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Chapter 6.

(1) Paramagnetism : 順磁.

Curie Law : Some magnetic materials exhibita magnetization (M) which is proportional to the applied magnetic field. (H) (or H_a) $M \sim H = \mu_B H = m \cdot H$

(2) Ferromagnetism : 鉄磁.

Metal materials (Iron, nickel, cobalt) exhibits a unique magnetic behavior which is called ferromagnetism.

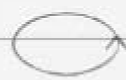
(3) anti-ferromagnetism : 反鉄磁.

$\Uparrow \Downarrow$ In atomic scale of electrons,

$4d^2 \rightarrow \uparrow\downarrow$

(4) Diamagnetism : 逆磁

Lenz's Law : When an external applied magnetic field, these magnetic dipole moment (current loops)



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tend to align in such way as to $\boxed{\text{oppose}}$ the field (H).

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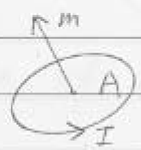
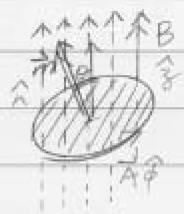
§6.1.2 Torques and Forces on $\vec{m}$.

Define the magnetic dipole moment.

(1) $m \equiv I \int da$, value

(2) 

, direction

(3) 

$\boxed{\text{applied magnetic field } B_a}$ $\hat{z}$

The magnetic moment can be considered to be a vector quantity $\hat{n}$, perpendicular to the right-hand-rule direction $\hat{z}$.

(4) The torques is given by

$$\tau = \vec{m} \times \vec{B_a} = m\hat{n} \times B_a\hat{z}$$

$$\boxed{\tau = mB \sin\theta \ (\hat{n} \times \hat{z})}$$

(5) The potential energy associated with the magnetic moment is

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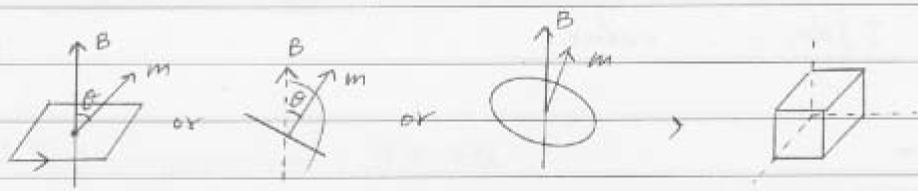
$$U(\theta) = -\vec{m}\cdot\vec{B} = -mB\cos\theta$$

(6) Force is defined (6.3)

$$\vec{F} = -\nabla U(\theta) = \underline{\nabla(m\cdot B)}$$

(7)



$$\vec{\tau} = \vec{m}\times\vec{B}\ ,\ \text{Torques} = mB\sin\theta$$

(8) Rotational work <u>W</u>

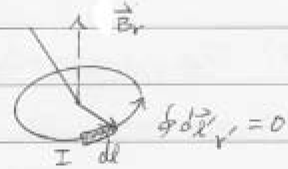
$$W = -\int_0^{\pi} \tau\, d\theta = -\int_0^{\pi} mB\sin\theta\, d\theta = 2mB.$$

The difference in energy between aligned and anti-aliged is

$$\Delta U = 2mB = W_{Rotational}.$$

(9) In a uniform field, <u>the net force</u> on a current loop is $\boxed{\text{zero}}$.

$$\vec{F} = I\oint d\vec{l}\times\vec{B} = (I\oint d\vec{l}')\times\vec{B} = 0$$



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* From the vector potential

$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}$, to magnetic field $\vec{B} = \nabla \times \vec{A}$

$$\vec{B} = \frac{\mu_0}{4\pi} \left[\frac{\vec{m}}{r^3} - \frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5} \right]$$

for example under applied field

In case of multi-magnetic moments.

$\vec{B}_a \longrightarrow \vec{m}_2$
$\vec{m} \longrightarrow \vec{m}_1$, A small coil of magnetic moment $\vec{m}_1$ has potential U, then the

$U = -\vec{m}_1 \cdot \vec{B}_2$, or $\tau = -\vec{m}_1 \times \vec{B}_2$

Two coils of magnetic dipole moments $\vec{m}_1$ & $\vec{m}_2$ separated by a distance $\vec{r}$

$$W = \vec{m}_1 \cdot \vec{B}_2 = \frac{\mu_0}{4\pi} \left[\frac{\vec{m}_1 \cdot \vec{m}_2}{r^3} - \frac{3\vec{m}_1 \cdot (\vec{m}_2 \cdot \vec{r})\vec{r}}{r^5} \right]$$

$$= \frac{\mu_0}{4\pi r^3} \left[\vec{m}_1 \cdot \vec{m}_2 - \frac{3(\vec{m}_1 \cdot \vec{r})(\vec{m}_2 \cdot \vec{r})}{r^2} \right]$$

if 

, W = ? 

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Obviously, the energy is $U_{12} = -\vec{m}_1 \cdot \vec{B}_2$,

For point dipole, the total force is related to the $-\nabla U$

$$\boxed{\begin{aligned} F &= -\nabla U = -\nabla(\vec{m}\cdot\vec{B}) \\ &= -(\vec{B}\cdot\vec{\nabla})\vec{m} + (\vec{m}\cdot\vec{\nabla})\vec{B} + \vec{B}\times(\vec{\nabla}\times\vec{m}) + \vec{m}\times(\nabla\times\vec{B}) \end{aligned}}$$

(In the case of m is fixed, $\vec{\nabla}\times\vec{B} \equiv 0$, then

$\vec{m}\times(\vec{\nabla}\times\vec{B}) = 0$, $\vec{B}\times(\nabla\times\vec{m}) = 0$, $\vec{\nabla}\cdot\vec{B} = 0$

We can obtain

$$\vec{F} = (\vec{m}_1 \cdot \vec{\nabla}_r)\vec{B}_2$$

$$= \left(m_x\frac{\partial}{\partial x} + m_y\frac{\partial}{\partial y} + m_z\frac{\partial}{\partial z}\right)\vec{B}$$

$\vec{m}_1 = m_x\hat{x} + m_y\hat{y} + m_z\hat{z}$

$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$

$\vec{B} = ?$ $\underline{\mu_0 J \times \hat{y}}$

$\neq 0$ ($\neq$ uniform)

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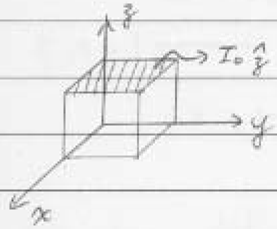
→ B-field.

Problem 6.5, A uniform current density $\boxed{J = J_0 \hat{z}}$ fills a slab stradding the y-z plane from $x=-a$ to $x=+a$.

$\nabla(-U)$

$\boxed{\vec{F} = \nabla(m\cdot B)}$

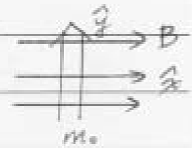
A dipole moment $m = m_0\hat{x}$



if $B = \mu_0 J_0 x\, \hat{y}$

$$\vec{F} = \nabla(m\cdot B) = \nabla\cdot[\mu_0 J x(\hat{y}\cdot m_0\hat{x})] = 0$$

(2) if $m = m_0\hat{y}$



$$F = \nabla(m\cdot B) = \nabla_x[\mu_0 J_0 x\; m_0] \to \hat{x}$$

$$= \underline{m_0\mu_0 J\,\hat{x}}$$

(3) $\vec{F} = \nabla(P\cdot E)$

$= (P\cdot\nabla)\vec{E}$ equal? $\nabla\cdot E \neq 0$

Prove $\nabla(m\cdot\vec{B}) = m\times\underbrace{(\nabla\times B)}_{\downarrow\ \mu_0 J} + \vec{B}\times(\nabla\times\vec{m}) + (m\cdot\nabla)\vec{B} + \boxed{\underset{=0}{(\vec{B}\cdot\nabla)\vec{m}}}$

$$= \mu_0(m\times\vec{J}) + 0 + \boxed{(m\cdot\nabla)\vec{B} + 0}$$

Prove $\nabla(m\cdot B) \neq (m\cdot\nabla)\vec{B}$ ↓

$$F = \nabla(m\cdot B) = \mu_0(m\times\vec{J}) + \underline{\underline{(\vec{m}\cdot\vec{\nabla})\vec{B}}}$$

Prove $(m\cdot\vec{\nabla})\vec{B} = 0$ by yourself.

* show that $B_0 y \hat{x} - B_0 x \hat{y} = A$

prove B-field is $B_0 \hat{z}$

$$B = \nabla \times A = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_0 x & 0 \end{vmatrix} = B_0 \hat{z} \quad , \quad B = \nabla \times A = \begin{vmatrix} \quad \\ \quad \\ -B_0 y \quad 0 \quad 0 \end{vmatrix} = B_0 \hat{z}$$

Then the total solutions just for x-y

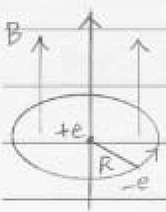
$\vec{A} = \frac{1}{2}(-B_0 y \hat{x} + B_0 x \hat{y})$, (x, y)-axis.

$\Rightarrow \vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})$, for (x, y, z)-axis.

Prove $\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})$

$\Rightarrow \nabla \times A = B = \nabla \times (\frac{1}{2} \vec{B} \times \vec{r})$?

§ 6.1.3 Magnetic dipole moment at the atomic level.



"atomic radius" R , r_n $\boxed{\text{Spin}}$

"current" $I = -e/T = \frac{e}{T}$

$\bar{v} = 2\pi r_n f = \frac{2\pi r_n}{T}$

⇒ For classical magnetic moment

$$\vec{m} = |\vec{m}| = IA = \frac{-e\vec{v}}{2\pi r_n} \cdot \pi r_n^2$$

$$= \frac{-e}{2m}(m\vec{v}\, r_n) = -\gamma \vec{L}$$

We can define the orbital angular moment.

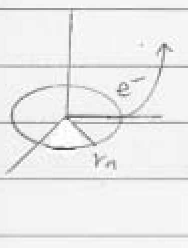
(a) $\vec{L} = m\vec{v}\, r_n$

(b) $\gamma = -e/2m$, gyomagnetic ratio.

圓周運動 $= m_e \frac{v^2}{R}$
庫倫力 $= \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$

Lorentz Force $= \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} + e(\vec{v}\times\vec{B})$
Lorentz Force $= m\frac{\vec{v}^2}{R}$

(1) $\vec{m} = \frac{1}{2}\frac{e}{m}(m\vec{v}\, r_n) = -\gamma\vec{L}$ orbital angular moment



(2) The negative sign indicates that $\vec{m}$ & $\vec{L}$ are in opposite directions, for a negative circulating change.

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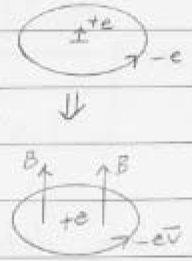
(3) Then the electron speeds up/slow down, dependending on B-field. Then the Force for Columb & Lorentz, respectively.

$\vec{m} \propto -I$



Eq 1. Colomb Force

$$\vec{F} = \frac{e^2}{4\pi\epsilon_0 R^2} = m_e \frac{v^2}{R}$$

Eq 2. Lorentz Force

$$\vec{F} = \frac{e^2}{4\pi\epsilon_0 R^2} + e(\bar{v} \times B) = m_e \frac{\bar{v}^2}{R}$$

$\Downarrow$ steady current

Eq 2 − Eq 1

$$\Rightarrow e(\bar{v} \times B) = \frac{m_e}{R}(\bar{v}^2 - v^2)$$

$$\boxed{e\bar{v}B = \frac{m_e}{R}(\bar{v} - v)(\bar{v} + v)}$$

if Δv is small enough $\Rightarrow v + \bar{v} = 2\bar{v}$

$$evB = \frac{m_e}{R}\Delta v \cdot 2v \Rightarrow \boxed{\Delta v = \frac{eRB}{2m_e}}$$

$\boxed{\text{if } \vec{m} = \frac{-1}{2} e\bar{v}R}$,

Then the Δm , $\Delta m = \frac{-1}{2} e \Delta v R = \frac{-1}{2} e\left(\frac{eRB}{2m_e}\right)R$

$$= -\frac{e^2R^2}{4m_e}B.$$

(4) Δm Notice that the change in m is opposite to the direction of B.

Summary

* Monopole existence

if $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{J(r')}{|r-r'|}\, dv' \Rightarrow \nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$

$\nabla\cdot\vec{A} = 0$

then prove $\nabla\times(\nabla\times A) = \mu_0 J$

* $\nabla\cdot B = \nabla\cdot(\nabla\times A) = A\cdot(\nabla\times\nabla) - \nabla\cdot(\nabla\times\vec{A})$

已知 $\nabla\cdot(\nabla\times A) = 0$, $\nabla\cdot B = 0$

* $-\nabla_r^2 A = \frac{-\mu_0}{4\pi}\int J(r')\, \nabla_r^2\left(\frac{1}{|r-r'|}\right) d^3r'$

$= \frac{+\mu_0}{4\pi}\int J(r')\, \boxed{\nabla_{r'}^2\left(\frac{1}{|r-r'|}\right)}\, d^3r'$

$= \frac{\mu_0}{4\pi}\int J(r')\, 4\pi\,\delta(r-r')\, d^3r'$

$= \mu_0/4\pi \cdot 4\pi J(r) = \mu_0 \cdot J(r)$

ch6.1 End

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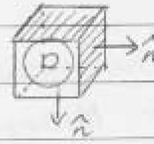
⊙ Dipole moment : $\vec{p} \to \vec{P} \to \vec{P}/V \to$ polarization (Chapter 4)

$\int M\,dV$ = 方向無関 ⟶ ⟵ $\int \vec{M}\cdot d\vec{s} = \int \vec{M}\cdot \hat{n}\, da$ 面積

体積



⊙ Volume density + Surface density under

Polarized $\vec{P}$

magnetized $\vec{M}$ ⟶ $\vec{A}$ vector potential

6.2 The field of a magnetized object. (磁化物体)

6.2.1 Bond current

if a small current loop is equivalent to a magnetic dipole of moment $\vec{m}$.

We define the magnetization $\vec{M}$, over a volume/surface:

$\vec{M} = \lim_{\Delta V \to 0} \sum \vec{m}_i / \Delta V_i$, where $\vec{m}_i$ is the magnetic moment of the i^{th} atom / element.

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We can rewrite the equation as $d\vec{m}' = \vec{M}\,dv'$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \qquad [分離座標系\ r' \& r]$$

$$\boxed{\vec{A}_{magnetized} = \frac{\mu_0}{4\pi}\int \frac{M(r') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,d^3r'}$$

if $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{M}' \times \boxed{(\vec{r}-\vec{r}')}}{\boxed{|\vec{r}-\vec{r}'|^3}}\,d^3r'$

then if $\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{-1}{|\vec{r}-\vec{r}'|^2}(\hat{r}-\hat{r}') = \frac{-(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

$$\boxed{\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)} = \boxed{\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \vec{M}' \times \nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) d^3r' \quad (統一座標系)$$

Math. method : $\nabla' \times (f\vec{A})$

$$\nabla' \times (f\vec{A}') = f(\nabla' \times \vec{A}) - \vec{A} \times (\nabla' f)$$

part 1 for $f(\nabla' \times \vec{A})$

$$\vec{M}' \times \nabla'\left(\frac{1}{|r-r'|}\right) = \boxed{f(\nabla' \times \vec{A})} - \nabla' \times (f\vec{A})$$

$$\Downarrow$$

$$\text{part 1} \equiv \frac{\mu_0}{4\pi}\int \frac{1}{|r-r'|}(\nabla' \times \vec{M}(r'))\,d^3r'$$

For part 1 as volume current

$$A_{volume} \equiv \frac{\mu_0}{4\pi}\int \frac{1}{|\vec{r}-\vec{r}'|}(\nabla' \times M')\, d^3r'$$

$$= \frac{\mu_0}{4\pi}\int \frac{J_b'}{|\vec{r}-\vec{r}'|}\, d^3r' \;, \quad J_b' = \nabla' \times M'$$

For part 2.

Note: $-\int \nabla' \times \frac{\vec{M}(r')}{|\vec{r}-\vec{r}'|}\, d^3r' = \oint \frac{M(r')}{|\vec{r}-\vec{r}'|} \times ds' = \int \frac{M(r') \times \hat{n}' da}{|\vec{r}-\vec{r}'|}$

Prove: if a vector $\overset{\vec{M}'}{}$ · function $\overset{\frac{1}{|r-r'|}}{}$ $\equiv \vec{\alpha}$

if $\boxed{-\int (\nabla' \times \vec{\alpha}')\, dv' = \oint \vec{\alpha} \times ds'}$

if $\int \nabla' \cdot \vec{A}'\, dv' = \oint \vec{A} \cdot d\vec{s}'$ $\boxed{\text{from divergence theorem}}$

then we can set $\vec{A} = \vec{\beta} \times \vec{\alpha}$. $\nabla \times \vec{\beta} = 0$

Using $\int \nabla \cdot \vec{A}\, dv' = \int \nabla \cdot (\vec{\beta} \times \vec{\alpha})\, dv'$

$$\nabla \cdot (\vec{\beta} \times \vec{\alpha}) = \vec{\alpha} \cdot \underset{\overset{\shortparallel}{0}}{(\nabla \times \vec{\beta})} - \vec{\beta} \cdot (\nabla \times \vec{\alpha})$$

then we can get $\nabla \cdot (\vec{\beta} \times \vec{\alpha}) = -\vec{\beta} \cdot (\nabla \times \vec{\alpha})$

$$\int \nabla' \cdot \vec{A}'\, dv' = \int \nabla' \cdot (\vec{\beta}' \times \vec{\alpha}')\, dv' = -\int \vec{\beta}' \cdot (\nabla' \times \vec{\alpha}')\, dv'$$

[Left term]

Right term : $\oint \vec{A}' \cdot d\vec{s}'$

$$= \oint \vec{\beta} \times \vec{\alpha} \cdot ds' \quad , \quad \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A})$$

$$= \vec{C} \cdot (\vec{A} \times \vec{B})$$

$$= \oint ds' \cdot (\vec{\beta} \times \vec{\alpha})$$

$$= \oint \vec{\beta} \cdot (\vec{\alpha} \times ds') \quad [\text{right term}]$$

$$-\int \vec{\beta}' \cdot (\nabla' \times \alpha')\, dv' = \oint \vec{\beta}' \cdot (\vec{\alpha}' \times ds')$$

$\vec{\beta}'$ 取出來

$$-\int (\nabla' \times \alpha')\, dv' = \oint (\vec{\alpha}' \times ds')$$

We know the $\vec{\alpha}$ is defined as vector • function as

$$\vec{\alpha}' \equiv \frac{\vec{M}'}{|\vec{r} - \vec{r}'|}$$

$$K_b' = M' \times \hat{n}'$$

$$\Rightarrow -\int \nabla' \times \frac{M'}{|\vec{r} - \vec{r}'|} d^3r' = \oint \frac{\vec{M}' \times ds'}{|\vec{r} - \vec{r}'|} = \oint \frac{M' \times \hat{n}'}{|\vec{r} - \vec{r}'|} da'$$

☆ • 座标系統一成 (r') 系統
• 分離 $\vec{A}$ vector potential 成体積與面積項.
• 條件是：被磁化的物体".

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \left[\int \frac{\nabla' \times M'(r')}{|\vec{r} - \vec{r}'|} d^3r' + \int \frac{M'(r') \times \hat{n}'}{|\vec{r} - \vec{r}'|} da' \right]$$

The integer are taken over the volume & surface of the magnetization matter.

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a. The vector potential $\vec{A}$ volume produced by volume current under magnetized

$\boxed{\vec{J}(r') = \nabla' \times M(r')}$, $\vec{A}_{volume} = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|\vec{r} - \vec{r}'|} d^3r'$

b. The vector potential $A_{surface}$ produced by surface current $\vec{K}(r')$ is given by

$\vec{A}_{surface}(r) = \frac{\mu_0}{4\pi} \int \frac{K(r')}{|\vec{r} - \vec{r}'|} da'$, $\boxed{K_b(r') = M(r') \times \hat{n}'}$

We see that the vector potential given by left equation would be produced by two magnetization currents ,

a volume density $\vec{J}_M$.

surface current density $\vec{K}_M$.

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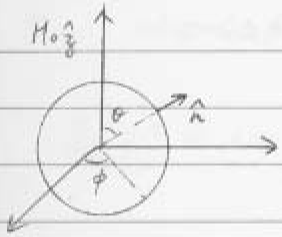
§ Example 6.1, Find the magnetic field of a uniformly magnetized sphere.



if we chose the z-axis along the direction $M_0\hat{z}$.

We can calculate the volume current as

<u>a.</u> $J_b = \nabla \times \vec{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & M_0 \end{vmatrix} = 0$, because M_0 is constant.

<u>b.</u> $K_b = \vec{M} \times \hat{n} = \vec{M} \times \frac{\vec{r}}{r}$,

$$\vec{r} = r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\phi\,\hat{z}$$

$$\vec{M} \times \hat{n} = M_0 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & M_0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\phi \end{vmatrix}$$

$$= \sin\theta\cos\phi\, M_0\,\hat{y} - M_0\sin\theta\sin\phi\,\hat{x}$$

$$= M_0\sin\theta\left[\cos\phi\,\hat{y} - \sin\phi\,\hat{x}\right]$$

$$K_b = M_0\sin\theta\,\hat{\phi}$$

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if a rotating spherical shell of uniform surface charge σ : the surface current.

$$\vec{K}_b = \sigma V = \sigma R \omega \sin\theta \, \hat{\phi} = \vec{M} = \text{magnetization}.$$

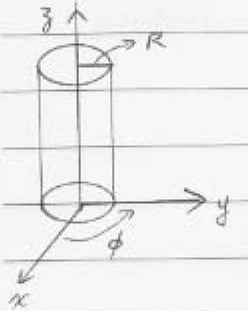
For Text book Ex 5.11

$$\vec{B} = \nabla \times \vec{A} = \frac{2}{3} \mu_0 \sigma R \omega = \frac{2}{3} \mu_0 M.$$

if a sphere $\vec{m} = M \cdot V = M_0 \frac{4}{3} \pi R^3$

$$= (\text{pure dipole moment}).$$

Problem 6.8 A long circular cylinder of radius R carries a magnetization $M = ks^2 \hat{\phi} = ks[-\sin\phi \, \hat{x} + \cos\phi \, \hat{y}]$, where k is constant, s is the distance from the origin.



Find the magnetic field due to $\vec{M}$

$$J_b = \nabla \times M = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\sin\phi \, s^2 & \cos\phi \, s^2 & 0 \end{vmatrix}, \quad \boxed{s^2 = x^2 + y^2}$$

$$= \cos\phi \frac{\partial}{\partial x}(s^2) \hat{z} - \sin\phi \overset{\overset{0}{\shortparallel}}{\frac{\partial}{\partial z}(s^2)} \hat{y} + \sin\phi \frac{\partial}{\partial y}(s^2) \hat{z}$$

$$- \cos\phi \underset{\underset{0}{\shortparallel}}{\frac{\partial}{\partial z}(s^2)} \hat{x}$$

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$= \left(\cos\phi \frac{\partial}{\partial x} s^2 + \sin\phi \frac{\partial}{\partial y} s^2\right)\hat{z}$

Then the calculations of

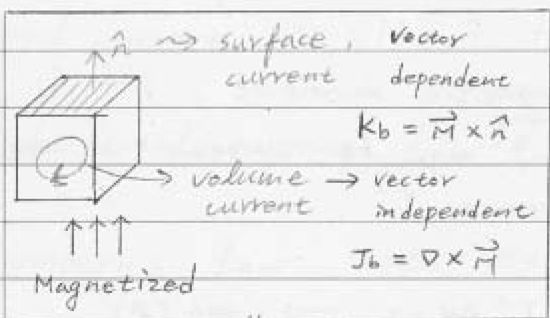
$\frac{\partial}{\partial x} s^2 = 2x \quad , \quad \frac{\partial}{\partial y} s^2 = 2y .$

Then $J_b = \nabla \times M = (2\cos\phi\, x + 2\sin\phi\, y)\,\hat{z}$

$$K_b = M \times \hat{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ ks^2(-\sin\phi) & ks^2\cos\phi & 0 \\ s\cdot\cos\phi & s\cdot\sin\phi & 0 \end{vmatrix}$$

4/24 (Ⅲ)

6.2.1



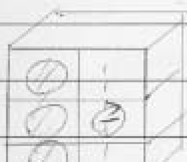
(review)

⇓

Bound current / uniformly

6.2.2 Physical Interpretation of Bound Current.

$\boxed{\text{for non-uniform magnetization}}$



volume current J_b

↑ ↓

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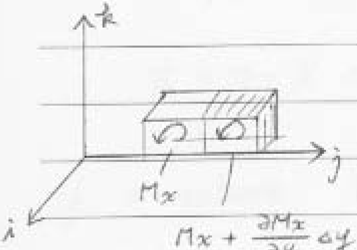
If the magnetization is nonuniform, the $\boxed{\text{cancellation}}$ is not complete. It is evident that between the two Broken lines there is more current moving down / up.



To find the relationship between $\vec{J}_M$ and $\vec{M}$. Let us consider two elements each volume $\Delta x\,\Delta y\,\Delta z$.

1st part : M_x (Magnetization)

2nd part of volume is :

$$M_x + \frac{\partial M_x}{\partial y}\Delta y.$$

The x-component of magnetic moment, 1st.

$\boxed{M_x\,\Delta x\,\Delta y\,\Delta z}$ $\left(M = \frac{\vec{m}}{V}\right)$ may be written in term of a circulating current I_c'.

1st $\quad M_x\,\Delta x\,\Delta y\,\Delta z = I_c'\,\Delta y\,\Delta z \longrightarrow (m = Ia)$

2nd $\quad \left(M_x + \frac{\partial M_x}{\partial y}\Delta y\right)\Delta x\,\Delta y\,\Delta z = I_c''\,\Delta y\,\Delta z$

The net upward current in the middle region of the two volume elements is

$$I_c' - I_c'' = -\frac{\partial M_x}{\partial y}\Delta x\,\Delta y, \quad J = \frac{I}{A}$$

the net upward current.

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(此節)課本 J_b 改為 J_M；J_M 來自不均勻的極化。 DATE 4/24

The net current, which comes about from non-uniform magnetization is the Magnetization current.

$(I_c' - I_c'')$ $\quad (J_M)_z = \frac{I_c' - I_c''}{\Delta A}$

$$= \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}$$

push to 3D-axis, $\boxed{J_M = \nabla \times \vec{M}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M_x & M_y & M_z \end{vmatrix}$

§6.3.1 Ampere's Law in Magnetized Materials.

a. Bound current $J_b = \nabla \times M$ for volume.
b. Bound current $K_b = M \times \hat{n}$ for surface.

We say the total current $J = J_b + J_f$

From ampere's Law $\nabla \times B = \mu_0 J = \mu_0 (J_b + J_f)$

$\Rightarrow \frac{1}{\mu_0}(\nabla \times B) - J_b = J_f$

$\Rightarrow \nabla \times \left[\frac{B}{\mu_0} - M\right] = J_f$

The quantity in parentheses is designated by

$H = \frac{B}{\mu_0} - M$.

Ampere's Law. $\boxed{\begin{array}{l} \nabla \times H = J_f \\ \oint H \cdot dl = I_f \end{array}}$ enclosed current $: I_f$ enclosed current density $: J_f$

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电場与介質　电場的旋度　高斯積分　3原

1.

静电場　$\vec{D} = \epsilon \vec{E}$　$\nabla \times E = 0$　$\oint \vec{D} \cdot d\vec{s} = Q_f$　Q

$\nabla \cdot E = Q/\epsilon_0$

2.

恆穩电場　$\vec{J} = \sigma \vec{E}$　$\nabla \times E = 0$　$\oint \vec{J} \cdot d\vec{s} = I_f$　I

dielectric

3.

静磁場　B　$\nabla \times B = \mu_0 J$　$\int B \cdot dl = \mu_0 I$

4.

恆穩磁場　$H = \frac{B}{\mu_0} - M$　$\nabla \times H = I_f$　$\int H \cdot dl = I_f$

(均勻磁化)

Magnetized

4/30 (三)

§ 6.3.1 Ampere's Law in Magnetized Materials

1. $H = \frac{B}{\mu_0} - M$

2. Ampere's law, $\nabla \times H = J_f$, $\oint \vec{H} \cdot d\vec{l} = I_f$ (enclosed)

3. In terms of Ampere's Law for the free current

→ Ampere's Law.

4. Magnes static, Electro static → free charge

→ Gauss's Law

封閉空間

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5. $\boxed{\text{If}}$ in enclosed space, $\boxed{\text{if}}$ the current $\vec{J} = 0$,

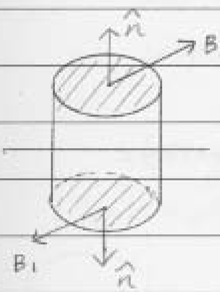
$$\begin{array}{ccc} \nabla \times \vec{H} = 0 & \longrightarrow & \nabla \cdot B = 0 \\ \Downarrow & & \\ \vec{\nabla} \times \vec{E} = 0 & & \end{array}$$

if the $\nabla \times H = 0$, we can find a potential φ_m in the region. $\vec{H} = -\nabla \varphi_m$ ($\vec{E} = -\nabla \varphi$), Then the Laplace's eq is $\nabla^2 \varphi_m = 0 \longrightarrow$ [Initial condition. Boundary condition.]

then in two magnetized matrials. (6.3.3 Boundary Condition.)

$$\varphi_{m_1} = \varphi_{m_2} \longleftrightarrow \mu_1 \frac{\partial \varphi_{m_1}}{\partial n} = \mu_2 \frac{\partial \varphi_{m_2}}{\partial n}$$

media ※1, media※2.



$\because \nabla \cdot B = 0$ $\left(\nabla \cdot E = \rho/\epsilon_0 \longrightarrow \int \nabla \cdot E \, dV = \int \rho/\epsilon_0 \, dV = Q/\epsilon_0\right)$

then $\int (\nabla \cdot B) \, dV = \int 0 \cdot dV = \int \vec{B} \cdot d\vec{s}$

$\longrightarrow$ free charge.

$= \int B \cdot \hat{n} \, da$, We now apply a small pillbox-shaped surface S, which intersects the interface.

We find $\int \vec{B} \cdot d\vec{s} = \int \vec{B}_1 \cdot d\vec{s}_1 + \int \vec{B}_2 \cdot d\vec{s}_2$

$$= \boxed{\vec{B}_2 \cdot \hat{n}_2 \Delta S + \vec{B}_1 \cdot \hat{n}_1 \Delta S} = 0$$

(if the pillbox is small)

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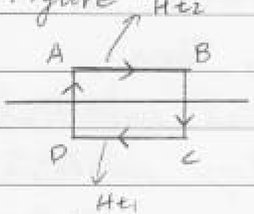
Now $\hat{n}_2 = -\hat{n}_1$, then

$$(\vec{B}_2 \hat{n}_2 + \vec{B}_1 \hat{n}_1)\Delta S = 0$$ (continuity in normal component of B-field.)

$\Rightarrow (\vec{B}_2 - \vec{B}_1)\cdot\hat{n}_2 = 0$. $B_{2n} = B_{1n}$, for $\nabla\cdot B = 0$

§ Figure



The boundary condition on $\vec{H}$ can be found by Ampere's Law.

$\overline{AB} = \overline{CD} = \Delta l$, $-\overrightarrow{AB} = \overrightarrow{CD}$

$$\oint H\cdot d\vec{l} = \vec{H}_{t_1}\cdot\overrightarrow{CD} + \vec{H}_{t_2}\cdot\overrightarrow{AB}$$

$$= (H_{t_1} - H_{t_2})\overrightarrow{CD} = (H_{t_1} - H_{t_2})\Delta l$$

if free current $\neq 0$, $(H_{t_1} - H_{t_2})\Delta l = I$.

$H_{t_1} - H_{t_2} = \frac{I}{\Delta l}$, if we define a surface current density $J\Delta l = I$, this gives ($J_b = \nabla\times M$; $K_b = M\times\hat{n}$)

$\Rightarrow \boxed{H_{t_1} - H_{t_2} = J}$, It means that is a discontinuity in the tangential component of H-field. equal to the surface current density in Ampere per meter at the boundary between two medias.

$B_{t_1} - B_{t_2} = ?$ $\quad B_{t_1}/\mu_1 - B_{t_2}/\mu_2 = J$

$(B_{t_1} - B_{t_2}) = \mu_0 (K_b\times\hat{n})$

Hw, P.6.10, P.6.15, next week 5/7 (三)交 DATE 4/30

if the current $J=0$, find a potential Ψ_m.

$H = -\nabla \Psi_m \Rightarrow B_{1n} = -\mu_1 \nabla_n \Psi_m = -\mu_1 \frac{\partial \Psi_{m1}}{\partial n_1}$, $B_{2n} = \frac{-\mu_2 \partial \Psi_{m2}}{\partial n_2}$

if $\nabla \cdot B = 0$ normal vector is continuous.

$B_{2n} - B_{1n} = 0$ ← (Gauss's law) free charge $=0$

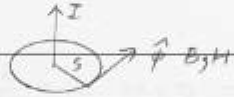
$\left(-\mu_2 \frac{\partial \Psi_{m2}}{\partial n_2}\right) - \left(-\mu_1 \frac{\partial \Psi_{m1}}{\partial n_1}\right) = 0$

$\boxed{\text{Magnetic potential for Boundary Condition}} \leftarrow \Rightarrow \boxed{\mu_2 \frac{\partial \Psi_{m2}}{\partial n_2} = \mu_1 \frac{\partial \Psi_{m1}}{\partial n_1}}$

§6.2 A long copper rod of radius R carried a uniformly distributed current I. We know copper is diamagnetic (逆磁)

(Magnetized materials)

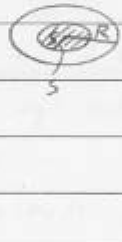
$\oint \vec{H} \cdot d\vec{l} = I_f$ (enclosed)



$H \cdot 2\pi s = I_f$ What's enclosed?

($\because H\hat{\phi}$)

$I_{enclosed} = \frac{\pi s^2}{\pi R^2} I_0$



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Then we can obtain

$$H \cdot 2\pi s = I_0 \frac{\pi s^2}{\pi R^2} \quad (\text{if } s < R)$$

$$H = \frac{I_0 s}{2\pi R^2} \hat{\phi}$$

if $s > R$, $I_{enclosed} = I_0$, $H = \frac{I_0}{2\pi s} \hat{\phi} \ (s > R)$

§ P. 6.12 必考 An infinity long cylinder of radius R, carries a "frozen-in" magnetization (Magnetized materials) parallel to the axis, $M = \underline{ks}\,\hat{z}$, $M_s = 0$, $M_\phi = 0$

Then we need to find out J_b. K_b.

$$J_b = \nabla \times M = \left[\frac{1}{s}\boxed{\frac{\partial M_z}{\partial \phi}} - \frac{\partial M_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial M_s}{\partial z}\boxed{- \frac{\partial M_z}{\partial s}}\right]\hat{\phi}$$

$$+ \frac{1}{s}\left[\frac{\partial}{\partial \phi} sM_\phi - \frac{\partial M_s}{\partial \phi}\right]\hat{z}$$

(1) $J_b = \nabla \times M = -\frac{\partial M_z}{\partial s}\hat{\phi}$

$$= -k\hat{\phi}$$

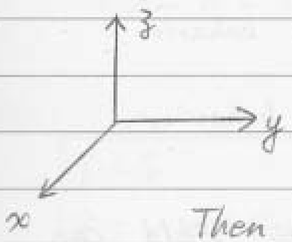
(2) $K_b = M \times \hat{n} = ks\,(\hat{z} \times \hat{n})$

We know normal vector is

$$\vec{n} = R\cos\phi\,\hat{x} + R\sin\phi\,\hat{y} \quad \underline{\text{at } s=R}$$

K_b existes at surface $s=R$

$$M = KR\,\hat{z}\ ,\ K_b = KR\,(\hat{z}\times\hat{n}) = KR\,\hat{z}\times[\cos\phi\,\hat{x} + \sin\phi\,\hat{y}]$$



$$\hat{z}\times\hat{x} = \hat{y}$$
$$\hat{z}\times\hat{y} = -\hat{x}$$

Then it gives $K_b = KR\,[\cos\phi\,\hat{y} - \sin\phi\,\hat{x}] = KR\,\hat{\phi}$

§ P. 6.13 Rotating sphere (σ) $\overset{\frac{2}{3}\mu_0 M}{B}$ → $\boxed{\text{Cavity}}$ 洞 in $\boxed{\text{magnetic material}}$
+ Magnetized materials M
B/μ_0

How a small spherical cavity is hollowed out of the magnetic material, find the H, B at the center of cavity in terms of B_0 & M.

$$H = \frac{B}{\mu_0} = \frac{1}{\mu_0}\left(B_0 - \frac{2}{3}\mu_0 M\right)$$

$$= \frac{B_0}{\mu_0} - \frac{2}{3}M = H_0 + M - \frac{2}{3}M$$

$$= H_0 + \frac{1}{3}M.$$

In case of solenoid
$B = \mu_0 K$
Cavity $= \mu_0 M$, $H = H_0$
$B = B_0 - \mu_0 M$

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DATE 5/1 (四) χ_m : susceptibility 磁化率

Chapter 6.4 , Linear $\boxed{\text{magnetized materials}}$. $\left(\underset{\downarrow\, 0}{H} = \frac{B}{\mu_0} - \underset{\downarrow\, 0}{M}\right)$

(1) As the B-field is removed.

(2) M disappers. (Decay) $M = \chi_m H$ $(y = \underline{ax+b})$ Linear
$\hookrightarrow$ a>0 ; a<0 Decay

(3) The magnetization is proportional to the field.

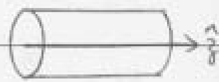
$$M = \chi_m H \longrightarrow \text{if} \quad \begin{array}{l} \chi_m > 0 : \text{paramagnetic behavior.} \\ \chi_m < 0 : \text{diamagnetic behavior.} \end{array}$$

if we consider in linear media, the magnetic field can be re-difined as

$$B = \mu_0 (H+M) = \mu_0 (H + \chi_m H) = \mu_0 (1+\chi_m) H$$

$\mu \equiv \mu_0 (1+\chi_m)$ permeability of the material. $\underline{(\chi_m, \rho)}$

Ex 6.3 , An linear solenoid $\left(n = \frac{N}{L} = \frac{\text{terms}}{\text{per length}}\right)$ $\nearrow I$



is filled with linear materials of χ_m.

Find the magnetic field inside.

① if $n = \frac{N}{l} \Rightarrow \int H \cdot dl = I$ (for 1 turn) $\Rightarrow H_0 = \frac{I}{l}$

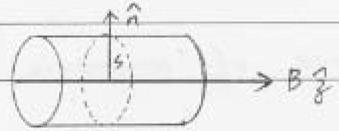
② for N turns , $H = N \frac{I}{l} = n I \hat{z} \Longrightarrow B = \mu_0 (1+\chi_m) H$

$$= \mu_0 (1+\chi_m) n I \hat{z}$$

if the material is paramagnetic $\chi_m > 0$. The field is slightly <u>enhanced</u> (diamagnetic $\longrightarrow$ reduced)

* Surface current density. $K_b = M \times \hat{n}$.

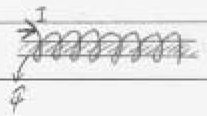
$$K_b = \chi_m \, n I \, (\hat{z} \times \hat{n})$$
$$= \chi_m \, n I \, (\hat{z} \times \hat{s})$$



Note:

$$\boxed{\begin{aligned} \hat{z} &= \hat{z} \\ \hat{\phi} &= -\sin\phi \, \hat{x} + \cos\phi \, \hat{y} \\ \hat{s} &= \cos\phi \, \hat{x} + \sin\phi \, \hat{y} \end{aligned}} \Rightarrow \hat{z} \times \hat{s} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ \cos\phi & \sin\phi & 0 \end{vmatrix} = \cos\phi \, \hat{y} - \sin\phi \, \hat{x} = \hat{\phi}$$

$$\begin{cases} K_b = \chi_m \cdot n \cdot I \, \hat{\phi} \\ K_b = \mu_0 \, n \, I \, \hat{\phi} \end{cases}$$

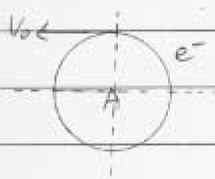


$\rightarrow$ Linear material.

$\oint B \cdot dl = \mu_0 I$

6.4 Diamagnetism (逆磁)

(a) electron arounds the atom $\left(\frac{-e}{T} \equiv I\right)$



$F_e = \frac{m_e V_0^2}{r}$, m_e : electron mass

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(b) magnetic moment

$$m = IA = I\pi r^2 = \frac{-e}{T}\pi r^2 = \frac{-eV_0}{2\pi r}\pi r^2$$

$$V_0 = \frac{2\pi r}{T} \qquad = \boxed{\frac{-eV_0}{2} r}$$

(c) The difference of magnetic moment

$$F_e \pm \underline{qVB} = \frac{m_e V^2}{r}$$

↓

offer magnetic field

↓

magnetic moment decreasing.

$$\pm qVB = \frac{m_e V^2}{r} - \frac{m_e V_0^2}{r}$$

$$= \frac{m_e}{r}(V+V_0)(V-V_0)$$

$$\pm qVB = \frac{m_e}{r}(2V)\cdot\Delta V \quad , \text{ if } V \simeq V_0$$

$$\Delta V = \pm \frac{qBr}{2m_e}$$

$$\Delta m = 2\times\left(\frac{e\Delta V}{2}\right) r = \left(\frac{er}{2}\cdot\frac{qBr}{2m_e}\right)\times 2 \quad (q=e)$$

$$= \frac{Be^2}{2m_e} r^2$$

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Then the diamagnetism is define as $M = -\chi_m H$

$$\chi_m = -\frac{M}{H} = -\frac{n\sigma m}{B/\mu_0} \quad , \quad m = \frac{M}{V}$$

$$\chi_m = -\frac{\mu_0 n e^2 r^2}{2 m_e}$$

5/7 (三) P. 6.21

Chapter 6 , B-field $^{(B)}$, Force (F)

magnetic moment$_{(IA)}$, Torque $(\tau = \vec{m} \times \vec{B})$

potential. $(U = -m \cdot B)$

$U = -m_1 \cdot B_2$

Find out the interation energy of two dipoles/magnement moment.

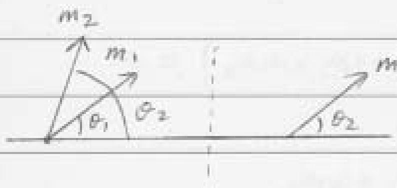
$$B_{dipole} = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m} \right]$$

Find out the $\vec{m}_1 \cdot \vec{B}$

$$\Rightarrow \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3\vec{m}_1 \cdot (\vec{m}_2 \cdot \hat{r})\hat{r} - \vec{m}_1 \cdot \vec{m}_2 \right]$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r}) - \vec{m}_1 \cdot \vec{m}_2 \right]$$

if



B-field B_z

Then $\vec{m}_1 \cdot \hat{r} = m_1 \cos\theta_1$

$\vec{m}_2 \cdot \hat{r} = m_2 \cos\theta_2$

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$\because \vec{m}_1 \cdot \vec{m}_2 = m_1 m_2 \cos(\theta_2 - \theta_1)$

$\cos(\theta_2 - \theta_1) = \cos\theta_2 \cos\theta_1 + \sin\theta_1 \sin\theta_2$

So the potential can be rewritten as

$$U = \frac{-\mu}{4\pi} \frac{1}{r^3} \left[3 m_1 m_2 \cos\theta_1 \cos\theta_2 - m_1 m_2 \cos\theta_1 \cos\theta_2 + m_1 m_2 \sin\theta_1 \sin\theta_2 \right]$$

$$= \frac{-\mu_0}{4\pi} \frac{1}{r^3} \left[2\cos\theta_2 \cos\theta_1 + \sin\theta_1 \sin\theta_2 \right] m_1 m_2$$

* Stable position occurs at minimum energy.

$$\frac{\partial U}{\partial \theta_1} = 0 \quad \text{or} \quad \frac{\partial U}{\partial \theta_2} = 0$$

1st term :

$$\frac{\partial U}{\partial \theta_1} = \frac{\mu_0 m_1 m_2}{4\pi r^3} \left[\cos\theta_1 \sin\theta_2 + 2\sin\theta_1 \cos\theta_2 \right] = 0$$

$$\Longrightarrow 2\sin\theta_1 \cos\theta_2 = -\cos\theta_1 \sin\theta_2$$

2nd term :

$$\frac{\partial U}{\partial \theta_2} = \frac{\mu_0 m_1 m_2}{4\pi r^3} \left[\sin\theta_1 \cos\theta_2 + 2\cos\theta_1 \sin\theta_2 \right] = 0$$

$$2\sin\theta_1 \cos\theta_2 = -4\cos\theta_1 \sin\theta_2$$

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Eithor $\sin\theta_1 = \sin\theta_2 = 0$

$\cos\theta_1 = \cos\theta_2 = 0$

$\theta_1 , \theta_2 = 0$

$\theta_1 = 0 \ , \ \theta_2 = 180^\circ$

$\theta_1 = 90^\circ , \ \theta_2 = 90^\circ$

$\boxed{\theta_1 = 90^\circ , \ \theta_2 = 270^\circ}$