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Chapter 8 , E. B - fields,
ϕ. A potentials and wave equation.
in vacuum.

$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$; $V = \frac{1}{\sqrt{\mu \epsilon}}$

① If $\vec{E}$ & $\vec{B}$ can be expressed in terms of the electro magnetic potential $\vec{A}$ & ϕ, $\vec{B} = \nabla \times \vec{A}$, $\vec{E} = -\nabla\phi - \frac{\partial A}{\partial t}$

$$\begin{cases} \nabla \times \vec{E} = -\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) = -\frac{\partial}{\partial t}(\nabla \times \vec{A} \boxed{-\nabla\phi}) \\ \nabla \times (\vec{E} - \frac{\partial A}{\partial t}) = 0, \quad \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}. \end{cases}$$

(假設不隨時變)

① put $\nabla \cdot \vec{E} = \rho/\epsilon_0$, $\nabla \cdot (-\nabla\phi - \frac{\partial A}{\partial t}) = \rho/\epsilon_0$

$$\nabla^2 \phi + \frac{\partial}{\partial t}(\nabla \cdot A) = -\rho/\epsilon_0$$

② $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\boxed{-\nabla^2 A + \nabla(\nabla \cdot A + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}) + \mu_0 \epsilon_0 \frac{\partial^2 A}{\partial t^2} = \mu_0 J}$$

$\nabla \cdot A = 0$ in EM static.

$\nabla \cdot A + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t} = 0$

$\nabla \cdot A = -\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}$ (Lorentz gauge condition)

③ $\boxed{-\nabla^2 A + \epsilon_0 \mu_0 \frac{\partial^2 A}{\partial t^2} = \mu_0 J}$ $\vec{A} \longrightarrow$ wave eq.

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④ $\nabla^2\phi + \frac{\partial}{\partial t}(\nabla\cdot A) = \nabla^2\phi + \frac{\partial}{\partial t}\left(-\mu_0\epsilon_0\frac{\partial\phi}{\partial t}\right) = -\rho/\epsilon_0$

$\Rightarrow \boxed{\nabla^2\phi - \mu_0\epsilon_0\frac{\partial\phi}{\partial t} = -\rho/\epsilon_0}$ $\quad \phi \rightarrow$ wave eq

In space where ρ & j vanish, (free space)

* $\nabla^2 A - \mu_0\epsilon_0\frac{\partial^2 A}{\partial t^2} = 0$

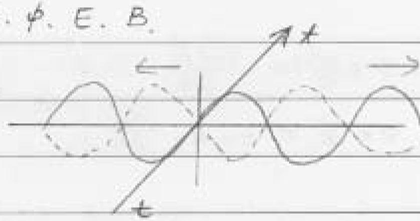
  $\nabla^2 \phi - \mu_0\epsilon_0\frac{\partial^2 A}{\partial t^2} = 0$

* E, B field , $\nabla^2 E - \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2} = 0$

  $\nabla^2 B - \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2} = 0$

* $\vec{E}$ and $\vec{B}$ as well as the scalar & vector potential all satisfy the same basic wave equation.

$$\nabla^2 U - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = 0, \quad U \equiv A, \phi, E, B.$$



$U(r,t) = f(-r,t) + f(r,t)$

$r' = r - ct \quad , r'' = r + ct$

$U(r,t) = f(r+ct) + f(r-ct)$

$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$

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* Energy & momentum in Electromagnetic wave.
(EM - fields)

We show that EM waves carry energy & the energy density u is calculated as

$u = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}$, For simply plane waves

$u = \epsilon_0 E_0^2 \sin^2(kx - \omega t)$, $\langle u \rangle = \frac{1}{2}\epsilon_0 E^2$

$B_0^2/\mu_0 \sin^2(kx - \omega t)$, $\langle u \rangle = \frac{B^2}{2\mu_0}$.

* We set up the vector.

$\vec{E}\times\vec{B}$, we can calculate the divergency of $\vec{E}\times\vec{B}$

$\nabla\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{B})$

Now $\left.\begin{aligned} \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{B} &= \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\} \rightarrow$ then

$\Rightarrow \nabla\cdot(\vec{E}\times\vec{B}) = -\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{E}\cdot\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = -\frac{\partial}{\partial t}\left[\frac{\mu_0\epsilon_0 E^2}{2} + \frac{B^2}{2}\right]$

$\nabla\cdot\underline{(\vec{E}\times\vec{B})}_{\mu_0} = -\frac{\partial}{\partial t}\left[\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}\right] = -\frac{\partial}{\partial t}u$

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* Poynting vector $\vec{S}$ can be defined as

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$\boxed{\nabla \cdot \vec{S} = -\frac{\partial}{\partial t} u}$ Integrated by volume.

$$\int \nabla \cdot \vec{S} \, dv = -\frac{\partial}{\partial t} \int u \, dv \quad , \quad \int \vec{S} \cdot d\vec{a} = -\frac{\partial}{\partial t} \int u \, dv$$

$$\left.\begin{aligned} \text{flux} \equiv\ & \vec{E} \cdot \vec{A} \\ & \vec{B} \cdot \vec{A} \\ & S \cdot \vec{A} \end{aligned}\right\} \Longrightarrow \boxed{\text{Energy-flux density}} \Downarrow \vec{S}$$

$$* \left.\begin{aligned} \frac{\partial}{\partial t} u &= -\nabla \cdot S \\ \frac{\partial \rho}{\partial t} &= -\nabla \cdot J \end{aligned}\right\}$$

The contributions of Maxwell

① Assume the change of electric field will induce the magnetic field.
(magnetic) ... (electric)

② The existance of EM wave and light is an EM wave.

③ Uniformlity (完整性): Given the distribution of source, initial condition, and boundary condition, E-M can be solved.

④ Integration: ($\vec{E}, \vec{D}, \vec{B}, \vec{H}, \vec{J}, \rho$) are the functions of time & space.
(t) (x, y, z)