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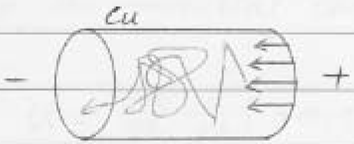
Chapter 7. Electrodynamics.

σ : conductivity ?

$\Rightarrow$ Microscopic view of Ohm's Law.



V_d, drift velocity.

The current density can be expressed in terms of the free electron density as $J = neV_d$

$$\left(I = \frac{q}{t} , \quad I/A = J \right)$$

$$n \equiv \frac{N\,(\text{atoms}/\text{mole})\ \rho\,(\text{kg}/\text{m}^3)}{A\,(\text{kg}/\text{mole})}$$

① $F = ma = m\,V_d/t = m\,V_d/(d/V_F)$

② $F = qE$

③ Kinetic energy, $E_i = \frac{1}{2} m \underline{V_F}^2$.

Then the current density

$$J = neV_d = \sigma E$$

$$F = ma = qE = \frac{mV_d}{d} V_F$$

$$\sigma = \frac{neV_d}{E} = \frac{ne}{E}\,\frac{eEd}{mV_F} = \boxed{\frac{ne^2 d}{mV_F}} = \sigma_c$$

$\underline{\sigma : \text{Conductivity.}}$

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§7.1.1 Force on charge will fast/slow their velocity.

so the current density J is proportional to the force per unit charge (f), $f = \frac{F}{q}$.

$$J \propto f = \sigma f \quad , \quad \rho = \frac{1}{\sigma} : \text{resistivity.}$$

if we consider the electrodynamics of electrical/magnetic force, the current density.

$$J = \sigma f = \sigma \frac{F}{q} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

Ohm's law $J = \sigma E$.

§7.1.2 Electromotive Force. (Emf)

It is the work per unit charge done by force/on charge.

Dimensions : energy per charge.

(1) The emf of a battery

$\boxed{\mathcal{E} = V = \int_a^b E \cdot dl}$,

$$qE(b-a)/q = \int E \cdot dl$$

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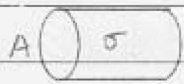
(2) $\mathcal{E} = \frac{1}{q}\int_a^b q\vec{E}\cdot d\vec{l}$ (Electromotive force)

$= \frac{1}{q}\int q(V\times B)\,dl = V\cdot B\cdot L$

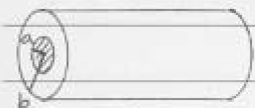
V ↓ Velocity.

Ex 7.1 A cylindrical resistor.



$I = JA = \sigma EA \Rightarrow I = \sigma A\frac{V}{L}$

Ex 7.2 Two long cylinders (radius a & b) conductivity σ maintained at a potential difference V.



§ The E-field between a/b radius.

① $E = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$ (P.2.35)

② The current is therefore

$I = \int J\cdot da = \int \sigma_c E\,da = \frac{\sigma_c\lambda}{2\pi\epsilon_0}\int\frac{1}{s}\,da$

$da = 2\pi s\cdot L$

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The total current

$$I = \frac{\sigma_c \lambda}{\epsilon_0} L$$

if The potential between b/a

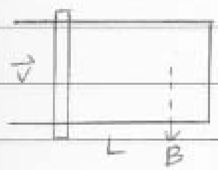
$$V = -\int \vec{E} \cdot d\vec{l} \quad , \quad -\nabla\phi = E$$

$$= -\int_a^b \frac{\lambda}{2\pi\epsilon_0 s} \cdot dl = \frac{-\lambda}{2\pi\epsilon_0} \ln\frac{b}{a}$$

Then the resistivity $\underline{V = IR}$

$$R = \frac{\ln b/a}{2\pi\sigma_c L} \quad , \quad \sigma_c = \frac{\ln b/a}{2\pi R L} = \frac{1}{\rho}$$

$$\mathcal{E}mf = \vec{v}\,\vec{B}\,L$$



We can discuss magnetic flux ($\underline{\int B \cdot d\vec{a}}$) in the term Δt for the charge it sweeps an area $V\Delta t \cdot L$ in time Δt.

* The charge in magnetic flux associated with this motion is $\Delta\Phi = BA = \underline{B\vec{v}}\,\Delta t\,\underline{L}$

* The rate of charge of magnetic flux.

$$|\mathcal{E}| = \vec{v} B L = \frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t}$$

Emf is the magnetic flux per time. (Lenz's Law)

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§7.1 Microscope Ohm's Law.

$\mathcal{E} = \frac{d\Phi_E}{dt}$ (E-field),

(1) E-field $\longrightarrow$ $J = \sigma E = \sigma f$, force per unit charge. $\Phi_E = EA$

(2) B-field $\longrightarrow$ $\mathcal{E} = -\frac{d\Phi}{dt}$, Electromotive force, work/per unit charge.

$\Phi_B = BA$

* Faraday's Law

(3) Lenz's Law : We have not specified the direction of the emf, which is provided by Lenz's Law

$$|\mathcal{E}| = \frac{d\Phi}{dt}.$$

* The induced emf acts in such way as to <u>oppose</u> (反抗) the change of time in flux, generate currents in circuit.

$$|\mathcal{E}| = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$ Faraday's Law

of electromagnetic <u>induction</u>.

$$F = L\frac{d^2 Q}{dt^2} \leftarrow L \leftrightarrow \text{mass}, \quad m\frac{d^2x}{dt^2} = F$$

7.2.1 Faraday's Law

"A changing magnetic field induce an electric field"

$$\frac{d\Phi_B}{dt} \longrightarrow \frac{d\Phi_E}{dt} \ ?$$

The emf is equal to the rate of change of the flux.

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$\mathcal{E} = \oint f_E \cdot d\vec{l}$, $f_E = \frac{F_E}{q} = \frac{qE}{q}$

$= \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

then E is related to the charge in B by the equation.

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t} da}$$

We can apply to the Stoke's theorem

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

Note : $\boxed{\frac{d\Phi}{dt} = \frac{d}{dt} \oint B \cdot dA}$, $\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int B \cdot d\vec{A}}$

The circuit can be through of any closed geometric path in space, not necessarily coincident with an electric field.

so $\frac{d}{dt} \int B \cdot dA = \int \frac{\partial B}{\partial t} dA$

then $\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t} dA$

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* The induced electric field.

Faraday's discovery tells us two kinds of electric fields.

(1) electric charges (e).

(2) changing magnetic field. $\frac{\partial B}{\partial t}$, (L) mass

The latter can be found by Faraday and Ampere's Law.

$$\left.\begin{aligned} \nabla \times E &= 0 \\ &= -\frac{\partial B}{\partial t} \end{aligned}\right] , \quad \left.\begin{aligned} \nabla \times B &= \mu_0 J \\ &= 0 \end{aligned}\right]$$

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Chapter 7. Ohm's Law

Faraday's Law

(1) The E-field can be originated from two sources.

A. Charges. Q. q

B. Changing magnetic field $\rightarrow$ $\boxed{\begin{array}{l} \text{magnetic flux BA} \\ \text{current changes with time.} \end{array}}$

$B(t)$, A - fixed

$I(t)$

$$\oint E \cdot dl = -\frac{d}{dt}\int B \cdot dA = -\frac{d}{dt}\Phi = -A\frac{dB}{dt}$$

$$= -\int \frac{\partial B}{\partial t} dA$$

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Pro. 7.12 A long solenoid of radius a, is driven by an alternating current, so that the field inside is $B(t) = B_0 \cos(\omega t)\,\hat{z}$. A circular loop of wire, radius $a/2$, resistance R. is placed inside the solenoid. Find the current induced in the loop.

1. If the magnetic flux is $\Phi = BA$, A : observed area $= \pi a^2/4$

2. Then the emf is

$$\underline{\mathcal{E}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[\frac{\pi a^2}{4} B_0 \cos(\omega t)\right]$$

$$= \frac{\pi \omega a^2}{4} B_0 \sin \omega t$$

3. $\boxed{\mathcal{E} = IR \;,\quad I = \frac{\pi \omega a^2}{4R} B_0 \sin \omega t}$

P.7.13 A square loop of wire, with sides of length a, lies in the first quadrant of the xy plane. In the region is a non-uniform time-dependent magnetic field $\boxed{B(y,t) = k y^3 t^2 \hat{z}}$. Find emf?

$$\underset{\text{axis}}{y^3} \quad \underset{\text{time}}{t^2}$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int B_{yz} \cdot dA = \frac{-d}{dt}\int_{yz} k y^3 t^2 dA \;,\quad dA = dy\,dz$$

$$= \frac{-d}{dt} k t^2 \frac{1}{4} y^4 \cdot z \Big|_{y,z=a}$$

$$= \frac{-d}{dt}\left[a \cdot \frac{1}{4} k t^2 a^4\right] = \frac{-k t a^5}{2}$$

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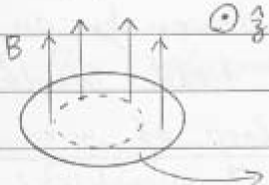


$F_E = qE$.

$W = \int F_e \cdot dl \Rightarrow \frac{W}{q} = \int \vec{E} \cdot d\vec{l} = \int \vec{f} \cdot d\vec{l}$

Ex 7.7, If the B-field is changing with time, what's the induced E-field?

From Faraday's Law of $B(t)$ $\left[\begin{array}{l} \text{B-field} \ \hat{\phi} \\ \text{E-field} \ -\hat{\phi} \end{array} \right.$

E clockwise

$\oint \vec{E} \cdot d\vec{l} = -A \frac{dB}{dt}$

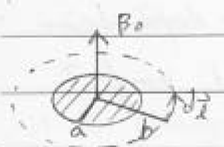
$E \cdot 2\pi s = -\pi s^2 \frac{dB}{dt}$. Therefore $\vec{E} = -\frac{s}{2} \frac{dB}{dt} \hat{\phi}$.

Ex 7.8, A line charge λ.

1. $\oint E \cdot d\vec{l} = -\pi a^2 \frac{dB}{dt}$

2. The torque is $\vec{r} \times \vec{F} = b \times qE$

$= b \times (\lambda dl) E = N$



Then the torque is

$N = \oint b\lambda \underline{E\, dl} = b\lambda \oint E \cdot d\vec{l} = +b\lambda(-\pi a^2)\frac{dB}{dt}$

$= -\pi\lambda a^2 b \frac{dB}{dt}$

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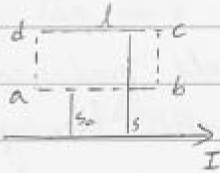
3. The total angular momentum is

$$\int N\,dt = -\lambda\pi a^2 b \int_{B_0}^{0} dB = \boxed{\lambda \pi a^2 b B_0}$$

Ex 7.9 , An infinity long straight wire carries a slowly varying current $I(t)$. Determine the induced electric-field as a function of the distance s , from the wire.



From Ampere's Law . $B(t) = \dfrac{\mu_0 I(t)}{2\pi s'}$,

the induced E - field

$$\oint E \cdot dl = \frac{-d\Phi}{dt} = -\frac{d}{dt}\int_{s_0}^{s} B(t)\,dA = -\frac{d}{dt}\int_{s_0}^{s} \frac{\mu_0 I}{2\pi s'} \boxed{ds' \cdot l}$$

$$= \frac{-d}{dt}\left(\frac{\mu_0 I l}{2\pi}\right) \ln s'$$

$$\int_{s_0}^{s} \frac{1}{s}\,ds = \ln s$$

$$\Rightarrow \int \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left(\frac{\mu_0 I l}{2\pi}\right)\left[\ln s - \ln s_0\right]$$

$$\boxed{E = \frac{\mu_0}{2\pi}\left(\ln s - \ln s_0\right)\frac{dI}{dt}}$$

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Example review

1. E-field $\propto \boxed{\frac{dI}{dt}}$

2. E-field $\propto \frac{-dB}{dt}$

3. $\mathcal{E}mf = -\frac{d\Phi}{dt}$

1.2.3 Inductance

1. For a given electrical circuit, the B-field produced by any point is proportional to the current flowing in the circuit, changing with time.

2. The magnetic flux Φ linking any closed path is proportional I, We may write as $\Phi \equiv LI \equiv BA$.

3. If the magnetic field, the flux varies with time, if $\Phi(t, I)$ function.

$m\frac{dv}{dt} = F$

$$\frac{d\Phi(t,I)}{dt} = \frac{d\Phi}{dI}\frac{dI}{dt} \equiv L\frac{dI}{dt} \quad (E \sim \frac{dI}{dt}) \text{ for magnetic field } B(t).$$

$$\equiv L\frac{d^2q}{dt^2} \text{ for charges} \quad \nabla \cdot E = \rho/\varepsilon_0$$

$\Rightarrow F = ma = m\frac{d^2x}{dt^2}$

4. A emf ε is also induced in the circuit as current I.

$$\varepsilon = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}.$$

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§ 7.2.3 Inductance

1. Magnetic flux $\Phi = L I_{(t)}$
2. Flux varies with time

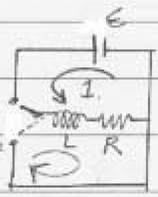
$$\mathcal{E} = \frac{d\Phi}{dt} = -L\frac{dI_{(t)}}{dt}$$

3. The negative sign reminds us that this induced emf tends to oppose the change of current.

4. The unit of L is called Henry

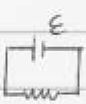
$$1 \text{ henry} = 1 \frac{\text{web}}{\text{Ampere}} = 1 \frac{\text{Voltage} \cdot \text{second}}{\text{Ampere}}$$

 Symbol L.



1. If I is the current flowing at any time $\underline{t}$ after the switch k_1 is closed, we have $\mathcal{E}_0 = \underline{L\frac{dI}{dt}} + RI$ or $L\frac{dI}{dt} + RI = \mathcal{E}$, slowing the differential eq, with the initial condition.

$$\begin{cases} t = 0, \; I = 0 \\ I = I_0\left(1 - \exp\frac{-Rt}{L}\right) = I_0 - \boxed{I_0 \exp\frac{-Rt}{L}} \end{cases}$$

2. where 

$I_0 = \mathcal{E}/R$ as $t \to \infty$. The time L/R is the time constant or relaxation time $\tau \equiv \frac{L}{R}$

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Then the eq can be expressed as

$$I = I_0 - I_0 \exp^{-\frac{t}{\tau}}$$

* When connected to K_2, we have

$$L\frac{dI}{dt} + RI = 0 \Rightarrow I = I_0 \exp^{\frac{-Rt}{L}}$$

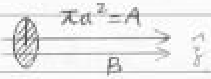
where the initial condition is $I = I_0$, at $t = 0$

Ex. Calculate the inductance L of a long solenoid.
Magnetic field inside a long solenoid is

$$\underline{B} = \frac{\mu_0 I N}{S} = n \mu_0 \underline{I}$$

$n = \frac{N}{S}$, S : length

$$\Phi = BA = \frac{\mu_0 N I}{S} \pi a^2$$



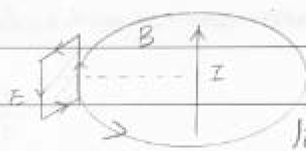
$$L = \frac{N\Phi}{I} = \frac{\mu_0 \pi N^2 a^2}{S} = \mu_0 \pi a^2 \underline{n}\, \underline{N}.$$

§ 7.2.3 Mutual inductance & Neumann's Formula
(互感)

If we consider $\boxed{\text{more than one circuit}}$. We generalized equation to

$$\frac{d\Phi_{kl}}{dt} = \frac{d\Phi_{kl}}{dI_l}\frac{dI_l}{dt} = M_{kl}\frac{dI_l}{dt}$$

where M_{kl} is the mutual inductance between circuit k & circuit $\underline{l}$.



$$\oint E \cdot dl = -\frac{d}{dt}\int B\, dA$$

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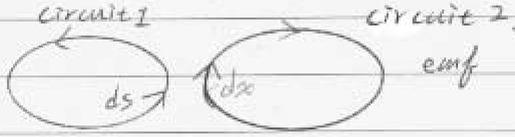
$\boxed{\text{The unit of } k \text{ is henry , } M_{kk} = \frac{d\Phi_{kk}}{dI_k} = L_k}$



$$\Phi_{21} = M_{21} I_1 \ , \quad M_{21} = \Phi_{21}/I_1$$

The induced emf in circuit 2 , $\mathcal{E} = -M_{21} \frac{dI_1(t)}{dt}$

To drive Biot - Savart + Neumanns formula

$\vec{B} = \nabla \times \vec{A}$, we can find

$$\mathcal{E} = -\int_{A_\lambda} \frac{\partial \vec{B}}{\partial t} \, \underline{dA_\lambda} = -\int \nabla \times \frac{\partial \vec{A}}{\partial t} \cdot dA_\lambda$$

Stoke's theorem

$$\boxed{\frac{\partial A}{\partial t} = \frac{\mu_0}{4\pi} \frac{dI}{dt} \oint \frac{ds}{r}}$$

$$\boxed{\mathcal{E} = -\oint \frac{\partial \vec{A}}{\partial t} \cdot d\lambda} \ , \text{ where } \vec{A} = \frac{\mu_0}{4\pi} \oint \frac{I \cdot ds}{r}$$

$$\mathcal{E} = -\oint \frac{\mu_0}{4\pi} \frac{dI}{dt} \oint \frac{ds}{dr} \cdot d\lambda$$

$$= \left[-\oint_\lambda \oint_s \frac{\mu_0}{4\pi} \frac{ds \, d\lambda}{r} \right] \frac{dI}{dt}$$

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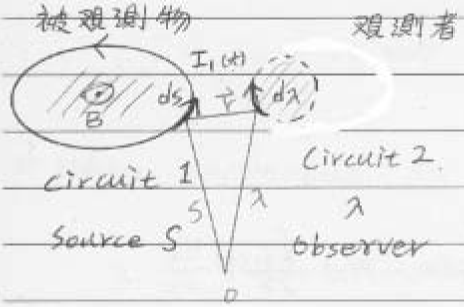
* 1.2 Mutual inductance 互感



* The <u>flux</u> is induced by circuit 1.

$$\Phi_{21} = M_{21} I_1 \quad , \quad M_{21} = \Phi_{21}/I_1$$

* Emf : $\varepsilon = -M_{21} \frac{dI_1(t)}{dt}$

$$\varepsilon = -\frac{d\Phi}{dt} = -\overset{M}{\frac{d\Phi}{dI}}\ \overset{L}{\frac{dI}{dt}}$$

* To derive Neumann's formula $\varepsilon = -\frac{d}{dt}\int_\lambda B_s \cdot dA_\lambda$

Then the B-field can be replaced as $\vec{B}_s = \nabla \times \vec{A}_s$ represent by vector potential $\vec{A}_s$. Then apply Stoke's Theorem

$$\varepsilon = \frac{-d}{dt}\int_\lambda \nabla \times A_s \cdot dA_\lambda \quad = \frac{-d}{dt}\oint \vec{A}_s \cdot d\vec{\lambda}$$

* The vector potential

$$A_s = \frac{\mu_0}{4\pi}\int \frac{I_1 \cdot ds}{r}$$

So the Emf : $-\frac{d}{dt}\oint_s\oint_\lambda \boxed{\frac{\mu_0}{4\pi}\frac{I_r}{r}\, d\vec{s}}\ d\vec{\lambda}$

$$\varepsilon = -\frac{dI(t)}{dt}\oint_s\oint_\lambda \frac{\mu_0}{4\pi}\frac{d\vec{s}\cdot d\vec{\lambda}}{r}$$

$$M_{21} = \oint\oint \frac{\mu_0}{4\pi}\frac{d\vec{s}\cdot d\vec{\lambda}}{r}$$

We could get the definition of M is $M = \frac{\mu_0}{4\pi} \oint\oint \frac{d\vec{s} \cdot d\vec{\lambda}}{r}$, where ds & $d\lambda$ are two elements of length & r is the distance between them.

A. M_{21} is purely geometrical quantity, having to do with the sizes (dA), shapes (s, λ), and relative position (r).

B. The mutual inductance is unchanged if we switch the roles of loop λ & s.

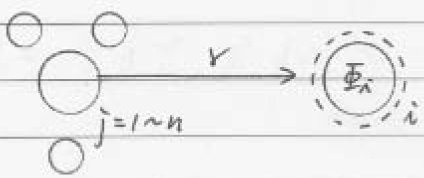
Then $M_{21} = M_{12}$

$$\Sigma M_{ij} \Rightarrow \left.\begin{matrix} M_{11} + M_{12} + M_{13} \cdots\cdots \\ M_{21} + M_{22} + M_{23} \cdots\cdots \end{matrix}\right] \frac{1}{2}$$

§ 7.2.4 Magnetic energy in terms of circuit parameter.

- We now apply it to n coupled circuits, then the flux changes are directed related with changes in the currents in the n circuits.

$$d\Phi_i = \Sigma \frac{d\Phi_{ij}}{dI_j} dI_j = \sum_j M_{ij}\, dI_j$$



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* for stationary circuit, no mechanical work, ($dw = 0$) is associated with the flux changes $d\Phi_i$.

Then dw is equal to dU (the change in magnetic flux).

* If all currents are built up at all the same fraction (α) of their final values.

$I_1' = \alpha I_1 \quad , \quad d\Phi = \Phi_1 \, d\alpha$

$\int dw = \int_0^1 \underline{d\alpha} \; \underline{\Sigma I_i \Phi_i} = \frac{1}{2} \sum_{i=1}^{n} I_i \Phi_i$

Thus the magnetic energy of the system is

$U = \frac{1}{2} \Sigma I_i \Phi_i$ Note that for linear media

$= \frac{1}{2} \Sigma M_{ij} I_i I_j$

$= \frac{1}{2} M I^2 , \; \boxed{\frac{1}{2} L I^2}$ 7.22

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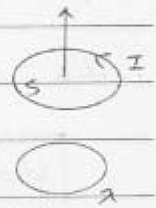
* Mutual Inductance For linear media.

$M_{ij} = M_{ji}$ & $M_{ii} = L_i$, $W = \frac{1}{2} M I^2$

* b Magnetic energy in terms of field vector.

We assume a single loop, then the flux Φ_i may be expressed as



$$\Phi_i = \int B \cdot \hat{n}\, ds = \int \underline{\nabla \times \vec{A}} \cdot \hat{n}\, ds \left(= \oint_\lambda \vec{A}_s \cdot dl_\lambda \right)$$

$$= \oint_c \vec{A} \cdot dl \quad \text{(for single)}$$

where, c is the enclose path

$$U = \frac{1}{2} \sum I_i \Phi_i$$

$$= \frac{1}{2} \sum_i \oint I_i \vec{A} \cdot d\vec{l}$$

Note : We can change $I_i dl_i$ with.

① $j \cdot da \cdot dl = \underline{j\, dv}$ $\left(J = \frac{I}{A} ,\ j\, da = I \right)$

② $\sum_i \oint_i \rightarrow \underline{\int d\alpha} \rightarrow \int_v dv$

sum fraction Integration

$\Rightarrow \boxed{U = \frac{1}{2} \oint \vec{J} \cdot \vec{A}\, dv}$

Apply $\vec{B} = \nabla \times \vec{A}$, $\nabla \times B = \mu_0 J$

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if $\nabla \cdot (\vec{A} \times \vec{B})$

$= \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$

$= \vec{B} \cdot \vec{B} - \vec{A} \cdot (\mu_0 \vec{J})$

$\mu_0 \vec{A} \cdot \vec{J} = \vec{B} \cdot \vec{B} - \nabla \times (\vec{A} \times \vec{B})$

We obtain

$$\boxed{u = \frac{1}{2\mu_0} \underbrace{\int_V \vec{B} \cdot \vec{B}\, dV}_{V} - \frac{1}{2\mu_0} \int_A \overset{\overset{0}{\parallel}}{\nabla \cdot (\vec{A} \times \vec{B})}\, dV}$$

$\downarrow$

$\nabla \times \left(\frac{1}{r} \times \frac{1}{r^2}\right)$

$\downarrow$

$\frac{1}{r^4} d r^3$

$$= \frac{1}{2\mu_0} \int \vec{B} \cdot \vec{B}\, dV.$$

$u_V = \frac{u}{V} = \frac{1}{2\mu_0} |\vec{B}|^2$

if $\int \vec{\nabla} \times (\vec{A} \times \vec{B})\, dV = \oint_S \vec{A} \times \vec{B} \cdot \underline{\hat{n}\, ds}$

when S is the surface bounds the volume.

$u_a = \frac{u}{A} \sim 0$ (太小了，量不到)

We can take any region larger than this for current density j is zero out there.

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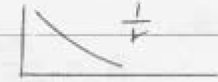
because ① $\vec{B}$ falls off at least as fast as $\frac{1}{r^2}$

 $\frac{1}{r}$

② $\vec{A} \sim \frac{1}{r}$ ③ Surface, r^2

Then the second term of surface integral vanishes

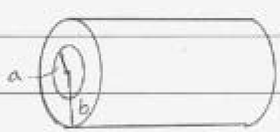
$$W_{mag} = U = \frac{1}{2\mu_0}\int \vec{B}\cdot\vec{B}\, dv = \frac{1}{2}\int \vec{H}\cdot\vec{B}\, dv$$

$\Rightarrow u = \frac{1}{2}\vec{H}\cdot\vec{B}$ for the case of isotropic

$u = \begin{bmatrix} u_{11} & \cdots & \\ & \ddots & \\ & & u_{33} \end{bmatrix}$ tensor for the anisotropic

$$W_{ele} = \frac{\epsilon_0}{2}\int \vec{E}\cdot\vec{E}\, dv = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, dv$$

Ex. A long coaxial cable carries current I. ☆ Important



the outter current I ←

inner current I →

Find the magnetic energy stored in a section of Length L.

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According to Ampere's Law the field between the cylinders,

$$B = \frac{\mu_0 I}{2\pi\rho}\hat{\phi}$$

The energy per unit volume is

$$u = \frac{1}{2\mu_0}B^2 = \frac{1}{2\mu_0}\left[\frac{\mu_0 I}{2\pi\rho}\right]^2 = \frac{\mu_0 I^2}{8\pi^2\rho^2}$$

total $U = \int u\,dv = \int \frac{\mu_0 I}{8\pi^2\rho^2} \cdot l\, 2\pi\rho\, d\rho$

$$= \frac{\mu_0 I^2 l}{4\pi}\int\frac{1}{\rho}\,d\rho = \frac{\mu_0 I^2 l}{4\pi}\ln\frac{b}{a}$$

The unit volume of cylinder is $(2\pi\rho\, d\rho)\, l$

* Find the inductance $U = \frac{1}{2}LI^2$

$$\boxed{L = \frac{2U}{I^2} = \frac{\mu_0 l}{2\pi}\ln\frac{b}{a}}$$

7.5mm×28行

7.3 Maxwell Equation

7.3.1 Electrodynamics before Maxwell. (old rules)

1. Gauss's Law $\nabla \cdot E = \rho/\varepsilon_0$ ✓
2. Gauss's Law $\nabla \cdot B = 0$
3. Ampere's Law $\nabla \times B = \boxed{\mu_0 J}$ $\mu_0 J'$
4. Faraday's Law $\nabla \times E = -\frac{\partial B}{\partial t}$ < charge. / changing B-field.

5. The divergency of curl is zero. $\nabla \cdot (\nabla \times A) = 0$
   E
   B

These eqs. represent the state of EM theory over a century.

連續性方程式在 Maxwell 之後出現.

* From the eqs of 5. the old rules that divergency of curl is alway zero.

1. $\nabla \cdot (\nabla \times E) = 0$ Electric statics.

equal ||

$$\nabla \cdot \left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\underbrace{(\nabla \cdot B)}_{=0} = 0$$

2. $\nabla' \cdot J' = 0$, $\nabla \cdot (\nabla \times A) = 0$

$\nabla \cdot (\nabla \times B) = 0$

? || ↗ $\nabla' \cdot J' + \text{const} = 0$

$\nabla \cdot (\mu_0 J) = \mu_0 \underline{(\nabla \cdot J)} = ?\ (0)$

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7.3.2 How Maxwell fixed Ampere's Law

Continuity eq. & Ampere's Law, they were all vaild even in time varying situations.

連續体方程與安培定律是不可分割的.

$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$

$\nabla \times B = \mu_0 J$

& realize Ampere's Law was in consistent with the continuity eq.?

if we taken the $\nabla \cdot (\nabla \times B)$ on both sides

$\nabla \cdot (\nabla \times B) = 0$

$\nabla \cdot (\mu_0 J) = 0$

This is indeed true of ρ does not change with time.

* But it is not true when ρ is changing with time. Maxwell's suggested a way out of this.

Using Gauss's Law

$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\nabla \cdot \epsilon_0 \vec{E})$

or

$\boxed{\nabla \cdot (\vec{J} + \frac{\partial}{\partial t} \epsilon_0 \vec{E}) = 0} \Longleftrightarrow \boxed{\vec{D} = \epsilon_0 \vec{E} + P}$ P.175

$\nabla \cdot (\nabla \times B) = \nabla \cdot (\mu_0 J) = \nabla \cdot \mu_0 \left[\underline{\vec{J} + \epsilon_0 \frac{\partial E}{\partial t}} \right] = 0$

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期末考 6/18 10:00　(30% 期中, 70% Ch 6.7)　DATE 5/29

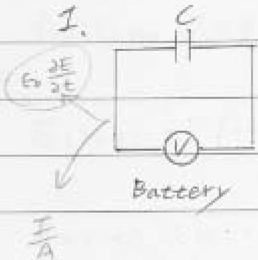
平時 40% +10%，期中·末 30%×2

The term of $\boxed{\epsilon_0 \frac{\partial E}{\partial t}}$ has the dimensions of the current density & called "displacement current" $= \epsilon_0 \frac{\partial E}{\partial t}$

$$\vec{D} = \epsilon_0 E + P \longrightarrow \frac{\partial \vec{D}}{\partial t} = \boxed{\epsilon_0 \frac{\partial E}{\partial t}} + \frac{\partial P}{\partial t}$$

Let consider a simple circuit such as parallel plates capacitor connected to a battery.

I.



① $I = \frac{dQ}{dt}$, $J = \frac{I}{A}$

② $\boxed{\epsilon_0 \frac{\partial E}{\partial t}}$ (New) $\Rightarrow$ $\boxed{\epsilon_0 \frac{\partial}{\partial t}\left(\frac{Q}{A\epsilon_0}\right) = \frac{1}{A}\frac{\partial Q}{\partial t} = \frac{I}{A}}$

$E = \sigma/\epsilon_0 = \frac{Q}{A\epsilon_0}$

6/4

Maxwell's eq

<1> The divergency of curl is zero. $\nabla \cdot (\nabla \times E) = 0$.

<2> From Ampere's Law, $\nabla \times B = \mu_0 J$

Drive $\nabla \cdot (\nabla \times B) = \nabla \cdot \mu_0 \left(\underline{\vec{J} + \epsilon_0 \frac{\partial E}{\partial t}}\right) = 0$

(3) $\epsilon_0 \frac{\partial E}{\partial t}$ is called as displacement current, if in the steady state

$\nabla \cdot j = -\frac{\partial \rho}{\partial t}$ continuity Eq.

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* if I/A gives the current density, then the quantity

$\epsilon_0 \frac{\partial E}{\partial t}$ can be interpreted as the density of the

current. $\boxed{J,\ \epsilon_0 \frac{\partial E}{\partial t},\ I/A,\ \rho,\ \sigma.}$

* if a charging sphere / Battery ε $\Rightarrow \frac{\partial E}{\partial t} \neq 0$

if the E-field is $\frac{q(t)}{4\pi\epsilon_0 r^2}$

$$\frac{\partial E}{\partial t} = \frac{1}{4\pi\epsilon_0 r^2}\frac{\partial q}{\partial t} \Rightarrow \frac{\partial q}{\partial t} = I, \quad 4\pi r^2 = A$$

$$\frac{\partial E}{\partial t} = \frac{I}{A\epsilon_0} \Rightarrow \text{prove} \quad \boxed{\epsilon_0 \frac{dE}{dt} = \frac{I}{A}} = J_D$$

J_D : Displacement current (density).

The vector function of the displacement current density Id of curl must be $\boxed{\text{Zero}}$.

$$\begin{cases} \nabla \times J_D = 0 = \epsilon_0 \nabla \times \frac{\partial E}{\partial t} = \epsilon_0 \frac{\partial}{\partial t}\underset{0}{\underbrace{(\nabla \times E)}} \\ \nabla \times B = \mu_0 J \end{cases}$$

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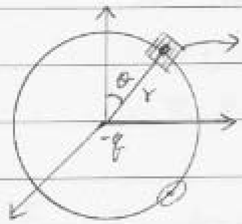
$*\ \nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow \nabla \times J_D = -\epsilon_0 \frac{\partial^2 B}{\partial t^2}$

B - field changing with time.



$E = \frac{-q}{4\pi\epsilon_0 r^2}$, q = constant

(P.321)

$$\boxed{\begin{array}{l} E(t, r) \\ \theta(v, t) = \theta(\underline{vt} - r) \end{array}}$$

Problem 7.34. Suppose

$$E(r,t) = \underbrace{\left(-\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\right)}_{E_r} \underbrace{\left[\theta(vt-r)\right]}_{E_\theta} \quad E_\phi$$

$\delta_\theta(vt-r)\,d\theta$

$B(r,t) = 0$

Show that these fields satisfy all of the Maxwell's equations and determine ρ, J_D. $J = \rho v$.

$\nabla \cdot B = 0$, $\nabla \cdot E = 0$?

$\nabla \times E = 0$, $\nabla \times B = 0$

$\Rightarrow$ Calculate $\nabla \cdot \vec{E}$

$$\nabla \cdot \left[\frac{-q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\right]\left[\theta(vt-r)\right]$$

$$= \boxed{\theta(vt-r)\ \nabla \cdot \left(\frac{-q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\right)} + \boxed{\frac{-q}{4\pi\epsilon_0}\frac{1}{r^2}\,\hat{r} \cdot \nabla_r \theta(vt-r)}$$

$r, \theta, \phi \rightarrow \delta^3(r, \theta, \phi)$

The 1^{st} term

$$\theta(vt-r)\left(\frac{-q}{4\pi\epsilon_0}\right)\left[\nabla\cdot\left(\frac{\hat{r}}{r^2}\right)\right] = \theta(vt-r)\left(\frac{-q}{4\pi\epsilon_0}\right)\left[4\pi\delta^3(r)\right]$$

$$= \frac{-q}{\epsilon_0}\theta(vt-r)\,\delta^3(r)$$

The 2^{nd} term

$$\frac{-1}{4\pi\epsilon_0}\frac{q}{r^2}\left[\hat{r}\cdot\nabla_r\left(\theta(vt-r)\right)\right] \longrightarrow \boxed{\text{See P.1.45} \quad \frac{d\theta}{dx} = \delta(x),\ x\frac{d}{dx}\delta(x) = -\delta(x)}$$

$$\boxed{\begin{aligned} \frac{d}{dr}\theta(vt-r) &= \frac{-d}{d(-r)}\theta(vt-r) = -\delta(vt-r) \\ \frac{d}{dt}\theta(vt-r) &= v\frac{d}{dvt}\theta(vt-r) = v\,\delta(vt-r) \end{aligned}}$$

$$\Rightarrow 2^{nd}\ \text{term} = \underline{+\frac{q}{4\pi\epsilon_0 r^2}\delta(vt-r)}$$

$$\nabla\cdot E = -\frac{q}{\epsilon_0}\delta^3(r)\,\theta(t) + \frac{q}{\epsilon_0 4\pi r^2}\delta(vt-r) = \rho/\epsilon_0$$

$$\rho = -q\,\delta^3(r)\,\theta(t) + \frac{q}{4\pi r^2}\delta(vt-r)$$

$$J_D = \epsilon_0\frac{\partial E}{\partial t} = \epsilon_0\left(-\frac{q}{4\pi\epsilon_0 r^2}\hat{r}\right)\underbrace{\frac{\partial}{\partial t}\theta(vt-r)}_{v\,\delta(vt-r)}$$

$$= \frac{-q}{4\pi r^2}\hat{r}\,v\,\delta(vt-r)$$

$$J = -\epsilon_0\frac{\partial E}{\partial t} \quad (\Rightarrow \nabla\times B = \mu_0 J)$$

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

§ 7.3.4 Magnetic charge. / Monopole

if has a magnetic charge density ρ_m

Case I, $\rho_e = 0$, $\rho_m = 0$, $J_e = 0$, $J_m = 0$

In free space, space-free

$$\nabla \cdot E = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0, \quad \nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Case II In free-space, But

Then we can replace

$\vec{E} \xrightarrow{by} \vec{B}$ ① $\nabla \cdot \vec{E} = 0 \Rightarrow \nabla \cdot \vec{B} = 0$

$\vec{B} \xrightarrow{by} -\mu_0 \epsilon_0 \vec{E}$ ② $\nabla \cdot B = 0 \Rightarrow \nabla \cdot (-\mu_0 \epsilon_0 \vec{E}) = 0$

③ $\nabla \times E = -\frac{\partial B}{\partial t}$

$\downarrow$

$\nabla \times B = -\frac{\partial}{\partial t}(-\mu_0 \epsilon_0 \vec{E}) = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

④ $\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

$\downarrow$

$\nabla \times (-\mu_0 \epsilon_0 \vec{E}) = \mu_0 \epsilon_0 \frac{\partial B}{\partial t} \Rightarrow \nabla \times \vec{E} = -\frac{\partial B}{\partial t}$.

Case III. There are something missing

From $\nabla \cdot B = 0$, $\nabla \times E = -\frac{dB}{\partial t}$

if we has the two eqs for electric charges.

1. $\boxed{\nabla \cdot E = \frac{\rho_e}{\epsilon_0}}$, $\boxed{\nabla \times B = \mu_0 J_e + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$ ok!

2. $\nabla \cdot B = \mu_0 \rho_m$ $\quad \nabla \times E = -\mu_0 J_m - \frac{\partial B}{\partial t}$

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$\rho_m, \rho_e \equiv$ Define as the $\boxed{\text{density}}$ of magnetic / electric charge

$I_m, I_e \equiv$ Define as the $\boxed{\text{current}}$ of magnetic / electric charge.

& Both charges would be conserved.

$$\nabla \cdot J_m = -\frac{\partial \rho_m}{\partial t}$$

$$\nabla \cdot J_e = -\frac{\partial \rho_e}{\partial t} \Rightarrow \nabla \cdot (J_m + J_e) = -\frac{\partial}{\partial t}(\rho_m + \rho_e) \Rightarrow ?$$

Maxwell's eq beg for the existance of magnetic charge. As far as we know, ρ_m is zero everywhere.

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7.3.4 Magnetic charge

continuity Eq. for magnetic charge

$$\nabla \cdot J_m = -\frac{\partial \rho_m}{\partial t} \quad , \; L_m . M_m \ldots$$

Problem 7.36

Suppose a magnetic monopole q_m passes through a resistanceless loop of wire with self-inductance L. What's the induced current in the loops.

7.5mm×28行

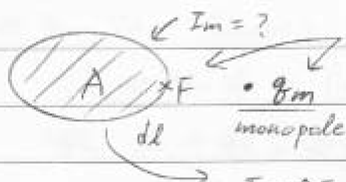
From Faraday's Law

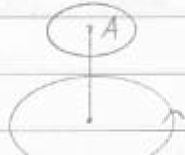
$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$, if there are one q_m , then

* $\nabla \times \vec{E} = \underline{-\mu_0 J_m} - \frac{\partial B}{\partial t}$ (modify Faraday's Law)

$\int \nabla \times \vec{E}\, da = -\int \mu_0 J_m\, da - \frac{\partial}{\partial t}\int B \cdot da$

$\int E \cdot dl = \boxed{-\mu_0 I_m - \frac{\partial \Phi}{\partial t} = \varepsilon}$ Eq of motion / cirwit





$F = qE$, $\int F \cdot dl = w$, $\int \frac{F}{q}\, dl = \varepsilon = \int f \cdot dl$

if we know $\varepsilon = -L\frac{dI}{dt}$

$\Rightarrow -L\frac{dI}{dt} = -\mu_0 \cdot I - \frac{\partial \Phi}{\partial t}$ then

$$\frac{dI}{dt} = \frac{\mu_0}{L}\frac{\overbrace{\Delta Q}^{=q_m}}{\Delta t} + \frac{1}{L}\underbrace{\frac{d\Phi}{dt}}_{=0} \rightarrow \phi$$

Per unit Δt

$$\boxed{I_m = \frac{\mu_0 q_m}{L}} + 0$$

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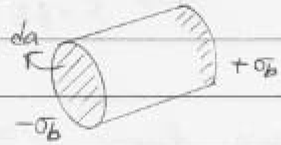
7.3.5 Maxwell's equations in Matter.

That an $\left.\begin{array}{l}\text{electric polarization } P \\ \text{magnetic magnetization } M\end{array}\right]$ produces

a $\left\{\begin{array}{l}\text{bound charge } \rho_b = -\nabla\cdot P \text{ , I} \\ \text{bound current } J_b = \nabla\times M \text{ , J}\end{array}\right.$

* Any charge in the electric polarization involves a flow charge, which will be induced in the total current.

The polarized introduces a charge density $\sigma_b = P$ at one end $-\sigma_b$
$+\sigma_b$



If P now increase a bit

$$dI = \frac{d\sigma_b}{dt}\, da_\perp = \frac{\partial P}{\partial t}\, da_\perp$$

The current density, therefore is

$$J_P = \frac{\partial P}{\partial t}, \quad \therefore \nabla\cdot J_P = \nabla\cdot\frac{\partial P}{\partial t}$$

$$= \frac{\partial}{\partial t}\boxed{\nabla\cdot P} = -\frac{\partial \rho_b}{\partial t}$$

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The continuity eq. is satisfied in fact.

The total charge density $\rho = \rho_f + \rho_b = \boxed{\rho_f - \nabla \cdot P}$

current density $J = J_f + J_b + J_P$

$$= \boxed{J_f + \nabla \times M + \frac{\partial P}{\partial t}}$$

So that the Gauss's law

$$① \ \nabla \cdot E = \frac{1}{\epsilon_0}(\rho_f - \nabla \cdot P) \ , \ \nabla \cdot B = 0$$

∗ So the Gauss's Law in matter

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_f - \nabla \times P$$

$$\Rightarrow \nabla \cdot (\epsilon_0 \vec{E} + P) = \rho_f$$

$$\nabla \cdot D = \rho_f \quad \boxed{P.175}$$

in the static-state

$$D = \epsilon_0 \vec{E} + P$$

Mean while the Ampere's Law in matter.

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad (\text{in space})$$

$$\Rightarrow \nabla \times B = \mu_0 (J_f + \nabla \times M + \frac{\partial P}{\partial t}) + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times \frac{B}{\mu_0} = (J_f + \nabla \times M + \frac{\partial P}{\partial t}) + \epsilon_0 \frac{\partial E}{\partial t} \quad (\text{if } H = \frac{B}{\mu_0} - M)$$

$$\nabla \times (\frac{B}{\mu_0} - M) = J_f + \boxed{\frac{\partial P}{\partial t} + \epsilon_0 \frac{\partial E}{\partial t}} ,$$

$$\Rightarrow \boxed{\nabla \times H = J_f + \frac{\partial D}{\partial t} \quad (\epsilon_0 \vec{E} + \vec{P} = \vec{D})}$$

Faraday's Law

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

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E. B in Matter

Maxwell

① $\nabla \cdot \vec{D} = \rho_f$

② $\nabla \cdot \vec{B} = 0$

③ $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

④ $\nabla \times \vec{H} = J_f + \frac{\partial P}{\partial t}$

* 7.3.5 Magnetic monopole

For Gauss Law for magnetic field $\nabla \cdot B = 0$?

Dirac shaved the existance of monopole would explain electric charge is quantized.

magnetic charge, $g = \frac{\hbar}{2\mu_0 e} \equiv 1.64 \times 10^{-9}\ A \cdot m$ existed

* Decay of free charge? "Important"

According to Maxwell's eq, a free charge should decay expmentially

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}, \quad J = \sigma \vec{E}$$

then, $\begin{cases} \nabla \times B = \mu_0 \sigma \vec{E} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \\ \nabla \cdot (\nabla \times B) = 0 \end{cases}$

$$\nabla \cdot (\nabla \times B) = \mu_0 \sigma\, \nabla \cdot \vec{E} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \nabla \cdot E$$

$$\nabla \cdot E = \rho / \epsilon_0$$

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$$\Rightarrow \boxed{\mu_0 \frac{\partial \rho}{\partial t} + \frac{\mu_0 \sigma}{\epsilon_0} \rho = 0} \Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon_0} \rho = 0}$$

电荷隨時变

Integration results

$\rho = \rho_0 e^{-t/\tau}$, $\boxed{\tau = \frac{\epsilon_0}{\sigma}}$ is know

as the relaxation time

The relation shows that any original distribution of charge decay exponentially at a rate that is independent of any other electromagnetic distribution.

(期末考至此)