

the x_1 column.

$$\begin{array}{c}
 \text{BV} \quad P \quad x_1 \quad x_2 \quad s_1 \quad \text{RHS} \\
 \hline
 x_1 \quad \left[\begin{array}{cccc|c}
 0 & 1 & 1 & 0 & 1 \\
 0 & 0 & 0.15 & 1 & 0.05 \\
 1 & 0 & 0.2 & 0 & 0.7
 \end{array} \right] \\
 s_1 \\
 P
 \end{array}$$

The new tableau represents a maximum problem in standard form. Since the objective row has a negative entry, we use the standard pivoting strategy. The pivot column is x_2 . We form the quotients and find the pivot row. It is row s_1 .

Step 5 Pivot

$$\begin{array}{c}
 \text{BV} \quad P \quad x_1 \quad x_2 \quad s_1 \quad \text{RHS} \\
 \hline
 x_1 \quad \left[\begin{array}{cccc|c}
 0 & 1 & 0 & 6.667 & 0.667 \\
 0 & 0 & 1 & 6.667 & 0.333 \\
 1 & 0 & 0 & 1.333 & 0.633
 \end{array} \right] \\
 x_2 \\
 P
 \end{array}$$

This is the final tableau. The maximum $P = -0.633$, so $C = 0.633$ and it is attained $x_1 = 0.667$ and $x_2 = 0.333$. The butcher should combine $\frac{2}{3}$ a pound of ground beef with $\frac{1}{3}$ pound ground pork for a meatloaf mixture of minimum cost of 63¢ per pound.

Chapter 4 Project

- The objective is to maximize the carbohydrates in the trail mix. The objective function is
 Maximize $C = 31.4x_1 + 114.74x_2 + 148.12x_3 + 33.68x_4$
- The objective function is subject to the following constraints

$$\begin{array}{rcl}
 x_1 + & x_2 + & x_3 + & x_4 \leq 10 \\
 x_1 & & & \geq 0.10(x_1 + x_2 + x_3 + x_4) \\
 & x_2 & & \geq 0.10(x_1 + x_2 + x_3 + x_4) \\
 & & x_3 & \geq 0.10(x_1 + x_2 + x_3 + x_4) \\
 & & & x_4 \geq 0.10(x_1 + x_2 + x_3 + x_4) \\
 854x_1 + 435x_2 + 1023.96x_3 + 162.02x_4 \leq 7000 \\
 x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0
 \end{array}$$

which we will rewrite as

$$\begin{array}{rcl}
 x_1 + & x_2 + & x_3 + & x_4 \leq 10 \\
 0.9x_1 - 0.1x_2 - & & 0.1x_3 - & 0.1x_4 \geq 0 \\
 -0.1x_1 + 0.9x_2 - & & 0.1x_3 - & 0.1x_4 \geq 0 \\
 -0.1x_1 - 0.1x_2 + & & 0.9x_3 - & 0.1x_4 \geq 0 \\
 -0.1x_1 - 0.1x_2 - & & 0.1x_3 + & 0.9x_4 \geq 0 \\
 854x_1 + 435x_2 + 1023.96x_3 + 162.02x_4 \leq 7000 \\
 x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0
 \end{array}$$

3. This is a maximum problem with mixed constraints, so we will rewrite all the constraints but the nonnegativity inequalities to be less than or equal to inequalities.

$$\begin{array}{rcl}
 x_1 + x_2 + x_3 + x_4 & \leq & 10 \\
 -0.9x_1 + 0.1x_2 + 0.1x_3 + 0.1x_4 & \leq & 0 \\
 +0.1x_1 - 0.9x_2 + 0.1x_3 + 0.1x_4 & \leq & 0 \\
 +0.1x_1 + 0.1x_2 - 0.9x_3 + 0.1x_4 & \leq & 0 \\
 +0.1x_1 + 0.1x_2 + 0.1x_3 - 0.9x_4 & \leq & 0 \\
 854x_1 + 435x_2 + 1023.96x_3 + 162.02x_4 & \leq & 7000 \\
 x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0 & &
 \end{array}$$

Since the problem is so large we will use Excel to find the solution.

As illustrated in the text

1. Enter the variables and their initial values of 0 (since they are nonbasic in the initial tableau).
2. Enter the objective function.
3. Enter the constraints, and the initial RHS
4. Go to the solver, set the target cell equal to the max(imum).
then enter the constraints including the nonnegativity constraints.
5. Check the options to be sure a “assume linear model” is checked.
6. Solve and highlight answer.

We find that the trail mix will maximize carbohydrates while meeting the constraints on the mix if 1 cup of peanuts, 3.7 cups of raisins, 4.3 cups of M&M’s, and 1 cup of pretzels are used in the mix. Then the maximum carbohydrates will be 1124.9 grams.

4. There are 112.5 carbohydrates in a cup of the mix.

There are $\frac{(854.10)(1) + (435)(3.7) + (1023.96)(4.3) + (162.02)(1)}{10} = 702.86$ calories in a cup of the mix.

5. To find the mix that will maximize the protein, we change the objective function. We

$$\text{maximize } P = 34.57x_1 + 4.67x_2 + 9.01x_3 + 3.87x_4$$

Again we use Excel, following the steps in the text, and we find that the trail mix which maximizes protein while meeting the constraints will contain 6.5 cups of peanuts, 0.9 cups of raisins, 0.9 cups of M&M’s, and 0.9 cups of pretzels. The maximum protein in the mix will be 239.06 grams.

This trail mix contains 9.2 cups of mixture. So there are $\frac{231.06}{9.2} = 25.9$ grams of protein per cup.

There are $\frac{(854.10)(6.5) + (435)(0.9) + (1023.96)(0.9) + (162.02)(0.9)}{9.2} = 762.01$ calories per cup of mix.

6. We now change the objective function to minimize the amount of fat in the mix and the constraints to insure a minimum of carbohydrates while ignoring the calories. We minimize $F = 72.5x_1 + 0.67x_2 + 43.95x_3 + 1.49x_4$ subject to the constraints

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &\leq 10 \\ -0.9x_1 + 0.1x_2 + 0.1x_3 + 0.1x_4 &\leq 0 \\ 0.1x_1 - 0.9x_2 + 0.1x_3 + 0.1x_4 &\leq 0 \\ 0.1x_1 + 0.1x_2 - 0.9x_3 + 0.1x_4 &\leq 0 \\ 0.1x_1 + 0.1x_2 + 0.1x_3 - 0.9x_4 &\leq 0 \\ -31.4x_1 - 114.74x_2 - 148.12x_3 - 33.68x_4 &\leq -1000 \\ x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad x_4 \geq 0 \end{aligned}$$

Using Excel we find that fat can be minimized at 120.65 grams while meeting the mixture and nutrition constraints if 1 cup of peanuts, 7 cups of raisins, 0.9 cup of M&M's, and 1 cup of pretzels are mixed.

7. The mix that minimizes the fat contains

$$\frac{(72.5)(1) + (0.67)(7) + (43.95)(1) + (1.49)(1)}{10} = 12.26 \text{ grams of fat per cup of mix.}$$

$$\frac{(34.57)(1) + (4.67)(7) + (9.01)(1) + (3.87)(1)}{10} = 8.0 \text{ grams of protein per cup of mix.}$$

$$\frac{(31.4)(1) + (114.74)(7) + (148.12)(1) + (33.68)(1)}{10} = 101.64 \text{ grams of carbohydrates}$$

per cup of mix.

8. The mix that minimizes fat contains

$$\frac{(854.10)(1) + (435)(7) + (1023.96)(1) + (162.02)(1)}{10} = 508.51 \text{ calories per cup of mix.}$$

Mathematical Questions from Professional Exams

- | | | | |
|--------|---------|---------|--------|
| 1. (c) | 2. (d) | 3. (c) | 4. (a) |
| 5. (b) | 6. (a) | 7. (c) | 8. (d) |
| 9. (d) | 10. (a) | 11. (d) | |

49. ~~The student is to amortize the \$4000 loan with $n = 4 \cdot 4 = 16$ quarterly payments. The interest rate per payment period of the loan is $i = \frac{0.14}{4} = 0.035$, and the quarterly payments are~~

$$P = V \frac{i}{1 - (1+i)^{-n}} = \$4000 \frac{0.035}{1 - (1.035)^{-16}} = \$330.74$$

50. ~~The present value of a 5 year annuity invested at 10% per annum with annual payments of \$20,000 is~~

$$V = P \frac{1 - (1+i)^{-n}}{i} = \$20,000 \frac{1 - (1.10)^{-5}}{0.10} = \$75,815.74$$

~~Since the present value of the annuity is less than the purchase price of the furniture, leasing is preferable.~~

Chapter 5 Project

1. The monthly payment for the 30 year mortgage with no points is

$$P = V \frac{i}{1 - (1+i)^{-n}} = \$120,000 \frac{\frac{0.0650}{12}}{1 - \left(1 + \frac{0.0650}{12}\right)^{-360}} = \$758.48$$

2. The monthly payment for the 30 year mortgage with 0.5 point is

$$P = V \frac{i}{1 - (1+i)^{-n}} = \$120,000 \frac{\frac{0.0638}{12}}{1 - \left(1 + \frac{0.0638}{12}\right)^{-360}} = \$749.04$$

3. The difference in the monthly payments between the loan with no points and the mortgage with 0.5 point is $\$758.48 - \$749.04 = \$9.44$

4. To determine how many months it will take to make back the \$600.00 spent on the points, we solve

$$\begin{aligned} \$9.44n &= \$600.00 \\ n &= 63.56 \text{ months} \end{aligned}$$

The \$600 is made back on the 64th monthly payment.

5. Answers will vary, but they should include justification.

6. The mortgage with 1.1 points:
The monthly payment

$$P = V \frac{i}{1 - (1+i)^{-n}} = \$120,000 \frac{\frac{0.0625}{12}}{1 - \left(1 + \frac{0.0625}{12}\right)^{-360}} = \$738.86$$

The difference in monthly payments between a loan with no points and the mortgage with 1.1 points is $\$758.48 - \$738.86 = \$19.62$

To determine how many months it will take to make back the \$1320.00 spent on the points, we solve

$$\begin{aligned} \$19.62n &= \$1320.00 \\ n &= 67.27 \text{ months} \end{aligned}$$

The \$1320 is made back on the 68th monthly payment.

The mortgage with 2.7 points:

The monthly payment

$$P = V \frac{i}{1 - (1+i)^{-n}} = \$120,000 \frac{\frac{0.0590}{12}}{1 - \left(1 + \frac{0.0590}{12}\right)^{-360}} = \$711.76$$

The difference in monthly payments between a loan with no points and the mortgage with 2.7 points is $\$758.48 - \$711.76 = \$46.73$

To determine how many months it will take to make back the \$3240.00 spent on the points, solve

$$\begin{aligned} \$46.73n &= \$3240.00 \\ n &= 69.33 \text{ months} \end{aligned}$$

The \$3240 is made back on the 70th monthly payment.

7. If the loan is decreased by the amount that would have been spent on points and the larger interest rate of 5.50% is used, the monthly payments become:

Rate	Points	Fee Paid	Loan Amount	Monthly Payment
6.5%	0	\$0.00	$\$120,000 - \$3240 = \$116,760$	$116,760 \frac{\frac{0.065}{12}}{1 - \left(1 + \frac{0.065}{12}\right)^{-360}} = \738.00
6.38%	0.50	\$600	$\$120,000 - \$2640 = \$117,360$	$117,360 \frac{\frac{0.0638}{12}}{1 - \left(1 + \frac{0.0638}{12}\right)^{-360}} = \732.56
6.25%	1.10	\$1320	$\$120,000 - \$1920 = \$118,080$	$118,080 \frac{\frac{0.0625}{12}}{1 - \left(1 + \frac{0.0625}{12}\right)^{-360}} = \727.04
5.90%	2.70	\$3240	\$120,000	$120,000 \frac{\frac{0.0590}{12}}{1 - \left(1 + \frac{0.0590}{12}\right)^{-360}} = \711.76

8. The lowest monthly payment is still with the 5.9% mortgage with 2.7 points.

Mathematical Questions from Professional Exams

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|----|-----|----|-----|----|-----|
| 1. | (b) | 2. | (c) | 3. | (b) |
| 4. | (b) | 5. | (d) | 6. | (a) |
| 7. | (c) | | | | |