➤ Applications

We now turn to the important concepts of **break-even** and **profit–loss** analysis, which we will return to a number of times in this book. Any manufacturing company has **costs,** C, and **revenues,** R. The company will have a **loss** if $R < C$, will **break even** if $R = C$, and will have a **profit** if $R > C$. Costs include **fixed costs** such as plant overhead, product design, setup, and promotion; and **variable costs,** which are dependent on the number of items produced at a certain cost per item. In addition, **price–demand** functions, usually established by financial departments using historical data or sampling techniques, play an important part in profit–loss analysis. We will let x, the number of units manufactured and sold, represent the independent variable. Cost functions, revenue functions, profit functions, and price–demand functions are often stated in the following forms, where a, b, m, and n are constants determined from the context of a particular problem:

Cost Function

$$\begin{aligned} C &= (\text{fixed costs}) + (\text{variable costs}) \\ &= a + bx \end{aligned}$$

Price–Demand Function

$$p = m - nx \quad \text{x is the number of items that can be sold at \$p per item.}$$

Revenue Function

$$\begin{aligned} R &= (\text{number of items sold}) \times (\text{price per item}) \\ &= xp = x(m - nx) \end{aligned}$$

Profit Function

$$\begin{aligned} P &= R - C \\ &= x(m - nx) - (a + bx) \end{aligned}$$

Example 7 and Matched Problem 7 explore the relationships among the algebraic definition of a function, the numerical values of the function, and the graphical representation of the function. The interplay among algebraic, numeric, and graphic viewpoints is an important aspect of our treatment of functions and their use. In Example 7, we also see how a function can be used to describe data from the real world, a process that is often referred to as *mathematical modeling.* The material in this example will be returned to in subsequent sections so that we can analyze it in greater detail and from different points of view.

EXAMPLE 7 Price–Demand and Revenue Modeling

Group 1

A manufacturer of a popular automatic camera wholesales the camera to retail outlets throughout the United States. Using statistical methods, the financial department in the company produced the price–demand data in Table 4, where p is the wholesale price per camera at which x million cameras are sold. Notice that as the price goes down, the number sold goes up.

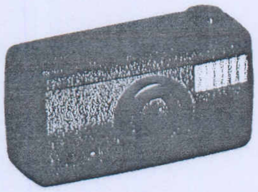

TABLE 4 Price–Demand

x (Millions)	p ($)
2	87
5	68
8	53
12	37

TABLE 5 Revenue

x (Millions)	$R(x)$ (Million $)
1	90
3	
6	
9	
12	
15	

Using special analytical techniques (regression analysis), an analyst arrived at the following price–demand function that models the Table 4 data:

$$p(x) = 94.8 - 5x \qquad 1 \le x \le 15 \qquad (5)$$

(A) Plot the data in Table 4. Then sketch a graph of the price–demand function in the same coordinate system.

(B) What is the company's revenue function for this camera, and what is the domain of this function?

(C) Complete Table 5, computing revenues to the nearest million dollars.

(D) Plot the data in Table 5. Then sketch a graph of the revenue function using these points.

Matched Problem 7

Group 4

The financial department in Example 7, using statistical techniques, produced the data in Table 6, where $C(x)$ is the cost in millions of dollars for manufacturing and selling x million cameras.

Using special analytical techniques (regression analysis), an analyst produced the following cost function to model the data:

$$C(x) = 156 + 19.7x \qquad 1 \le x \le 15 \qquad (6)$$

TABLE 6 Cost Data

x (Millions)	$C(x)$ (Million $)
1	175
5	260
8	305
12	395

(A) Plot the data in Table 6. Then sketch a graph of equation (6) in the same coordinate system.

(B) What is the company's profit function for this camera, and what is its domain?

(C) Complete Table 7, computing profits to the nearest million dollars.

TABLE 7 Profit

x (Millions)	$P(x)$ (Million $)
1	−86
3	
6	
9	
12	
15	

(D) Plot the points from part (C). Then sketch a graph of the profit function through these points.

EXAMPLE 6 **Purchasing** A company that rents small moving trucks wants to purchase 25 trucks with a combined capacity of 28,000 cubic feet. Three different types of trucks are available: a 10-foot truck with a capacity of 350 cubic feet, a 14-foot truck with a capacity of 700 cubic feet, and a 24-foot truck with a capacity of 1,400 cubic feet. How many of each type of truck should the company purchase?

Group 2

Matched Problem 6 A company that rents small moving trucks wants to purchase 16 trucks with a combined capacity of 19,200 cubic feet. Three different types of trucks are available: a cargo van with a capacity of 300 cubic feet, a 15-foot truck with a capacity of 900 cubic feet, and a 24-foot truck with a capacity of 1,500 cubic feet. How many of each type of truck should the company purchase?

Group 5

EXAMPLE 3 **Medicine** We now convert Example 5 from the preceding section into a linear programming problem. A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B, each day (assume that an overdosage of either drug is harmless). Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now, suppose that both M and N contain an undesirable drug D, 3 units per gram in M and 1 unit per gram in N. How many grams of each of substances M and N should be mixed to meet the minimum daily requirements and at the same time minimize the intake of drug D? How many units of the undesirable drug D will be in this mixture?

Group 3

Matched Problem 3 *Agriculture* A chicken farmer can buy a special food mix A at 20¢ per pound and a special food mix B at 40¢ per pound. Each pound of mix A contains 3,000 units of nutrient N_1 and 1,000 units of nutrient N_2; each pound of mix B contains 4,000 units of nutrient N_1 and 4,000 units of nutrient N_2. If the minimum daily requirements for the chickens collectively are 36,000 units of nutrient N_1 and 20,000 units of nutrient N_2, how many pounds of each food mix should be used each day to minimize daily food costs while meeting (or exceeding) the minimum daily nutrient requirements? What is the minimum daily cost? Construct a mathematical model and solve using the geometric method.

Group 6