## Chapter 1 Project

**1.** Since Avis charges a flat rate that does not depend on the number of miles, $x$, driven the equation is that of a horizontal line.

$A = 64.99$

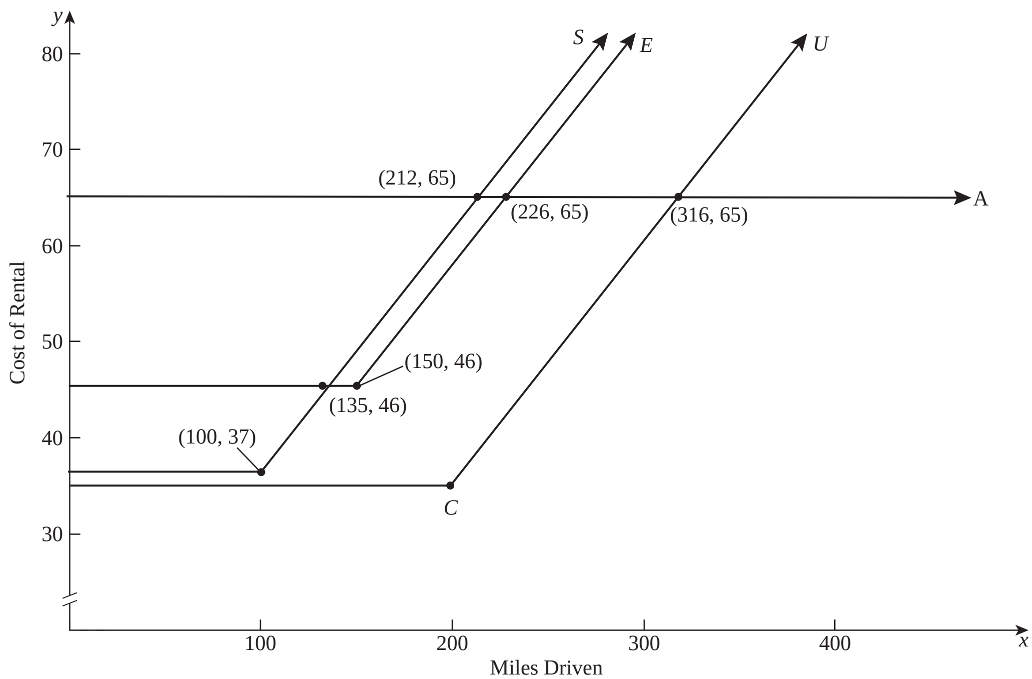
**2.** If $x \le 150$ miles, the cost of renting an Enterprise car is constant, and we have

$E = 45.87$

If $x > 150$ miles, the cost of renting an Enterprise car is the daily rate plus a mileage charge of $0.25 per mile driven over 150 miles.

$$\begin{aligned} E &= (x - 150)(0.25) + 45.87 \\ &= 0.25x - 37.5 + 45.87 \\ &= 0.25x + 8.37 \end{aligned}$$

**3.**




**4.** The point of intersection $(x_0, y_0)$ is a point on both equations and E = A. $x_0 > 150$, since for $x_0 < 150$, $E \neq A$. So we have,

$$\begin{aligned} 0.25x_0 + 8.37 &= 64.99 \\ 0.25x_0 &= 56.62 \\ x_0 &= 226.48 \end{aligned}$$

Avis is more economical if the car is driven more than 226 miles.

**5.** By planning your sightseeing carefully and figuring how far you intend to drive, the information in question 4 will help you to decide which car to rent. If you are going to drive more than 226 miles, renting from Avis would be more economical. Otherwise, you would save money by renting from Enterprise.

**6.** AutoSaveRental has a two tiered rental rate similar to Enterprise. The cost of renting a car from AutoSaveRental can be calculated from the following two equations,

when $x \le 100$ $\quad S = 36.99$

when $x > 100$ $\quad S = 36.99 + 0.25(x - 100)$

$= 36.99 + 0.25x - 25$

$= 0.25x + 11.99$

**7.** See graph in Problem 3.

**8.** To decide when, if ever, Avis or Enterprise offers a better deal than AutoSaveRental, find the points of intersection of Avis and AutoSave and of Enterprise and AutoSave.

*Avis and AutoSave*: We know that at the point of intersection, $S = A$. Since $S \ne A$ when $x < 100$, $x_0$ must be greater than 100.

$$\begin{aligned} S &= A \\ 0.25x_0 + 11.99 &= 64.99 \\ 0.25x_0 &= 53 \\ x_0 &= 212 \end{aligned}$$

Avis become less expensive than AutoSave after 212 miles have been driven.

*Enterprise and AutoSave*: We know that at the point of intersection, $S = E$. Since $S \ne E$ when $x < 100$, $x_0$ must be greater than 100. However, this still leaves two possibilities:

$100 < x_0 \le 150$ and $x_0 > 150$

When $100 < x_0 \le 150$, $E = 45.87$ and $S = 0.25x + 11.99$. If the point of intersection is between 100 and 150 miles then $S = E$.

$$\begin{aligned} S &= E \\ 0.25x_0 + 11.99 &= 45.87 \\ 0.25x_0 &= 33.88 \\ x_0 &= 135.52 \end{aligned}$$

If $x_0$ is greater than 150 miles, then $S = E$.

$$\begin{aligned} S &= E \\ 0.25x_0 + 11.99 &= 0.25x_0 + 8.37 \\ 11.99 &= 8.37 \end{aligned}$$

which is meaningless.

We can conclude that AutoSave rents the cheapest car if you are driving fewer than 135 miles. If you are driving more than 135 miles, but fewer than 226 miles, Enterprise provides the least expensive rental, but if you plan to drive more than 226 miles, rent from Avis. It will save you money.

**9.** Usave Car Rental also has a two tiered pricing policy similar to Enterprise and AutoSave. The equations describing the cost of renting a Usave car are:

$$\begin{aligned} U &= 35.99 && \text{when } x \le 200 \\ U &= 35.99 + 0.25(x - 200) && \text{when } x > 200 \\ &= 35.99 + 0.25x - 50 \\ &= 0.25x - 14.01 \end{aligned}$$

Usave Car Rental is always less expensive than Enterprise and AutoSave. Usave's base rate is lower than either of the two, and it offers more free miles than both the other companies. There will be a point, however, where Avis again becomes more economical. To determine at which mileage Avis is the better deal, find the point of intersection of $A$ and $U$. Consider only $x_0 > 200$.

$$\begin{aligned} U &= A \\ 0.25x_0 - 14.01 &= 64.99 \\ 0.25x_0 &= 79 \\ x_0 &= 316 \end{aligned}$$

Therefore, if you drive more than 316 miles, the Avis car which costs $64.99 is the least expensive to rent.

**10.** Answers vary. They might include only price, but perhaps might include convenience, reliability, or other concerns.

## Mathematical Questions Form Professional Exams

**1.** The break-even point is the value of $x$ for which the revenue equals cost. If $x$ units are sold at price of $2.00 each, the revenue is $R = 2x$.

Cost is the total of the fixed and variable costs. We are told that the fixed costs are $6000, and that the variable cost per item is 40% of the price. So the cost is given by the equation:

$$\begin{aligned} C &= (0.40)(2)x + 6000 \\ &= 0.80x + 6000 \end{aligned}$$

Setting $R = C$ and solving for $x$ yields,

$$\begin{aligned} R &= C \\ 2x &= 0.8x + 6000 \\ 1.2x &= 6000 \\ x &= 5000 \end{aligned}$$

**Answer**: (b) 5000 units

**2.** Profit is defined as revenue minus cost. If $x$ rodaks are sold for $6.00 each, the revenue equation is:

$$R = 6x$$

We are told that to manufacture rodaks costs $2.00 per unit and $37,500 in fixed costs. So the cost equation for producing $x$ rodaks is:

$$C = 2x + 37{,}500$$

The Breiden Company wants to realize a before tax profit equal to 15% of sales, which is:

$$P = (.15)R = (.15)(6x) = 0.9x$$

To find the number of rodaks that need to be sold to meet Breiden's goal, we solve the equation:

$$\begin{aligned} P &= R - C \\ 0.9x &= 6x - (2x + 37{,}500) \\ 0.9x &= 4x - 37{,}500 \\ 3.1x &= 37{,}500 \\ x &= 12{,}096.77 \end{aligned}$$

**Answer**: (d) sell 12,097 units

**3.** The break-even point is the number of units that must be sold for revenue to equal cost. Using the notation given, we have:

$$R = SPx \quad \text{and} \quad C = VCx + FC$$

To find the sales level necessary to break even, we set $R = C$ and solve for $x$.

$$\begin{aligned} R &= C \\ SPx &= VCx + FC \\ SPx - VCx &= FC \\ (SP - VC)x &= FC \\ x &= \frac{FC}{SP - VC} \end{aligned}$$

**Answer:** (d)

**4.** At the break-even point of $x$ = 400 units sold, cost equals revenue. We are told cost, C = \$400 + 200 = 600, so revenue R = \$600. Since R = price quantity, the price of each item is $\frac{\$600}{400} = \$1.50$. The variable cost per unit is \$1.00, so the 401$^{st}$ unit contributes

\$1.50 – \$1.00 = \$0.50 to profits.

**Answer**: (b)

**5.** Since straight-line depreciation remains the same over the life of the property, its expense over time will be a horizontal line. Sum-of-year's-digits depreciation expense decreases as time increases.
**Answer:** (c)

**6.** **Answer:** (c); $Y$ = \$1000 + \$2$X$ is a linear relationship.

**7.** **Answer:** (b); $Y$ is an estimate of total factory overhead.

**8.** **Answer:** (b); In the equation \$2 is the estimate of variable cost per direct labor hour.

$$\xrightarrow{R_2=-\frac{1}{2}r_2}\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & 0.5 & 0.5 & -0.5 & 0\\ 0 & 0.031 & 0.073 & -0.09 & 0 & 1\end{array}\right]\xrightarrow[R_3=-0.031r_2+r_3]{R_1=-r_2+r_1}\left[\begin{array}{ccc|ccc}1 & 0 & 0.5 & 0.5 & 0.5 & 0\\ 0 & 1 & 0.5 & 0.5 & -0.5 & 0\\ 0 & 0 & 0.0575 & -0.1055 & 0.0155 & 1\end{array}\right]$$

$$\xrightarrow{R_3=\frac{1}{0.0575}r_3}\left[\begin{array}{ccc|ccc}1 & 0 & 0.5 & 0.5 & 0.5 & 0\\ 0 & 1 & 0.5 & 0.5 & -0.5 & 0\\ 0 & 0 & 1 & -1.8348 & 0.2696 & 17.3913\end{array}\right]$$

$$\xrightarrow[R_2=-0.5r_3+r_2]{R_1=-0.5r_3+r_1}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1.4174 & 1.3652 & -8.6957\\ 0 & 1 & 0 & 1.4174 & -0.6348 & -8.6957\\ 0 & 0 & 1 & -1.8348 & 0.2696 & 17.3913\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}1.4174 & 1.3652 & -8.6957\\ 1.4174 & -0.6348 & -8.6957\\ -1.8348 & 0.2696 & 17.3913\end{bmatrix}=\begin{bmatrix}\frac{163}{115} & \frac{42}{115} & -\frac{200}{23}\\ \frac{163}{115} & -\frac{73}{115} & -\frac{200}{23}\\ -\frac{211}{115} & \frac{31}{115} & \frac{400}{23}\end{bmatrix}$$

(a) When the desired rate of return is 11%, matrix $B$ is

$$B=\begin{bmatrix}10{,}000\\ 2{,}000\\ 1{,}100\end{bmatrix}\text{ and } X=A^{-1}B=\begin{bmatrix}5339.13\\ 3339.13\\ 1321.74\end{bmatrix}$$

You should invest \$5339.13 in the Large Growth fund, \$3339.13 in the Small Growth fund, and \$1321.74 in the Large Value fund.

(b) When your desired rate of return is 12%, matrix $B$ is

$$B=\begin{bmatrix}10{,}000\\ 2{,}000\\ 1{,}200\end{bmatrix}\text{ and } X=A^{-1}B=\begin{bmatrix}4469.57\\ 2469.57\\ 3060.87\end{bmatrix}$$

You should invest \$4469.57 in the Large Growth fund, \$2469.57 in the Small Growth fund, and \$3060.87 in the Large Value fund.

(b) When your desired rate of return is 14%, matrix $B$ is

$$B=\begin{bmatrix}10{,}000\\ 2{,}000\\ 1{,}400\end{bmatrix}\text{ and } X=A^{-1}B=\begin{bmatrix}2730.43\\ 730.43\\ 6539.13\end{bmatrix}$$

You should invest \$2730.43 in the Large Growth fund, \$730.43 in the Small Growth fund, and \$6539.13 in the Large Value fund.

## Chapter 2 Project

**1.** Matrix $A$ represents the amounts of materials needed to produce 1 unit of a product. The entries in the matrix are quotients. Column 1 is found by dividing each entry in the agriculture column of the table by total gross output of agriculture. Column 2 entries are the quotients found by dividing each entry in the manufacturing column by total gross output of manufacturing, and column 3 entries are the quotients found by dividing entries by total gross outcome of services. $D$ is the consumer demand. It is labeled Open Sector in Table 1.

$$A = \begin{bmatrix} 0.410 & 0.030 & 0.026 \\ 0.062 & 0.378 & 0.105 \\ 0.124 & 0.159 & 0.192 \end{bmatrix} \qquad D = \begin{bmatrix} 39.24 \\ 60.02 \\ 130.65 \end{bmatrix}$$

**2.** The production vector $X$ is the total output necessary to meet producer and consumer needs. We let $x$, $y$, and $z$ represent the total output needed by agricultural, manufacturing, and service sectors respectively.

$$X_0 = [I_3 - A]^{-1} D_0$$

First we will find $[I_3 - A]^{-1}$.

$$[I_3 - A]^{-1} = \begin{bmatrix} 1-0.410 & -0.030 & -0.026 \\ -0.062 & 1-0.378 & -0.105 \\ -0.124 & -0.159 & 1-0.192 \end{bmatrix}^{-1} = \begin{bmatrix} 0.590 & -0.030 & -0.026 \\ -0.062 & 0.622 & -0.105 \\ -0.124 & -0.159 & 0.808 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1.720 & 0.100 & 0.068 \\ 0.223 & 1.676 & 0.225 \\ 0.308 & 0.345 & 1.292 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 1.720 & 0.100 & 0.068 \\ 0.223 & 1.676 & 0.225 \\ 0.308 & 0.345 & 1.292 \end{bmatrix} \begin{bmatrix} 39.24 \\ 60.02 \\ 130.65 \end{bmatrix} = \begin{bmatrix} 82.379 \\ 138.740 \\ 201.593 \end{bmatrix}$$

**3.** To calculate the new $X_1$, we need to solve the equation $X_1 = [I_3 - A]^{-1} D_1$

$$X_1 = \begin{bmatrix} 1.720 & 0.100 & 0.068 \\ 0.223 & 1.676 & 0.225 \\ 0.308 & 0.345 & 1.292 \end{bmatrix} \begin{bmatrix} 40.24 \\ 60.02 \\ 130.65 \end{bmatrix} = \begin{bmatrix} 84.099 \\ 138.963 \\ 201.901 \end{bmatrix}$$

**4.** $$X_1 - X_0 = \begin{bmatrix} 84.099 \\ 138.963 \\ 201.901 \end{bmatrix} - \begin{bmatrix} 82.379 \\ 138.740 \\ 201.593 \end{bmatrix} = \begin{bmatrix} 1.72 \\ 0.223 \\ 0.308 \end{bmatrix}$$

**5.** Using the interpretation from Problem 4, we find that service production would need to increase by 0.308 unit for a one unit increase in agriculture.

**6.** Manufacturing output needs to increase by 0.223 unit for a one unit increase in agriculture.

**7.** $$A = \begin{bmatrix} 0.2424 & 0.0005 & 0.0058 & 0.0366 & 0.0001 & 0.0012 & 0.0045 & 0.0354 & 0.0005 \\ 0.0013 & 0.2131 & 0.0073 & 0.0207 & 0.0419 & 0.0000 & 0.0000 & 0.0000 & 0.0025 \\ 0.049 & 0.0318 & .0009 & 0.0073 & 0.0379 & 0.0082 & 0.0261 & 0.0083 & 0.0214 \\ 0.1744 & 0.0982 & 0.2976 & 0.3492 & 0.0564 & 0.0438 & 0.0076 & 0.0980 & 0.0145 \\ 0.0446 & 0.0856 & 0.0247 & 0.0455 & 0.1607 & 0.0439 & 0.0207 & 0.0347 & 0.0189 \\ 0.0492 & 0.0237 & 0.0812 & 0.0583 & 0.0121 & 0.0211 & 0.0019 & 0.0196 & 0.0022 \\ 0.0729 & 0.2251 & 0.0164 & 0.0180 & 0.0322 & 0.0698 & 0.1749 & 0.0701 & 0.0066 \\ 0.0318 & 0.0396 & 0.1031 & 0.0607 & 0.1156 & 0.1412 & 0.0751 & 0.1526 & 0.0112 \\ 0.0006 & 0.0002 & 0.0011 & 0.0035 & 0.0026 & 0.0072 & 0.0111 & 0.0071 & 0.0025 \end{bmatrix}$$

$$D_0 = \begin{bmatrix} 34940 \\ -39241 \\ 787208 \\ 1611520 \\ 586248 \\ 1103110 \\ 1520718 \\ 2214382 \\ 1032052 \end{bmatrix}$$

**8.** $$X_0 = [I_9 - A]^{-1} D_0 = \begin{bmatrix} 434735.4858 \\ 135766.5949 \\ 1009113.182 \\ 3906797.394 \\ 1300135.663 \\ 1566308.293 \\ 2531704.074 \\ 3715940.555 \\ 1119064.336 \end{bmatrix}$$

**9.** $D_1 = \begin{bmatrix} 34940 \\ -39241 \\ 787209 \\ 1611520 \\ 586248 \\ 1103110 \\ 1520718 \\ 2214382 \\ 1032052 \end{bmatrix} \quad X_1 = [I_9 - A]^{-1} D_1 = \begin{bmatrix} 434735.5291 \\ 135766.6225 \\ 1009114.196 \\ 3906797.921 \\ 1300135.743 \\ 1566308.417 \\ 2531704.148 \\ 3715940.758 \\ 119064.343 \end{bmatrix}$

If demand for construction increases by 1 million dollars, demand for transportation, communication, and utilities increases by \$80,000.00.

$$1300135.74 - 1300135.66 = 0.08$$

**10.** The effects of an increase of \$1 million of demand for construction on other segments of the economy are found by comparing the difference between $X_1$ and $X_0$.

$$X_1 - X_0 = \begin{bmatrix} 435029.32 \\ 136878.01 \\ 1010011.69 \\ 3909435.51 \\ 1320320.97 \\ 1565286.57 \\ 2533020.11 \\ 3719165.67 \\ 1119148.07 \end{bmatrix} - \begin{bmatrix} 435029.27 \\ 136877.98 \\ 1010010.67 \\ 3909434.99 \\ 1320320.89 \\ 1565286.45 \\ 2533020.04 \\ 3719165.47 \\ 1119148.06 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.03 \\ 1.02 \\ 0.52 \\ 0.08 \\ 0.12 \\ 0.07 \\ 0.20 \\ 0.01 \end{bmatrix}$$

The three sectors most affected by an increase of \$1 million in construction demand are construction, it increases by \$1.01 million; manufacturing, it increases by \$530,000; and services, it increases by \$200,000.

**11.** An increase of \$1 million in demand for construction, produces an composite increase of \$2.1 million in the economy.

**12.** If the $(i, j)$ entry in $(I - A)^{-1}$ is 0, it means that a 1 unit increase in the demand for product $j$ has no effect on the demand for product $i$.

## Mathematical Questions From Professional Exams

**1.** (b) **2.** (b) **3.** (d)

**4.** (c) We have $A_e = 190{,}000 + 0.8B_e + 0.7C_e$

$B_e = 170{,}000 + 0.15C_e$

$C_e = 230{,}000 + 0.25\,A_e$

Rewriting the equations we get

$$\begin{aligned} A_e - 0.8B_e - 0.7C_e &= 190{,}000 \\ B_e - 0.15C_e &= 170{,}000 \\ -0.25A_e + C_e &= 230{,}000 \end{aligned}$$

or

$$\begin{bmatrix} 1 & -0.8 & -0.7 \\ 0 & 1 & -0.15 \\ -0.25 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_e \\ B_e \\ C_e \end{bmatrix} = \begin{bmatrix} 190{,}000 \\ 170{,}000 \\ 230{,}000 \end{bmatrix}$$

$$\begin{bmatrix} A_e \\ B_e \\ C_e \end{bmatrix} = A^{-1}D = \begin{bmatrix} 1 & -0.8 & -0.7 \\ 0 & 1 & -0.15 \\ -0.25 & 0 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 190{,}000 \\ 170{,}000 \\ 230{,}000 \end{bmatrix} = \begin{bmatrix} 647295.60 \\ 228773.58 \\ 391823.90 \end{bmatrix}$$

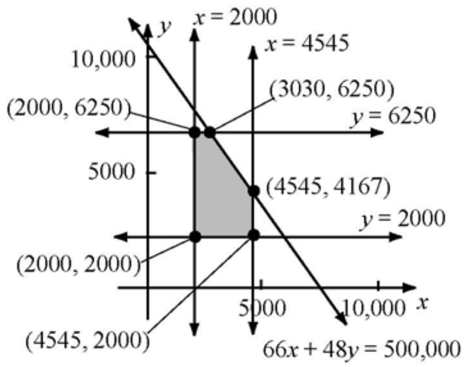
$B_e$ = \$228,773.58, and its minority interest in consolidated net income is (0.2)(\$228,773.58) = \$45,754.72.

$$Y = 0.52x + 0.50y$$

subject to the constraints

$$\begin{cases} 66x + 48y \le 500{,}000 & (1) \\ x \ge 2{,}000 & (2) \\ y \ge 2{,}000 & (3) \\ 66x \le 300{,}000 & (4) \\ 48y \le 300{,}000 & (5) \end{cases}$$

The constraints are graphed, the set of feasible points is shaded, and the corner points are identified.




The objective function is evaluated at each the corner points, and the maximum value of $R$ is chosen.

| Corner Point | $Y = 0.52x + 0.50y$ |
|---|---|
| $(2000, 2000)$ | $0.52 \cdot 2000 + 0.50 \cdot 2000 = 2040$ |
| $(4545, 2000)$ | $0.52 \cdot 4545 + 0.50 \cdot 2000 = 3363.4$ |
| $(4545, 4167)$ | $0.52 \cdot 4545 + 0.50 \cdot 4167 = 4446.9$ |
| $(2000, 6250)$ | $0.52 \cdot 2000 + 0.50 \cdot 6250 = 4165$ |
| $(3030, 6250)$ | $0.52 \cdot 3030 + 0.50 \cdot 6250 = 4700.6$ |

To maximize the projected dividend the fund should purchase 3030 shares of Chevron and 6250 shares of Bank of America. The projected dividend is $4700.60.

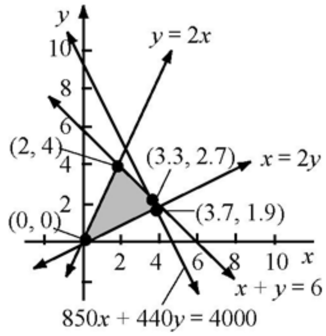
## Chapter 3 Project

**1.** Objective function: $c = 31.4x + 114.74y$

**2.** Constraints:

$$\begin{cases} x + y \le 6 & \text{total quantity constraint} & (1) \\ x \ge \frac{1}{2}y & \text{proportion peanuts} & (2) \\ y \ge \frac{1}{2}x & \text{proportion raisins} & (3) \\ 850x + 440y \le 4000 & \text{calorie limit constraint} & (4) \\ x \ge 0 & \text{non-negativity constraint} & (5) \\ y \ge 0 & \text{non-negativity constraint} & (6) \end{cases}$$

**3.**




The corner point (3.3, 2.7) is the solution to the system

$$\begin{cases} x + \quad y = \quad 6 & (1) \\ 850x + 440y = 4000 & (2) \end{cases}.$$

Substituting $6 - x$ for $y$ in (2), gives $410x = 1360$ or $x = 3.3$. Back-substitution into (1) gives $y = 2.7$.

The point (3.7, 1.9) is the solution to the system

$$\begin{cases} 850x + 440y = 4000 \\ x = 2y \end{cases}$$

**4.** The objective function $c$ is evaluated at the corner points and the largest value is chosen.

| Corner Point | $c = 31.4x + 114.74y$ |
|---|---|
| (0, 0) | $c = 31.4(0) + 114.74(0) = 0$ |
| (3.7, 1.9) | $c = 31.4(3.7) + 114.74(1.9) = 334.19$ |
| (3.3, 2.7) | $c = 31.4(3.3) + 114.74(2.7) = 413.42$ |
| (2, 4) | $c = 31.4(2) + 114.74(4) = 521.76$ |

Using 2 cups of peanuts and 4 cups of raisins maximizes the amount of carbohydrates in the mix at 521.76 grams.

**5.** The total mix contains 6 cups, so there are $521.76 \div 6 = 86.96$ grams of carbohydrates per cup.

Since each cup of raisins contain 440 calories and each cup of peanuts have 850 calories, the calorie count per cup is $(4 \cdot 440 + 2 \cdot 850) \div 6 = 576.67$ calories per cup of mix.

**6.** To find the mix that will maximize the protein we use the same corner points, but we change the objective function to $p = 34.57x + 4.67y$. The objective function $p$ is evaluated at each corner point and the largest value is chosen.

| Corner Point | $p = 34.57x + 4.67y$ |
|---|---|
| (0, 0) | $p = 34.57(0) + 4.67(0) = 0$ |
| (3.7, 1.9) | $p = 34.57(3.7) + 4.67(1.9) = 136.78$ |
| (3.3, 2.7) | $p = 34.57(3.3) + 4.67(2.7) = 126.69$ |
| (2, 4) | $p = 34.57(2) + 4.67(4) = 87.82$ |

Using 3.7 cups of peanuts and 1.9 cups of raisins maximizes the amount of protein in the mix at 136.78 grams.

In a cup of this protein rich mix, there are $136.78 \div (3.74 + 1.87) = 24.43$ grams of protein.

The new mix has $(3.7 \cdot 850 + 1.9 \cdot 440) \div 5.61 = 710.89$ calories per cup.

**7.** To determine the recipe needed to minimize the fat in the mix, construct a new objective function and evaluate it at the corner points.

Objective function: $f = 72.5x + 0.67y$

| Corner Point | $f = 72.5x + 0.67y$ |
|---|---|
| (0, 0) | $f = 72.5(0) + 0.67(0) = 0$ |
| (3.7, 1.9) | $f = 72.5(3.7) + 0.67(1.9) = 269.52$ |
| (3.3, 2.7) | $f = 72.5(3.3) + 0.67(2.7) = 241.06$ |
| (2, 4) | $f = 72.5(2) + 0.67(4) = 147.68$ |

Since you intend to eat at least 3 cups of mix, we cannot use the corner point (0, 0). So you would minimize the fat content by making a mix that consists of 2 cups of peanuts and 4 cups of raisins.

**8.** This newest mix has 521.76 grams of carbohydrates. See problem 4.

**9.** This low–fat mix has 87.82 grams of protein. See problem 6.

## Mathematical Questions from Professional Exams

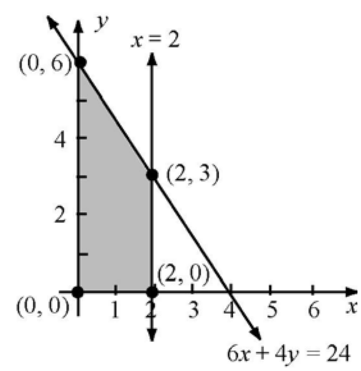
**1.** (b) **2.** (a) **3.** (c)

**4.** Linear Programming problem:

Maximize $P = 2X + 1Y$

subject to the constraints

$$\begin{cases} 6x + 4y \le 24 & \text{machine 1 constraint} \\ 12x \le 24 & \text{machine 2 constraint} \\ x \ge 0 & \text{non-negativity constraint} \\ y \ge 0 & \text{non-negativity constraint} \end{cases}$$




The constraints are graphed, and the set of feasible points is shaded.

Corner points: (0, 0), (2, 0), (0, 6), and (2, 3).

| Corner Point | $P = 2x + y$ |
|---|---|
| (0, 0) | $P = 2(0) + 0 = 0$ |
| (2, 0) | $P = 2(2) + 0 = 4$ |
| (0, 6) | $P = 2(0) + 6 = 6$ |
| (2, 3) | $P = 2(2) + 3 = 7$ |

Maximum profit is \$7, and is attained when **(c)** 2 units of product $X$ and 3 units of product $Y$ are produced.

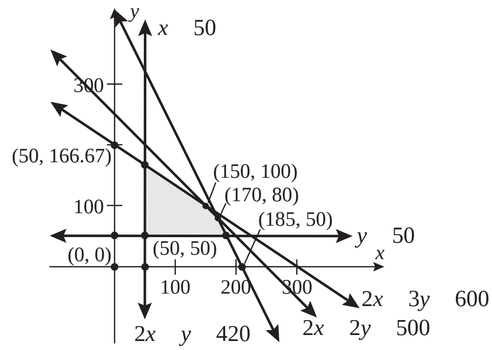
**5.** Profit is the difference between revenue and cost, so profit for product $Q = 20 - 12 = 8$, and profit for product $P = 17 - 13 = 4$. The objective function is **(d).**

**6.** Linear Programming problem:

Maximize $P = 4X + 2Y$

subject to the constraints

$$\begin{cases} 2x + \ \ y \le 420 & \text{machining constraint} \\ 2x + 2y \le 500 & \text{assembling constraint} \\ 2x + 3y \le 600 & \text{finishing constraint} \\ \quad x \ge 50 & \\ \quad y \ge 50 & \end{cases}$$




The constraints are graphed, and the set of feasible points is shaded.

Corner points: (50, 50), (185, 50), (170, 80), (150, 100) and (50, 166.67).

| Corner Point | $P = 4x + 2y$ |
|---|---|
| (50, 50) | $P = 4(50) + 2(50) = 300$ |
| (185, 50) | $P = 4(185) + 2(50) = 840$ |
| (170, 80) | $P = 4(170) + 2(80) = 840$ |
| (150, 100) | $P = 4(150) + 2(100) = 800$ |
| (50, 166.67) | $P = 4(50) + 2(166.67) = 533.34$ |

**(c)** producing 170 of product *X* and 80 of product *Y* maximizes profit.

**7.** **(c)**

**8.** Evaluate the objective function at the corner points.

| Corner Point $(x, y)$ | Value of Objective Function $P = 3x + 4y$ |
|---|---|
| (0, 20) | $P = 3(0) + 4(20) = 80$ |
| (30, 0) | $P = 3(30) + 4(0) = 90$ |
| (20, 10) | $P = 3(20) + 4(10) = 100$ |

**(b)** producing 20 units of product *A* and 10 units of product *B* maximizes profit.

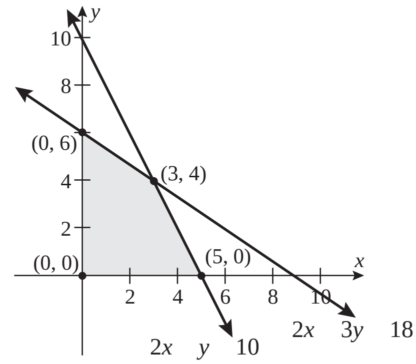
**9.** **(b)**

**10.** Linear Programming problem:

Maximize $P = 4x + 2y$

subject to the constraints

$$\begin{cases} 2x + 3y \le 18 & \text{machine 1 constraint} \\ 2x + \ \ y \le 10 & \text{machine 2 constraint} \\ x \ge 0 & \text{non-negativity constraint} \\ y \ge 0 & \text{non-negativity constraint} \end{cases}$$




The constraints are graphed, and the set of feasible points is shaded.
Corner points: (0, 0), (5, 0), (0, 6), and (3, 4).

| Corner Point $(x, y)$ | Value of Objective Function $P = 4x + 2y$ |
|---|---|
| (0, 0) | $P = 4(0) + 2(0) = 0$ |
| (5, 0) | $P = 4(5) + 2(0) = 20$ |
| (0, 6) | $P = 4(0) + 2(6) = 12$ |
| (3, 4) | $P = 4(3) + 2(4) = 20$ |

**(a)** maximum profit is $20

**11.** **(b)**

**12.** **(e)**

**13.** **(c)**

**14.** **(b)**