$
$$40x_1 + 20x_2 \geq 120{,}000$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

We change the problem to a maximization problem and write the constraints as less than or equal to inequalities.

Maximize $P = -C = -x_1 - x_2$

subject to

$$-50x_1 - 70x_2 \leq -100{,}000$$
$$-40x_1 - 20x_2 \leq -120{,}000$$
$$x_1 \geq 0, \; x_2 \geq 0, \; x_3 \geq 0$$

We introduce nonnegative slack variables and set up the initial tableau. Since there are negative entries in the RHS, we use the alternate pivoting strategy.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & \boxed{-50} & -70 & 1 & 0 & -100{,}000 \\ 0 & -40 & -20 & 0 & 1 & -120{,}000 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \end{array}\right] \end{array} \xrightarrow{\text{Alternative Pivoting Strategy}} \begin{array}{c} \text{BV} \\ x_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & \frac{7}{5} & -\frac{1}{50} & 0 & 2{,}000 \\ 0 & 0 & 36 & \boxed{-\frac{4}{5}} & 1 & -40{,}000 \\ \hline 1 & 0 & -\frac{2}{5} & \frac{1}{50} & 0 & -2{,}000 \end{array}\right] \end{array}$$

$$\xrightarrow{\text{Alternative Pivoting Strategy}} \begin{array}{c} \text{BV} \\ x_1 \\ s_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{40} & 3{,}000 \\ 0 & 0 & -45 & 1 & -\frac{5}{4} & 50{,}000 \\ \hline 1 & 0 & \frac{1}{2} & 0 & \frac{1}{40} & -3{,}000 \end{array}\right] \end{array}$$

Maximum $P = -3000$, so minimum $C = 3000$, which is obtained when $x_1 = 3000$ and $x_2 = 0$. The appliance store will spend the least on advertising, \$3000, and reach the intended audience, if they spend all \$3000 on newspaper advertising and nothing on radio advertising.

Group 4: Salespeople at a Trade Show Suppose that a particular computer company has 12 sales representatives based in New York City and 18 sales representatives based in San Francisco. To staff an upcoming trade show in Dallas and another trade show in Chicago, this company wants to send at least 5 of these representatives to Dallas and at least 5 of these sales representatives to Chicago. According to Travelocity, for March 2003, the lowest round-trip airfares (including taxes and fees) for a one-week stay, with tickets purchased two weeks in advance, are as follows: San Francisco to Dallas: \$340; San Francisco to Chicago: \$180; New York (LaGuardia) to Dallas: \$280; New York (LaGuardia) to Chicago: \$180. For these two trade shows, this computer company has a total airfare budget of \$4830. Formulate and solve a linear programming problem to determine how many sales representatives this company should send from New York City to each trade show and how many sales representatives should be sent from San Francisco to each trade show in order to satisfy the staffing requirements and stay within the company's airfare budget while maximizing the total number of sales representatives that are sent.

Let x_1 denote the number of representatives sent from New York to Dallas,
x_2 denote the number of representatives sent from New York to Chicago,
x_3 denote the number of representatives sent from San Francisco to Dallas,
x_4 denote the number of representatives sent from San Francisco to Chicago.
We want to

Maximize $P = x_1 + x_2 + x_3 + x_4$

subject to the constraints

$$\begin{aligned} x_1 \quad + \quad x_3 \quad &\geq 5 \\ x_2 \quad + \quad x_4 &\geq 5 \\ x_1 + x_2 \quad &\leq 12 \\ x_3 + x_4 &\leq 18 \\ 280x_1 + 180x_2 + 340x_3 + 180x_4 &\leq 4830 \\ x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 &\geq 0 \end{aligned}$$

This problem does not need the simplex method. Since the company wants to maximize the number of people sent to the trade shows, but limit the costs. We need to send 5 representatives from New York to Dallas at a cost of $5 \cdot 280 = \$1400$, leaving \$3430 left to send the other representatives to a trade show. If the company sends them all to Chicago (since it is cheaper), they can send $3430 \div 180 = 19.06$. They can send any combination of 19 people to Chicago.

Group 5: Capital Expenditures In an effort to increase productivity, a corporation decides to purchase a new piece of equipment. Two models are available; both reduce labor costs. Model A costs \$50,000, saves \$12,000 per year in labor costs, and has a useful life of 10 years. Model B costs \$42,000, saves \$10,000 annually in labor costs, and has a useful life of 8 years. If the time value of money is 10% per annum, what piece of equipment provides a better investment?

We first find the equivalent annual cost of each machine.
Machine A: Machine A has an expected life of 10 years. We determine that at 10% interest an annuity for 10 years has a present value

$$V = P\,\frac{1-(1+i)^{-n}}{i} = P\cdot\frac{1-(1+0.10)^{-10}}{0.10} = 6.14457P$$

The equivalent annual cost of machine A is then $\frac{\$50{,}000}{6.14457} = \$8137.27.$

Machine B: The present value of P for 8 years is

$V = P\,\frac{1-(1+i)^{-n}}{i} = P\cdot\frac{1-(1+0.10)^{-8}}{0.10} = 5.33493P$, and the equivalent annual cost of machine B is $\frac{\$42{,}000}{5.33493} = \$7872.65.$

The net annual savings of each machine is

Machine	A	B
Labor Savings	$12,000.00	$10,000.00
Equivalent Annual Cost	8,137.27	7,872.65
Net Savings	$ 3,862.73	$ 2,127.35

Machine A has a greater net savings, so Machine A is preferable.

Group 6: House Mortgage Mr. and Mrs. Ostedt have just purchased an $400,000 home and made a 25% down payment. The balance can be amortized at 10% for 25 years.

(a) What are the monthly payments?

(b) How much interest will be paid?

(c) What is their equity after 5 years?

Mr. and Mrs. Ostedt's new house cost \$400,000. They made a \$100,000 down payment, and are financing \$300,000 at 10% for 25 years.

(a) The monthly payments are found by evaluating $P = V\dfrac{i}{1-(1+i)^{-n}}$. In this problem $V = \$300{,}000$, $n = 12\cdot 25 = 300$ months, and the interest rate per month is $i = \dfrac{0.10}{12}$.

$$P = \$300{,}000\frac{\frac{0.10}{12}}{1-\left(1+\frac{0.10}{12}\right)^{-300}} = \$2726.10$$

The Ostedt's monthly payments are \$2726.10.

(b) The Ostedt's will pay (300)(\$2726.10) – \$300,000 = \$517,830 in interest.

(c) The equity in a house is the sum of the down payment and the amount paid on the loan. After 5 years, the Ostedt's have made 60 payments. The present value of the remaining 240 payments is

$$V = P\frac{1-(1+i)^{-n}}{i} = \$2726.10\frac{1-\left(1+\frac{0.10}{12}\right)^{-240}}{\frac{0.10}{12}} = \$282{,}491.07$$

which indicates that the Ostedt's have paid \$300,000 – \$282,491.07 = \$17,508.93 of the principal. Their equity is then

\$100,000 + \$17,508.93 = \$117,508.93