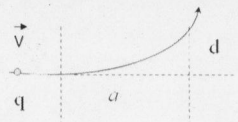
Department of Physics, National Dong Hwa University

DEP. of PHYS.- Electrodynamics, Midterm Exam. - April 22, 2009

1. A particle of charge q enters a region of uniform magnetic field B. The field deflects the particle a distance d above the original line of flight. Is the charge positive or negative? (2%) In terms of a, d, B and q, find the momentum of the particle. (8%)



2. A uniformly charged solid sphere of radius R carries a total charge Q, and is set spinning with angular velocity ω about the z-axis.
   a) What is the magnetic dipole moment of sphere? (10%)
   b) Find the average magnetic field within the sphere. (5%)
   c) Find the approximate vector potential at a pint (r, θ) where r>>R. (10%)

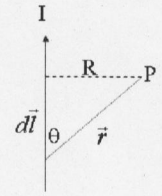
   Hint: for rotating spherical shell, the vector potential is

$$A(r,\theta,\phi)=\frac{\mu_o R\omega\sigma}{3}r\sin\theta,(r\le R)$$

$$A(r,\theta,\phi)=\frac{\mu_o R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}(r\ge R)$$

3. If the vector potential $\vec{A}=\frac{\mu_o}{4\pi}\frac{1}{r^3}\vec{m}\times\vec{r}$, show that the magnetic field of a dipole in coordinate-free form? (15 %)

4. The magnetic field due to a current flowing in a long straight wire, Find the vector potential at point P? (10%) If R<<L, calculate the magnetic field of B(ρ, ϕ, z). (10%)



5. What current density would produce the vector potential, $A=k\hat{\phi}$ (where k is a constant) in cylindrical coordinates? (10%)

6. A current I is uniformly distributed over a wire of circular cross section, which radius is *a*,

a. Find the volume current density *J* (5%)
b. Suppose the current is proportional to ks^2,
   1. Find the total current? (5%)
   2. Calculate the magnetic fields B(r) at outside(r>*a*)and inside the wire (r<*a*)? (10%)

Note:

$$\nabla\cdot(A\times B)=B\cdot(\nabla\times A)-A\cdot(\nabla\times B);$$

$$\nabla\times(A\times B)=(B\cdot\nabla)A-(A\cdot\nabla)B+A(\nabla\cdot B)-B(\nabla\cdot A)$$