NO 下學期　　office hour (三) PM 1:00 ~ PM 2:00

DATE 2/20 (三)　　ch5 ~

Chapter 2 ~ 4 Review

$\vec{E}(\vec{r}-\vec{r}')$	Columb $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qq'(\vec{r}-\vec{r}')}{\lvert\vec{r}-\vec{r}'\rvert^3}$
$\vec{B}(\vec{r})$	Ampere

∴ 磁場比电場容易

Gauss theorem

$\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\epsilon_0}$, $\nabla\cdot\vec{E} = \rho/\epsilon_0$

Electrostatics

$\oint \vec{E}\cdot d\vec{l} = 0$, $\nabla\times\vec{E} = 0$

⇓ in matter

$\oint \vec{D}\cdot d\vec{s} = Q_f$, $\nabla\cdot\vec{D} = \rho$

Gauss's law

Faraday Law, $\varepsilon = \frac{-\partial\phi}{\partial t}$

$\oint \vec{E}\cdot d\vec{l} = -\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}$

Bio - Savare Law

ch5+6

⇓

Stokes theorem

||

$\oint \vec{E}\cdot d\vec{l} = -\int\frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

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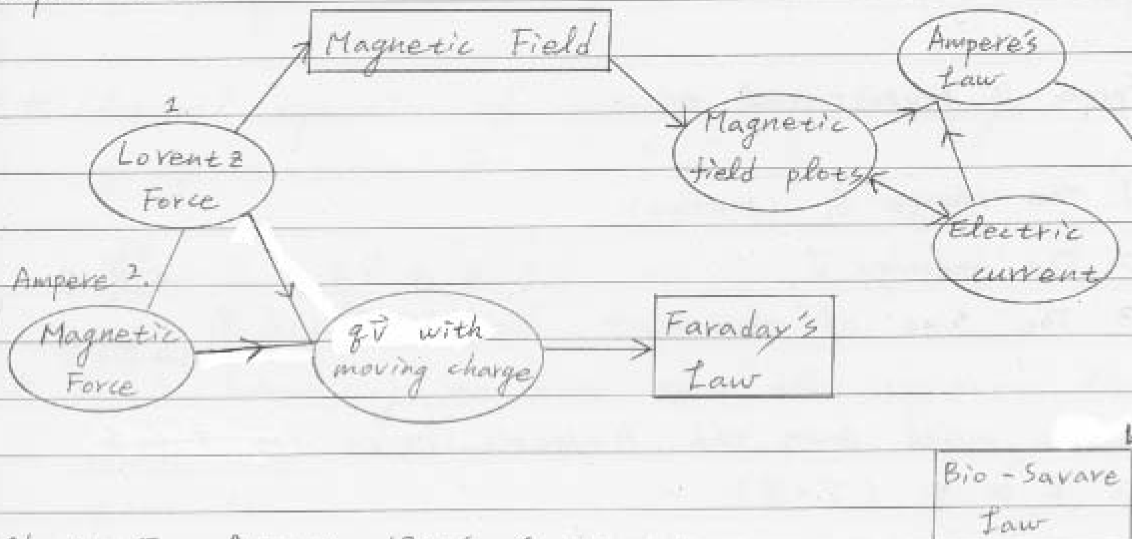
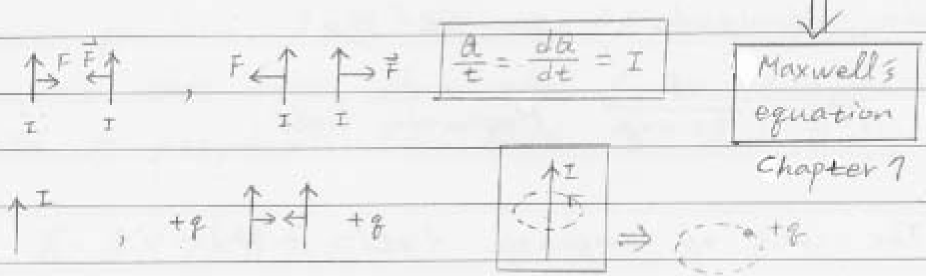
Chapter 5 + 6



Chapter 5, Ampere 1820's Experment

Case I :

Case II :



* The definition of Magnetic field , $\vec{F}(q, \vec{v}, \vec{B})$

Magnetic force only acting on moving charges , so it is nature to use the magnetic force $\vec{F}$ exerted on a positive charge q whose velocity is $\vec{v}$ to define $\vec{B}$.

NO.

DATE 2/20

Ampere's conclusions :

Force is proportional to

1. The charge q (positive)
2. The velocity $\vec{v}$
3. The Sine of the angle between $\vec{v}$ and $\vec{B}$ → $\vec{v} \times \vec{B}$, → $\vec{B} \times \vec{v}$

So we write down the Ampere's Force as $\vec{F} \propto q\,(\vec{v} \times \vec{B})$

the magnitude is specified by

$$|\vec{B}| = \frac{F}{qv\sin\theta} \quad \text{Magnetic field.}$$

* The unit of magnetic field in SI is

$$\text{Newtons}/\text{ampere-meter} = \text{Newton-Second}/\text{Columb-meter}$$

$$= \text{Tesla (T)}$$

1 * The earth's magnetic field is about $\sim 5\times10^{-5}\,T$. $(1T \sim 10^{4}\text{ Gauss})$

~ 0.5 Gauss

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NO.

DATE 2/20

2.* Superconductor at Low Temperature $\sim 10\,T$

3.* Classical equation of motion : Thus a charge / particle q of momentum $\vec{P}$

$$\frac{d\vec{P}}{dt} = \vec{F} = q\vec{E} + \boxed{q\vec{v}\times\vec{B}}$$

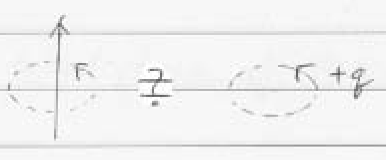
* It is interesting to note that the magnetic field <u>DOSE NOT</u> mechanical work on a moving charged particle.

4.* The vector product $\vec{v}\times\vec{B}$ in the lorentz force incorporate the fact that the force due to the $\vec{B}$ field is perpendicular both $\vec{B}$ and $\vec{v}$, & so

$$\vec{B}\cdot q(\vec{v}\times\vec{B}) = 0$$

$\vec{F} = \frac{dw}{dl}$ 、 $dW_{\text{magnetic work}} = \vec{F}\cdot d\vec{l}$

$$= q(\vec{v}\times\vec{B})\cdot\vec{v}\,dt$$
$$= [\vec{v}\cdot(\vec{v}\times\vec{B})]\,q\,dt$$
$$= 0.$$



$$qvB = m\frac{V^2}{R}$$

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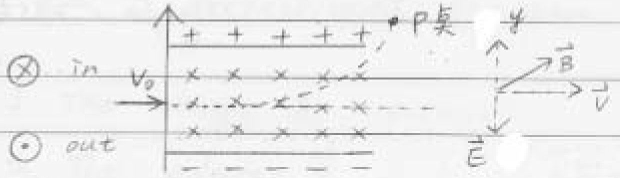
DATE 2/20

* 1897, Thomson, The measurement of e/m,
Cambridge Canvendish Lab.



using lorentz force, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

① if the sum of forces $= 0 \Rightarrow \vec{E} = \vec{v} \times \vec{B}$, $V = E/B$

② if the moving direction is close to y-axis.

$$x = Vt, \quad y = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}t^2$$
$$\boxed{= \frac{1}{2}\frac{eE}{m}t^2}$$

③ $y = \frac{eE}{2mV^2}x^2$

$+ V = \frac{E}{B}$, then we can get the relation of

$$\boxed{e/m = \frac{2yE}{B^2x^2}}$$

Thomson's result $e/m \sim 1.7 \times 10^{11}$ C/kg

1912 Millikan's results $\boxed{e \sim 1.602 \times 10^{-19}\ C}$

→ electron's mass $m = 9.1 \times 10^{-31}$ kg.

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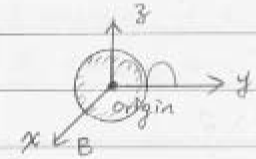
Cyclone Ex 5.2

① $\vec{B} \perp \vec{v}$

② $\vec{E} \perp \vec{B}$



* a charged particle with momentum $\vec{p}$ then the equation of motion can be expressed as

$$\frac{d\vec{p}}{dt} = q\vec{E} + q(\vec{v} \times \vec{B}),$$

A particle at rest is released from the origin.

* if B points in the x-direction.
  E points in the z-direction.

$P(x, y, z, t) = (0, \dot{y}(t), \dot{z}(t))$

$V_p = (0, \dot{y}, \dot{z})$ is the velocity of particle then we can calculate the magnetic force

$$F_B = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \dot{y} & \dot{z} \\ B_x & 0 & 0 \end{vmatrix}, \quad F_E = qE_z\hat{z}$$

$$= qB_x(\dot{z}\hat{y} - \dot{y}\hat{z})$$

The total force, $F \equiv ma = m(\ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z})$

$$= qB_x\dot{z}\hat{y} + (q\vec{E}_z - qB_x\dot{y})\hat{z}$$

NO.

DATE 2/21

then we can know $\ddot{x} = 0$

$$\left\{\begin{array}{l} m\ddot{y}\,\hat{y} = qBx\,\dot{z}\,\hat{y} \\ m\ddot{z}\,\hat{z} = (qE_z - qBx\,\dot{y})\,\hat{z} \end{array}\right.$$

$$\boxed{\begin{array}{l} \frac{dx}{dt} = v = \dot{x} \\ \frac{d^2x}{dt^2} = a_x = \ddot{x} \end{array}}$$

(equation of motion)

(1) Let $qB\dot{z} = m\ddot{y}$, $\frac{qB}{m} = w$

Eq of motion $\Longrightarrow$ $\boxed{w\dot{z} = \ddot{y}}$, $\dddot{y} = w\ddot{z}$

(2) $qE - qB\dot{y} = m\ddot{z}$

$\boxed{\ddot{z} = w\left(\frac{E}{B} - \dot{y}\right)}$

$\rightarrow \dddot{y} = w^2\left(\frac{E}{B} - \dot{y}\right)$

Let $\dot{y} = s$, then $\ddot{s} = w^2\left(\frac{E}{B} - s\right)$

then we can get out the solution

$$s = A\cos wt + B\sin wt + \frac{E}{B} = \dot{y}$$

$$\therefore\ y(t) = \int \dot{y}\,dt = \frac{A}{w}\sin wt + \frac{-B}{w}\cos wt + \frac{E}{B}t + C$$

代入 $\ddot{z} = w\left(\frac{E}{B} - \dot{y}\right)$

$$= w\left(E/B - A\cos wt - B\sin wt - E/B\right)$$

$$\ddot{z} = -w\,(A\cos wt + B\sin wt)$$
$$\dot{z} = -A\sin wt + B\cos wt + C'$$
$$z = \frac{A}{w}\cos wt + \frac{B}{w}\sin wt + C''$$

$\boxed{\dot{y}(0) = 0\ ,\ \dot{z}(0) = 0\ ,\ y(0) = 0\ ,\ z(0) = 0 \quad \text{initial conditions}}$

$$y(t) = E/wB\,(wt - \sin wt)\ ,\ z(t) = E/wB\,(1 - \cos wt)$$

A comparison between Electrostatics and Magneostatics.
It should be compared between <u>two static fields</u>.

non-statics $E(t)$, $B(t)$

1. Both $\vec{B}$ & $\vec{E}$ are defined in terms of force.
   (Columb & Ampere) $q\vec{E}$ & $q(\vec{v}\times\vec{B})$.

θ, I

2. Where $\vec{E}$ is defined via $\vec{F} = q\vec{E}$ on a $\boxed{\text{stationary}}$ $\boxed{\text{change}}$ & is consequently a polar vector.

3. $\vec{B}$ is defined via $\vec{F} = q(\vec{v}\times\vec{B})$. the cross-product of two polar vectors is an axial vector or $\boxed{\text{pseudo-vector}}$.

   New $\vec{B}$ is defined via Lorentz force $\vec{F} = q(\vec{v}\times\vec{B})$ on a $\boxed{\text{moving charge}}$.

NO.

DATE 2/27 (三)

3 $\vec{F} = q(\vec{v}\times\vec{B})$, $\vec{B}$ is a psedo- vector. $\boxed{A_x B_y \hat{z}}$ ⊕⊕ ⊖⊖

a polar vector product a polar vector.

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix} = \underline{A_x B_y \hat{z}} \text{ , if } \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -A_x & 0 & 0 \\ 0 & -B_y & 0 \end{vmatrix} = \underline{A_x B_y \hat{z}}$$

Orgin system   reflect system

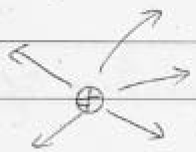
4 Gauss's Law appear different for these two static fields (E, B)

$\nabla\cdot\vec{E} = \rho/\epsilon_0$, $\nabla\cdot B = 0$

5. The B-field is divergence at all points & its solenoid $\nabla\cdot B = 0$

It's field lines from closed loops.

6. $\vec{E}$ is cur-free. ($\nabla\times E = 0$)



solenoid E-field is easy to make

But we could need to have a $\boxed{\text{magnetic charge}}$, known as magnetic monopole.

$\nabla\cdot B \neq 0$? (Yes or No)

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NO

DATE 2/27

7. The magnetic charge would also has all the properties of a scalar.

8. The electric & magnetic dipole moments are related in a similar way as $\vec{E}$ & $\vec{B}$.

$\vec{p} = \int \rho \, \vec{r} \, d^3r$, $\vec{m} = \frac{1}{2} \int \vec{r} \times \vec{j} \, d^3r$

9. Time reversal or inversion ?

The time reversal operaction consist of replacing (t) in all equations by $(-t)$. under time reversal operation, posistion vector x.
acceleration a.
Force F.

But $\vec{v} = d\vec{r}/dt$ is reversed

$-\vec{v} = d\vec{r}/-dt$

Because $\vec{E}$-field is defined as force per unit charge. q.

→ it's invariant with respert to the time reversal operation.

But $\vec{B}$ charged sign $(\pm)$ under time reversal.

NO.

DATE 2/27

5.1.3 Current v.s. charge

A. Electric current I (Ampere)

$\longleftrightarrow$ Charge at rest Q.

Definition : Electric current is the flow of electric charge. $I = \frac{Q}{t}$ or $I = \frac{\Delta Q}{\Delta t}$.

B. The magnitude of an electric charge at a point is defined at the time derivative of electric charge.

$$\boxed{I_{(t)} = \frac{dQ_{(t)}}{dt} \Longleftrightarrow Q_{(t)} = \int_{I_0}^{I} I_{(t)}\,dt + Q_0}$$

Q . L

$V = ?\ \frac{I}{\lambda}$

$\lambda = \frac{Q}{L}$, $I = \frac{Q}{t}$ charge velocity

C. In a metal media / current [為何金属会導电]

In conductive metal, <u>with no</u> applied force (Voltage), there exist a random motion of "mobile" or "free" electrons created by the thermal Energy $k_B T$ (300K, 25meV) which electrons gained from surround medium. ($k_B T$).

D. When a metal is connected across the terminals of a DC voltage, the free electrons are forced to drift toward the positive terminal under the influence of this field, $I = Q/t$.

columb with 6.242×10^{18} electrons

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NO.

DATE 2/27

E. Current density

Is a measure of the density of electrical current. It's defined a vector, whose magnitude is the electric current per cross-sectional area.

$$\vec{J} = \vec{I}/A = \rho\vec{v} \quad , \quad \underline{I = \lambda V} \;\; (Q, L, \lambda) \;\; \boxed{1D}$$

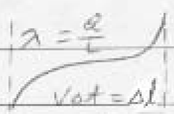
The total charges pass through the frame A in a time interval Δt, then the charge is

$$\Delta Q = q\,\underbrace{(v\Delta t)}_{L}\,\underset{\vec{A}}{\underline{\vec{A}}}\,n \quad , \quad nq \equiv \rho$$

$$\text{If } \vec{A} \parallel \vec{v} \, , \quad I = \frac{\Delta Q}{\Delta A} = nqA\vec{v}.$$

$$\vec{J} = I/A = \boxed{nq\vec{v}} = \rho\vec{v}$$

5.1.3 electric current measurement along a wire.



$$I = \lambda V$$
$$= \lambda_+ V_+ + \lambda_- V_-$$

Mag. force

Then the magnetic force can be written from the Loertz force.

$$F_{mag} = \int (\vec{v} \times \vec{B})\, dq = \int (\vec{v} \times \vec{B})\, \lambda \, dl$$

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NO

DATE 2/27

(3/5)
下星期三小考　5.1 ~ P.211

$F_{mag} = \int (\vec{I} \times \vec{B})\, d\vec{l}$

$= \int I (d\vec{l} \times \vec{B})\ , \quad \vec{I} \parallel d\vec{l}$

1. IF $\vec{B}$ depends on position
2. IF $\vec{B}$ is uniform, then is position indenpendent.

$\vec{F}_{mag} = I \oint d\vec{l} \times \vec{B} = 0.$

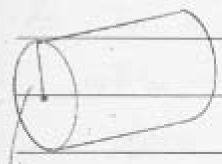
3/5 (三)

Ex 5.4, ρ = constant & $\rho = kr$ charge density.

$J = I/A$ current density.

J = constant & $J = ks$

if J = constant



radius a

$J = I/A = I/\pi a^2$

if $J = ks$, suppose the current density is $\propto ks$.

then we can write the integrated equation.

$I = JA = \int J \cdot d(A) = \int_{s=0}^{a} J(s)\, d(\pi s^2)$

$= \int_0^a ks \cdot 2\pi s\, ds = 2\pi k \left. \frac{s^3}{3} \right|_0^a = \frac{2\pi k a^3}{3}$

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P.S. Continuity Equation.

1. Because charge (continuous) is conserved. $\boxed{Q}$

2. if the current crossing a surface flows out through the surface. $\boxed{J}$

3. $I = \int \vec{J} \cdot d\vec{a} = \int \nabla \cdot \vec{J}\, dv = -\frac{d}{dt} \int \rho\, dv$

$I = \frac{dQ}{dt} = \frac{d}{dt} \int \rho\, dv$ (charge, continuous charge)

4. $\nabla \cdot J = -\frac{\partial \rho}{\partial t} \Rightarrow \nabla \cdot J + \partial\rho/\partial t = \underline{0}$ (P.214, 5.29)

5.2 The Law of Biot-Savart.

1. From Ampere's Law of Force. $\vec{F}_{mag} = I \int d\vec{l} \times \vec{B}$ open line

after performing & analyzing many experiments on the force by current-carrying circuits.

closed loop

"origin"

To confirm Newton's third Law

2. Ampere found the following Law of force for $\boxed{\text{two circuits}}$

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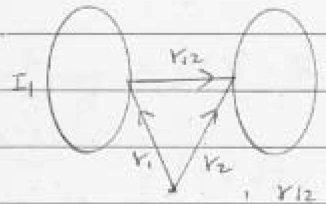
NO.

DATE 3/5



, $r_{12} = \vec{r}_1 - \vec{r}_2$

$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

$F_{12} = F_2$

$$\vec{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_{12} d\vec{l}_2 \times \frac{(d\vec{l}_1 \times \vec{r}_{12})}{r_{12}^3} \quad , \quad r_{12}^3 = |r_1 - r_2|^3 = r_{21}^3$$

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_{12} d\vec{l}_1 \times \frac{(d\vec{l}_2 \times \vec{r}_{21})}{r_{12}^3}$$

To demonstrate the relation of $\vec{F}_{12} = -\vec{F}_{21}$, we expand the equation to give.

$$A \times (B \times C) = B\,(A \cdot C) - C\,(A \cdot B)$$

$$F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \Big[\underbrace{\frac{(d\vec{l}_2 \cdot \vec{r}_{12})\, d\vec{l}_1}{r_{12}^3}}_{=0} - \frac{(d\vec{l}_1 \cdot d\vec{l}_2)\,\vec{r}_{12}}{r_{12}^3} \Big]$$

Further reduce as

$$\oint \frac{d\vec{l}_2 \cdot \vec{r}_{12}}{r_{12}^3} = \oint \underline{\nabla_2 \left(\frac{1}{r_{12}}\right)} d\vec{l}_2 = 0$$

$$\boxed{F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_{12} \left(\frac{-d\vec{l}_1 \cdot d\vec{l}_2}{r_{12}^3} \right) \underline{\underline{\vec{r}_{12}}}} = -F_{21}$$

Demonstrats the 3rd Law of Newton

Biot - Savart rewritten

$$+ \quad \boxed{\vec{F}_2 = I_2 \oint_2 d\vec{l}_2 \times \vec{B}_2}$$

$$\boxed{B = \frac{\mu_0}{4\pi} I_1 \int \frac{d\vec{l}_1 \times \vec{r}_{12}}{r_{12}^3}}$$

NO.

DATE 3/5

where the $\vec{B}$ component is the magnetic field by $\underline{I_1}$ is known as Bio-Savart Law which is orginally expressed in different form, i.e.

$$d\vec{B} = \frac{\mu_0}{4\pi} I_1 \frac{dl_1 \times \vec{r}_{12}}{r_{12}^3}$$

1. $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$ permeability of free space.

2. Tesla, $B = \dfrac{\text{Newton}}{\text{Ampere} \cdot \text{meter}}$ (T) Gauss (G) Oster (Oe)

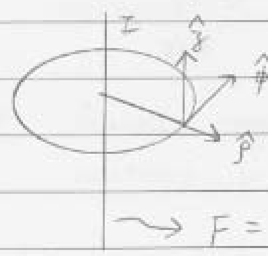
目前最大 五十多 Tesla

台灣 12 Tesla，百萬音響 1 Tesla，法國現在花四百億建一百 Tesla

5.2. The magnetic field of steady-current
Ampere's circuital Law.

Ampere discovered the $\boxed{\frac{1}{\rho}}$ dependence of the magnetic field due to a steady current $\boxed{I}$, flowing a long straight wire of circuit cross section.

magnetic field $\boxed{B = \mu_0 I / 2\pi\rho}\ \hat{\phi}$



$\rightarrow F = I \int d\vec{l} \times \vec{B}$

In cylindrical coordinates (ρ, ϕ, z), with the wire as z-axis at point $\vec{r}$ outside the wire $\boxed{\vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{\vec{I} \times \hat{\rho}}{\rho}}$.

I along z-axis

plane direction ρ-axis $\quad \hat{I} \times \hat{\rho} = I(\hat{z} \times \hat{\rho}) = I\hat{\phi}$

$$B = \frac{\mu_0 I}{2\pi\rho}\hat{\phi}$$

$$\boxed{\oint B \cdot d\vec{l} = \mu_0 \cdot I}$$

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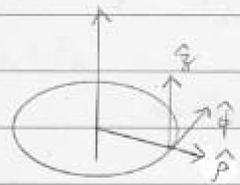
DATE 3/6 (四)

* Ampere's circuital Law

In cylindical coordinates



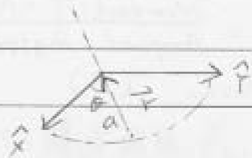
A

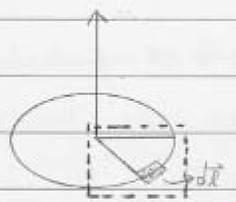
$$\left(= \frac{\mu_0 I}{2\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} \right)$$

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \vec{\rho}}{\rho} \quad \text{or} \quad \vec{B} = \frac{\mu_0}{2\pi} I \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

图A Ampere

图B Biot-Savart





B

B field out of plane

$\Rightarrow d\vec{l} \times \hat{r}$ (inner)

if the B-field is $\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \vec{\rho}}{\rho}$.

Let us find the line integral of $\oint \vec{B} \cdot d\vec{l}$ for a path Γ, that enclose the wire. ($\neq 0$)

non-enclose the wire. ($= 0$)

where $d\vec{l} = d\rho\, \hat{\rho} + \rho\, d\phi\, \hat{\varphi} + dz\, \hat{z}$

then $\frac{\vec{I} \times \hat{\rho}}{\rho} = \frac{I \hat{\varphi}}{\rho}$

so we can calculate the

$$\vec{B} \cdot d\vec{l} = \frac{\mu_0}{2\pi} \frac{I}{\rho} (\hat{\varphi} \cdot d\vec{l}) = \frac{\mu_0 I}{2\pi \rho} \rho\, d\phi$$

DATE 3/6

So the enclose path

$$\oint \vec{B}\cdot d\vec{l} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi\rho}\,\rho\, d\phi = \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\phi = \mu_0 I$$

For a path does not enclose the wire

$$\oint_{\Gamma_2} I\, d\phi = \oint 0\, d\phi = 0 \,, \quad \oint_{\Gamma_2} \vec{B}\cdot d\vec{l} = 0$$

Then the equation $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ is often called the circutial form of Ampere's Law.

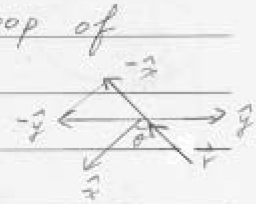
* Important Note:

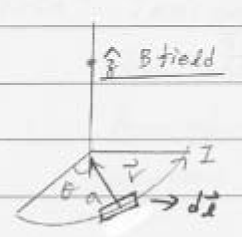
The origin of magnetic field,

(1) Ampere's Law recoghizes only one source of magnetic field: **a moving electric charges or steady current.**

(2) The other second orgin of a magnetic field: a changing electric field, so called a displacement current.

Fig B. Axial magnetic field of a circular loop of current wire:





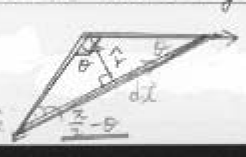
(1) The vector of $\vec{r}$

$$\vec{r} = -\hat{x}\, a\cos\theta - \hat{y}\, a\sin\theta + z\hat{z}$$

(2) The vector of $d\vec{l}$

$$d\vec{l} = (a\,d\theta)\left[-\hat{x}\sin\theta + \hat{y}\cos\theta\right]$$ (P.218, Ex 5.6)

$|dl| = a\,d\theta$



NO.

DATE 3/6

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(3) $d\vec{l} \times \vec{r}$

$$= a^2 d\theta \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -\cos\theta & -\sin\theta & z \end{vmatrix}$$

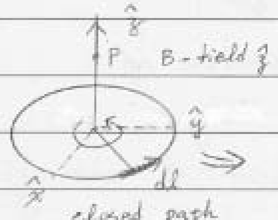
$$= (\hat{x}\, z a\cos\theta + \hat{y}\, z a \sin\theta + \boxed{a^2 \hat{z}})\, d\theta$$

3/12 (三)

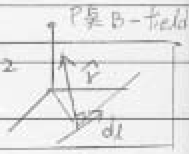
(1) Ampere's circuitual Law, right-hand rule.

(2) Bio - Savart Law. $\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{I\, \boxed{d\vec{l}} \times \hat{r}}{r^2}$ → $d\vec{l}_2$

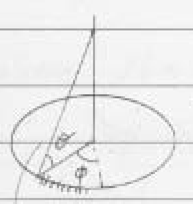
→ Ampere's magnetic force law



⇒ $\nearrow dl \xrightarrow{\text{from rule 1}} \odot \hat{z}$ B along $\hat{z}$ + $d\vec{l} \times \hat{r} \xrightarrow{\text{from rule 2}}$



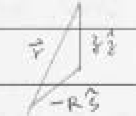
* Axial magnetic field of a circular loop, with radius R.



(1) $d\vec{l} = R\,d\phi\,\hat{\phi}$

(2) $\vec{r} = z\hat{z} + (-R)\hat{\rho}$

(ρ 徑向)



(3) Then the B-field $\frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R\,d\phi\,\hat{\phi}\times\vec{r}}{r^3}$

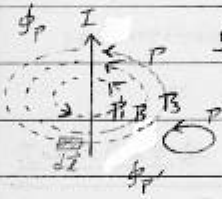
$$= \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R\,d\phi\,(z\hat{\rho} + R\hat{z})}{r^3}$$

$$= \boxed{\frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R^2 d\phi\, \hat{z}}{r^3}} + \underbrace{\frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{Rz\,d\phi\,\hat{\rho}}{r^3}}_{0\,(?)}$$

NO.

DATE 3/12 (三)

§5.3 The divergence & curl of $\underline{B}$



5.3.1 Straight-line currents.

* It's clear that the field has a non-zero curl.

* According to the field is axis dependent $B(r) \Rightarrow \int \vec{B}\cdot d\vec{l} = \mu_0 I$

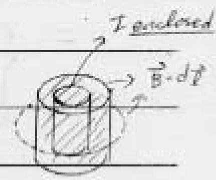
$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi s} d\vec{l} = \mu_0 I \,,\ \boxed{\text{if } \vec{B} \text{ is } 1/s \text{ dependent}}\ \star$$

For cylindrical coordinates $\underline{(s, \phi, \hat{z})}$ with the current flowing along the $\hat{z}$-axis.

if $B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$. then the $\vec{B}\cdot d\vec{l}_B = \frac{\mu_0 I}{2\pi} d\phi$

$d\vec{l}_B = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$, $\oint B\cdot d\vec{l}_B = \int_0^{2\pi} \frac{\mu_0 I}{2\pi} d\phi = \mu_0 I$.

Then the line integral will be $\underline{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}}$ where $I_{enclosed}$ stands for the total current enclose by the integration path. $I_{enc} = \int \vec{J}\cdot da$.



Apply to Stoke's theorem $\int \vec{B}\cdot d\vec{l} = \int \nabla\times B\cdot d\vec{a} = \mu_0 \int \vec{J}\cdot d\vec{a}$,

$$\boxed{\nabla\times B = \mu_0 J}$$

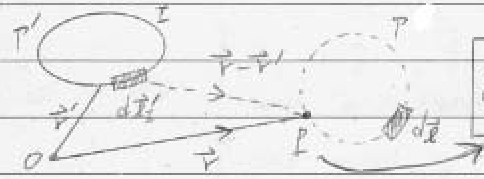
Note: Junction to 5.3 curl & divergence

We need to prove the theorem of Ampere

According to Bio-Savart law in a closed path Γ' of current I in point P.

NO.

DATE 3/12



at P

$$\boxed{B = \frac{\mu_0 I}{4\pi} \oint_{\Gamma'} \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}},$$

if integration of $\oint_\Gamma \vec{B}\cdot d\vec{l} \Rightarrow \nabla \times B = \mu_0 J$

Then we write down the line integrated of $\vec{B}$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \oint_\Gamma \left[\oint_{\Gamma'} \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \right] \cdot d\vec{l}$$

$$\begin{cases} = \mu_0 I, & \text{if } \Gamma \text{ covers } \Gamma' \\ = 0, & \text{if } \Gamma \text{ did not cover } \Gamma' \end{cases}$$

Γ' Γ I $\frac{4\pi\delta(r-r')}{\vec{B} = 4\pi}$

I Γ' Γ $= 0$

$$\Rightarrow \oint_\Gamma \boxed{\oint_{\Gamma'} \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}} \cdot d\vec{l}$$

$$= \begin{cases} 0, & \Gamma \text{ No} \\ 4\pi, & \Gamma \text{ cover } \Gamma' \end{cases}$$

, if we set the Gradient of $\frac{1}{|\vec{r}-\vec{r}'|}$ we can write the equation as

$$\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\nabla_r \frac{1}{|\vec{r}-\vec{r}'|} = \underline{\underline{\nabla_{r'} \frac{1}{|\vec{r}-\vec{r}'|}}}$$

Let the integral formula can be re-writen as

$$\oint_\Gamma \oint_{\Gamma'} d\vec{l}' \times \left(\nabla_{r'} \frac{1}{|\vec{r}-\vec{r}'|}\right) \cdot d\vec{l}, \quad \text{if } \vec{f} = \nabla_{r'} \frac{1}{|\vec{r}-\vec{r}'|}$$

(1) $\underline{\oint\oint d\vec{l}' \times \vec{f} \cdot dl}$, $\oint d\vec{l} \times \vec{f} = \int \nabla(f\cdot d\vec{s}) - \int d\vec{s}\,(\nabla\cdot f)$

P.S. $\nabla(\vec{f}\cdot d\vec{s}) = (ds\cdot\nabla)\vec{f} + d\vec{s}\times(\nabla\times\vec{f})$

(2) $\int d\vec{l}\times\vec{f} = \oint (d\vec{s}\cdot\nabla) f\cdot dl + \int ds\times(\nabla\times f)\cdot d\vec{l} - \int ds\,(\nabla\cdot f)\cdot d\vec{l}$

re-written

$$= \oint_P \oint_{S'} (d\vec{s}'\cdot\nabla')\overbrace{\left[\nabla'\frac{1}{|\vec{r}-\vec{r}'|}\right]}^{=0}\cdot d\vec{l} + \oint_P\oint_{S'} d\vec{s}'\times\underbrace{\left[\nabla'\times\left(\nabla'\frac{1}{|\vec{r}-\vec{r}'|}\right)\right]}_{=0}\cdot d\vec{l}$$

$$-\boxed{\oint_P\oint_{S'}\nabla'\cdot\left(\nabla'\frac{1}{|\vec{r}-\vec{r}'|}\right)ds'\cdot d\vec{l}}$$

$$= \underline{-4\pi\,\delta(r-r')}$$

3/13 (四)

$$\oint_P\oint_{P'} d\vec{l}'\times\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)\cdot d\vec{l}$$

$$= \underbrace{\oint_P\int_{S'}(d\vec{s}'\cdot\nabla')\left[\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)\right]}_{(1)}\cdot d\vec{l} + \oint_P\int_{S'} d\vec{s}'\times\left[\nabla'\times\nabla'\left(\underline{\frac{1}{|\vec{r}-\vec{r}'|}}\right)\right]d\vec{l}$$

$$\nabla'\times\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = -\nabla\times 4\pi\epsilon_0 E = 0$$

$$-\boxed{\oint_P\int_{S'}\nabla'\cdot\left(\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)\right)ds'\cdot d\vec{l}}$$

(1) $\Rightarrow \oint\int d\vec{s}'\cdot\nabla'\left(\nabla'\frac{1}{|\vec{r}-\vec{r}'|}\right)\cdot d\vec{l}$, $\boxed{\text{if } \frac{df}{dx}dx = df}$

$$\Rightarrow \oint\int d\vec{s}'\cdot\nabla\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)\cdot d\vec{l} = \int_{S'} -d\vec{s}'\cdot\underbrace{\boxed{\oint_P d\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)}}_{=0}$$

The Last term　代表Bio-Savart law 之 B 無散度

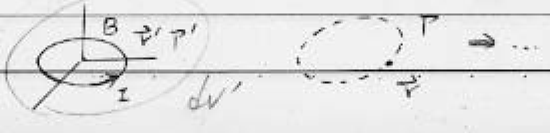
$$\Rightarrow \nabla'\cdot\nabla'\left(\frac{1}{|r-r'|}\right) = \nabla'^2\left(\frac{1}{|r-r'|}\right) = \underline{-4\pi}\,\delta(r-r')$$

$$P = P'$$

Eq 5.4.1

$$\oint \vec{B}\cdot \boxed{d\vec{l}} = \begin{cases}\mu_0 I\\ 0\end{cases} \Rightarrow \begin{matrix}\nabla\cdot B = 0\\ \nabla\times B = \mu_0 J\end{matrix}$$

Bio-Savart　　　Observer



$B = \frac{\mu_0 I}{4\pi}\int\frac{dl\times\hat{r}}{r^2}$,

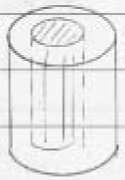
$\oint B\cdot d\vec{l}$ = (Lest term)

$= \mu_0 I$

(Prove B)

§5.3.2 From Biot-Savart Law to Divergency of B

<1> Volume current density.



$$\underline{\vec{B}(\vec{r})} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,\underline{dv'}$$

$$= \frac{\mu_0}{4\pi}\int J(r') \times \left[\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right] dv'$$

<2>

$$\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = \nabla'_{r'}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = -\nabla_r\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) \quad \text{special skill}$$

$$B = \frac{\mu_0}{4\pi}\int J \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,dv' = \frac{\mu_0}{4\pi}\int J(r') \times \left[-\nabla_r \frac{1}{(\vec{r}-\vec{r}')}\right] dv'$$

$$= \frac{\mu_0}{4\pi}\int \left(\nabla_r\left(\frac{1}{\vec{r}-\vec{r}'}\right) \times \vec{J}(\vec{r}')\,dv'\right.$$

<3> $\nabla \times (f\vec{A}) = f(\nabla \times \vec{A}) + \boxed{-\vec{A} \times (\nabla f)}$

$$\Rightarrow \quad = \nabla \times \left(\frac{J(r')}{|\vec{r}-\vec{r}'|}\right) - \frac{1}{|\vec{r}-\vec{r}'|}(\nabla \times \vec{J}(r'))$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \nabla_r \times \left(\frac{J(\vec{r}')}{|\vec{r}-\vec{r}'|}\right) dv' = \nabla_r \times \left[\frac{\mu_0}{4\pi}\int \frac{J(r')}{|\vec{r}-\vec{r}'|}\,dv'\right]$$

Let the vector operator $\frac{\mu_0}{4\pi}\int \frac{J(r')}{|\vec{r}-\vec{r}'|}\,dv' = \vec{A}$

then $\vec{B} = \nabla_r \times \vec{A}$

Next weak, test, P.215 ~ P.220

NO.

DATE 3/13

<4> We can get two results of Divergency of B-field.

① $\nabla_r \cdot B = \nabla_r \cdot (\nabla_r \times \vec{A}) = 0$

vector identity

Divergency of Observer

② $\vec{\nabla}_r \times \vec{B}_r$ curl of Observer

$$\nabla_r \times (\nabla_r \times \vec{A}) = \nabla(\nabla A) - \vec{J} \cdot \vec{A} = \mu_0 \vec{J}$$

③ Cover the eq of

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I \quad \text{, for 2D/Area.}$$

$$= \int \nabla \times \vec{B} \cdot d\vec{a} = \mu_0 \int J \cdot d\vec{a}$$

3/19 (三)

$$\longrightarrow \vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{r}}{r^2} \quad (5.2)$$

Review: From Biot – Savart Law to Divergency & Curl of B. (5.3)

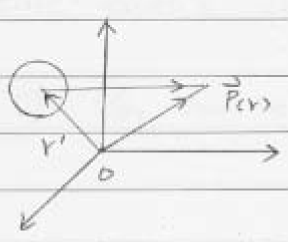
Conclusion: $\vec{B}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r') \times \hat{r}}{r^2} dv' \quad (5.3) \rightarrow$ Ampere's Law $\underline{\underline{\nabla \cdot B = 0}}$

$\nabla \times B(r) = \mu_0 J$

$$\vec{B}(r) = \nabla \times \vec{A} \quad , \quad \boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|\vec{r} - \vec{r}'|} d^3 r'} \rightarrow \text{Vector Potential.}$$

§ 5.3.2 Reverse divergency & Curl of B.

Take $\boxed{\nabla \cdot}$ or $\boxed{\nabla \times}$ of B



From the form of

$$B(r) = \frac{\mu_0}{4\pi} \int \frac{J(r) \times \hat{r}}{r^2} d^3 r' \quad , \quad \vec{r} = \vec{r} - \vec{r}'$$

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3/13 的筆記做成 PDF

DATE 3/19 (三)

(1) Take $\nabla \cdot B = \frac{\mu_0}{4\pi} \int \nabla \cdot \left(\vec{J}(r') \times \frac{\hat{\mathscr{r}}}{\mathscr{r}} \right) d^3r'$

$$\left(\begin{array}{l} \text{Math. method} \\ \nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} (\nabla \times \vec{A}) - \vec{A} (\nabla \times \vec{B}) \end{array} \right) = 0$$

$$= \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \underbrace{(\nabla \times \vec{J})}_{=0} - \vec{J} \cdot \underbrace{\left(\nabla \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)}_{=0}$$

$\boxed{\text{Pro. 1.62}}$

(2) $\nabla \times B = \nabla \times \nabla \times \vec{A}$ (so, way 1 is better.)

$\Rightarrow \nabla \times \nabla \times A = \nabla (\nabla \cdot \vec{A}) - \nabla^2 A$

$$= \nabla_r \left[\frac{\mu_0}{4\pi} \nabla_r \cdot \int \frac{J(r')}{|\vec{r} - \vec{r}'|} d^3r' \right] - \nabla_r^2 \frac{\mu_0}{4\pi} \int \frac{J(r')}{|\vec{r} - \vec{r}'|} d^3r'$$

$$= \nabla_r \left[\frac{\mu_0}{4\pi} \int J(r') \left(\nabla \cdot \frac{1}{|\vec{r} - \vec{r}'|} \right) d^3r' \right] - \frac{\mu_0}{4\pi} \int J(r') \boxed{\nabla^2 \left(\frac{1}{|\vec{r} - \vec{r}'|} \right)} d^3r'$$

$\hookrightarrow -4\pi \delta(r - r')$

The Second Term.

$(\because \nabla \times \vec{B} = 0)$

$$-\frac{\mu_0}{4\pi} \int J(r') \cdot (-4\pi \delta^3(r - r')) d^3r' = \boxed{\mu_0 J(r)}$$

if $\boxed{\int f(x') \delta(x - x') = f(x) \text{, if } x = x'}$

The First Term.

for $\int f (\nabla \cdot \vec{A}') = -\int \vec{A}' \cdot \nabla f \, dv + \oint f \vec{A}' \cdot d\vec{a}$

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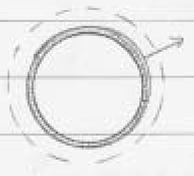
$\vec{A}' = J(r')$, $f = \frac{1}{|\vec{r} - \vec{r}'|}$

$$\therefore \int J(r')\, \nabla' \left(\frac{1}{|r - r'|} \right) = - \int \frac{\nabla' \cdot J}{|r - r'|} dv' + \oint \frac{J(r')}{|r - r'|} da = 0$$

$\nabla \cdot J(r) = 0$, $\nabla \times B = \mu_0 J$, $\nabla \cdot (\nabla \times B) = \nabla \cdot J \mu_0 = 0$

for $\boxed{\text{Steady current}}$ I

§ 5.4 Magnetic Vector Potential.



divergency less

Support the original potential A_0 is not divergenceless. We add to it the gradient of λ, so the new vector potential $A = A_0 + \nabla \lambda$.

The new divergence is $\nabla \cdot \vec{A} = \nabla \cdot (A_0 + \nabla \lambda)$

$$= \underline{\nabla \cdot A_0} + \underline{\nabla^2 \lambda}$$

if $\boxed{\nabla^2 \lambda = - \nabla \cdot A_0}$, then $\nabla \cdot A = 0$

Compare with Poisson eq. $\nabla^2 V = \boxed{-\rho/\epsilon_0}$

But that's identical to Poisson eq. $\nabla^2 V = -\rho/\epsilon_0$

① ρ/ϵ_0 for electrical potential as the source.

NO

DATE 3/19

② Then the E potential torm $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho'}{|\vec{r}-\vec{r}'|}\, d^3r'$

③ Then the λ can be rewritten as

$$\boxed{\lambda = \frac{1}{4\pi}\int \frac{\nabla\cdot A_0}{|\vec{r}-\vec{r}'|}\, d^3r'}$$

④ $\nabla\cdot A_0$ does not go to zero at Infinity

$\boxed{\nabla^2 A = -\mu_0 J}$ (From $\nabla\times B = \mu_0 J$)

3/20 (四)

* Dirac Delta function 1920

① The origin of δ-function.

(力学, 电学, 磁学)

② The function of potential $\underset{\text{are}}{\underline{\text{is}}}$ proportional to $\frac{1}{|r-r'|}$.

③ The Question of $\underset{\substack{=-\rho/\epsilon_0 \\ =-\mu_0 J}}{\nabla^2 V = ?}$ in spherical coordinates?

④ The spherical coordinates of $\underline{3D}$

$$\boxed{\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}}$$

⑤ if $f = \frac{1}{r}$, for $\hat{r}$-direction

$$\boxed{\longrightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\left(\frac{1}{r}\right)\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot\left(-\frac{1}{r^2}\right)\right)}$$

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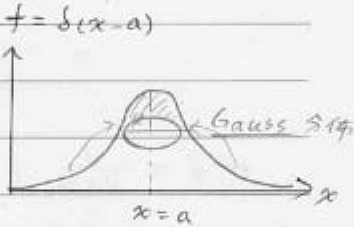
NO

DATE 3/20 (四)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{-1}{r^2}\right) = \frac{-1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{r^2}\right) = 0$$

⑥ if $r = 1$, then $\nabla_r^2 f = 0$

if $r = 0$, $\frac{r^2}{r^2} = \frac{0}{0}$ = undetermined



⑦ Definition of δ − fuction.

a. The sum of probability = 1

b. The position of $x = a$

c. $\int_{-\infty}^{\infty} f(x)\,dx = \int \delta(x-a)\,dx = 1$

d. if a function has the distribution.

Properties :

$$\int f(x)\,\delta(x-a)\,dx = f(a)$$

* Prove $\nabla^2\left(\frac{1}{r}\right) = ?$ by integrated function

$$\int \nabla^2\left(\frac{1}{r}\right) d^3r = \int \nabla\cdot\left(\nabla\frac{1}{r}\right) d^3r$$

Then we can write as

$$\int \nabla\cdot\left(\nabla\frac{1}{r}\right) d^3r = \int \nabla\left(\frac{1}{r}\right)\hat{e}_r\, d\vec{a}$$

$\Rightarrow d\Omega = \frac{da}{r^2}$ for solid angle.

$$= \int \nabla\left(\frac{1}{r}\right)\hat{e}_r\, r^2 d\Omega = \int -\frac{1}{r^2}\hat{e}_r \cdot r^2 d\Omega = \underline{-\int d\Omega = -4\pi}$$

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DATE 3/20

Derive $\nabla^2\phi = -\rho/\epsilon_0$, $\phi = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(x')}{|x-x'|} d^3x'$

$\Rightarrow \nabla^2\phi = \frac{1}{4\pi\epsilon_0}\int \boxed{\nabla^2\left(\frac{1}{|x-x'|}\right)} \rho(x')\, d^3x'$

$\Downarrow$

$= \underline{-\rho(x)/\epsilon_0}$ undetermined function.

$\nabla^2\phi = \frac{1}{4\pi\epsilon_0}\int \underline{-4\pi\,\delta(x-x')}\,\rho(x')\, d^3x'$ 把物理的矛盾，用数学解决。

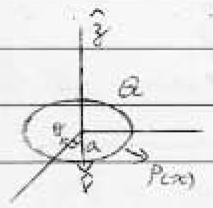
$= \frac{1}{4\pi\epsilon_0}(-4\pi)\,\rho(x'=x) = \underline{-\rho(x)/\epsilon_0}$

Skill

$\nabla_r\left(\frac{1}{|r-r'|}\right) = \frac{\partial}{\partial r}\left(\frac{1}{|r-r'|}\right) = \frac{\partial}{\partial r'}\left(\frac{-1}{|r-\underline{r'}|}\right)$ $+(-r')$

$= -\nabla'_{r'}\left(\frac{1}{|r-r'|}\right)$

§ A total charge Q is uniformly distributed around a circular ring of radius $\underline{a}$. $\boxed{\text{infintesimal thickness}}$.



$\rho(x)$ in cylindrical coordinates.

$\rho(x) = k\,\delta(r-a)\,\delta(z)$ charge density of ring

what's the k constant?

7.5mm×28行

$$\int \rho(x)\, d^3x = Q = \int k\, \delta(r-a)\, \delta(z)\, d^3x$$

$$= \int k\, \delta(r-a)\, r\, dr \underbrace{\int d\theta}_{= 2\pi} \underbrace{\int \delta(z)\, dz}_{= 1}$$

$$= \int k\, \delta(r-a)\, r\, dr \cdot 2\pi \cdot 1$$

$$= 2\pi k \int \delta(r-a)\, r\, dr, \quad r = f(r)$$

$$\Rightarrow 2\pi k \int \delta(r-a) \underset{\Downarrow\ r}{f(r)}\, dr = 2\pi k\, f(a) = 2\pi k a$$

$Q = 2\pi k a \Rightarrow$ Then we can obtain the k constant as

$$k = \frac{Q}{2\pi a}, \quad \boxed{\rho(x) = \frac{Q}{2\pi a}\, \delta(r-a)\, \delta(z)}$$

3/26 (三)

* Magnetic Vector Potential (5.4)

(1) $A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|\vec{r}-\vec{r}'|}\, d^3r'$ for volume.

(2) $A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{k(r')}{|\vec{r}-\vec{r}'|}\, da'$ $\boxed{\text{for surface.?}}$

(3) $A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\lambda(r')}{|r-r'|}\, dr'$ for line.

(4) Special skill. The definition of $J = \frac{I}{A} = \frac{Q/t}{A}$ if plus d.

$$= \frac{d \cdot Q/t}{d \cdot A} = \frac{\frac{d}{t} \cdot Q}{\text{Volume}} = \frac{V \cdot Q}{\text{Volume}}.$$

$$J, k, \lambda = \text{Velocity} \cdot \text{charge} / \text{Volume} = \boxed{(\omega \times r)\, \sigma}$$

$$= v\sigma$$

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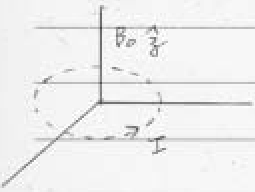
DATE 3/26 (三)

if $\vec{B} = \nabla \times \vec{A}$, $\vec{A}$ is an unique solution?

Ex : Vector potential of a uniform magnetic field $B_0 \hat{z}$ in the direction of the z-axis.



(1) $\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}$.

(2) Drwen the $\vec{A}$, $B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0$$

$$\boxed{B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B_0 \hat{z}}$$

It is clear for any partical magnetic field $\vec{B}$.

a. $A_y = x B_0$, $A_x = 0$, $A_z = 0$.

b. $A_x = -y B_0$, $A_y = 0$, $A_z = 0$.

c. $A_x = -\frac{1}{2} y B_0$, $A_y = \frac{1}{2} x B_0$, $A_z = 0$.

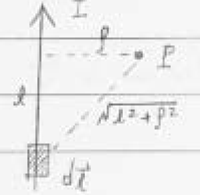
The 3rd solution is $|\vec{A}| \propto \sqrt{x^2+y^2}$.

Thus $\boxed{\vec{A} = \frac{1}{2} \vec{B} \times \vec{r}}$. it's magnitude is $\frac{1}{2} Br$ & rotates about z-axis.

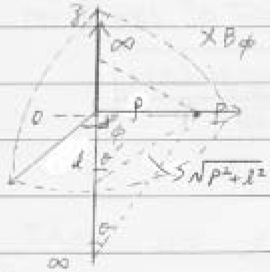
7.5mm×28行

$\vec{B} = \nabla \times \vec{A} \longrightarrow \vec{A} = \frac{1}{2}\vec{B}\times\vec{r} \longrightarrow \cdots$

Ex : The magnetic field due to a current flowing in a thin long straight wire.



where $d\vec{A} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}}{r}$ $\left(\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r}dv\right)$



$A_z = 2\cdot\frac{\mu_0}{4\pi}\int_0^L \frac{I\,dl}{\sqrt{\rho^2+l^2}} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{\sqrt{L^2+\rho^2}+L}{\rho} - 1\right)$ (P.S. I)

if $\rho^2 \ll L^2$, $(L\to\infty)$ $A \approx \frac{\mu_0 I}{2\pi}\ln\frac{2L}{\rho}$

Note: 三角化換法

what's the magnetic field.

$\sqrt{x^2+a^2}$, x, θ, a

let $x = a\tan\theta$

$\vec{B} = \nabla\times\vec{A}_z = \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix}$, $B_\rho = 0$, $B_z = 0$

$B_\phi = -\frac{\partial A_z}{\partial\rho} = \frac{\mu_0 I}{2\pi\rho}$, for $\rho^2 \ll L^2$. $(L\to\infty)$

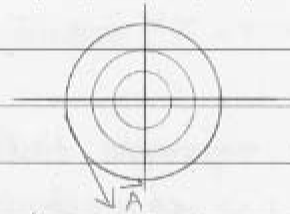
* Vector potential & magnetic field for a slowly moving charge. For a charge q moving with a constant $\vec{v}$, may put $q\vec{v}$ for <u>$J\,dV$</u>.

$\boxed{\begin{array}{l}\vec{J} = \vec{v}\cdot\rho = \vec{v}\cdot\frac{Q}{\text{Volume}} \\ \vec{J}\cdot d(\text{Volume}) = \vec{v}\cdot Q\end{array}}$ $\boxed{\vec{A} = \frac{\mu_0}{4\pi}\frac{q\vec{v}}{r}}\left(= \frac{\mu_0}{4\pi}\int\frac{\vec{v}\sigma\,da}{r}\right)$

NO.

DATE 3/26

given by equation.



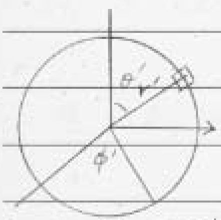
The magnetic field

$$\vec{B} = \nabla\times\vec{A} = \nabla\times\left[\frac{\mu_0 q}{4\pi}\frac{\vec{v}}{r}\right] = \frac{\mu_0 q}{4\pi}\left(\frac{\vec{v}\times\hat{r}}{r^2}\right).$$

§5.11 A spherical shell of radius R, carrying a uniform surface charge σ is set spinning at $\boxed{\text{angular velocity } \omega}$ $\vec{v} = \vec{\omega}\times\vec{r}'$

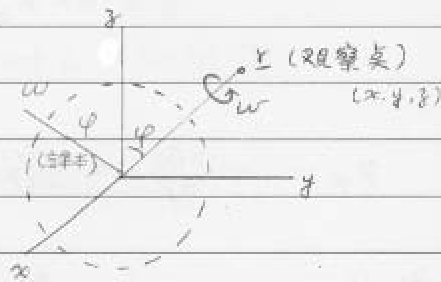
Find the vector potential it produce at r.

$$A(r) = \frac{\mu_0}{4\pi}\int\frac{K'(r)}{|\vec{r}-\vec{r}'|}da' \Rightarrow \boxed{\begin{matrix}\vec{v}\rho = \vec{J}\\ \vec{v}\sigma = \vec{K}\end{matrix}} \Rightarrow \boxed{A(r) = \frac{\mu_0}{4\pi}\int\frac{\omega\times\vec{r}'\cdot\sigma}{|\vec{r}-\vec{r}'|}da'}$$



(被觀察点)

if $\frac{1}{|r-r'|}$, $r>r'$, $r<r'$



(1) $\vec{r}-\vec{r}' = \sqrt{R^2+r^2-2Rr\cos\theta'}$

(2) $da' = R^2 \sin\theta'\, d\theta'\, d\phi'$ (area)

$\int \sin\phi'\, d\theta' = 0$, $\int\cos\phi'\, d\phi' = 0$

(3) Note the velocity of a point r' in a rotating rigid body is given by $\omega\times\vec{r}' = \vec{v}$.

NO.

DATE 3/26

(4) $w = w\sin\phi\,\hat{x} + 0 + w\cos\phi\,\hat{z}$

$$\vec{r}' = R\sin\theta'\cos\phi'\,\hat{x} + \sin\theta'\sin\phi'\,\hat{y} + \cos\theta'\,\hat{z}$$

$$\vec{v} = w\times\vec{r}' = Rw\{-(\cos\phi\sin\theta'\sin\phi')\,\hat{x} + \sin\phi\sin\theta'(\sin\phi')\,\hat{y} + [\cos\phi\sin\theta'\cos\phi' - \sin\phi\cos\theta']\,\hat{z}\}$$

$$A(r) = \frac{\mu_0}{4\pi}\int\frac{V\sigma\,da'}{|r-r'|}$$

$$= \frac{\mu_0}{4\pi}\int_{\theta',\phi'}\frac{\sigma Rw(-\sin\phi\cos\theta')\,\hat{y}}{\sqrt{R^2+r^2-2Rr\cos\theta'}}\,R^2\sin\theta'\,d\theta'\,d\phi'$$

$$\boxed{\begin{aligned} &\text{let } u\equiv\cos\theta' \Big|_{-1}^{+1} \\ &du = -\sin\theta'\,d\theta' \Big|_{0}^{\pi}\end{aligned}}$$

Then we can write as

$$\int\frac{u\,du}{\sqrt{R^2+r^2-2Rru}} = \int\frac{\cos\theta'(-\sin\theta')\,d\theta'}{\sqrt{R^2+r^2-2Rr\cos\theta'}}$$

$$-\frac{R^2+r^2-Rru}{3R^2r^2}\sqrt{R^2+r^2-2Rru}\,\Big|_{-1}^{+1}$$

$$= \frac{-1}{3R^2r^2}\left[(R^2+r^2+Rr)\,\underline{|R-r|} - (R^2+r^2-Rr)\,\underline{(R+r)}\right]$$

if the point $\vec{r}$ lies inside the sphere, then $R > r$

$\Rightarrow$ The above term is

$$\frac{-1}{3R^2r^2}\left[(R^2+r^2+Rr)\,\underline{(R-r)} - (R^2+r^2-Rr)(R+r)\right]$$

$$= \frac{2r}{3R^2} \quad (r<R)$$

7.5mm×28行

if $R < r$, then

$$\frac{1}{3R^2r^2}\left[(R^2+r^2+Rr)(r-R)-(R^2+r^2+Rr)(R+r)\right]$$

$$= \frac{2R}{3r^2} \quad (r > R)$$

§ 5.11

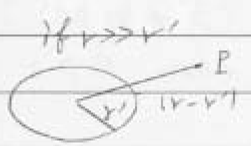
$$A(r) = \begin{cases} \dfrac{-\mu_0 R^3 \sigma w \sin\phi}{2} \cdot \dfrac{2r}{3R^2}, & \text{if } r<R \\ \dfrac{-\mu_0 R^3 \sigma w \sin\phi}{2} \cdot \dfrac{2R}{3r^2}, & \text{if } r>R \end{cases}$$

if $w \times \vec{r} = -wr\sin\phi\, \hat{y}$

$$A(r) = \frac{\mu_0 R \sigma}{3}(w \times \vec{r}), \text{ inside}.$$

$$= \frac{\mu_0 R^4 \sigma}{3r^3}(w \times \vec{r}), \text{ outside}.$$

* 5.4.3 Find of a small object/current loop.



The vector potential $\vec{A}$ may be used to advantage in determing the magnetic field.

Taylor Expanson for $\frac{1}{|r-r'|}$.

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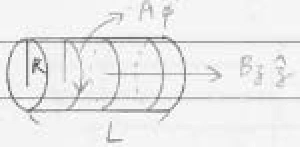
Ex 5.12

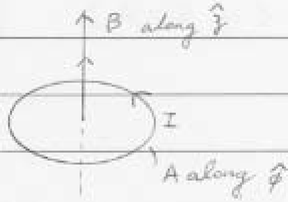


<1> if we calculate the magne field along $\hat{z}$-axis.

$\int B \cdot dl = \mu_0 \cdot I_{enclosed}$.

$\int B \cdot dl \longrightarrow$ along $\hat{z}$-axis. (L) (2πs)

<2> Then A along $\hat{\phi}$.



$\int A\, da = A \cdot \pi s^2$

$\int A \cdot dl \longrightarrow \underline{2\pi s}$, $s \equiv$ radius. (L)

Find the vector potential of an finite solenoid with n turns per unit length, with radius R & current I, (a) if s < R , (b) s > R .

$n \equiv \frac{N}{L}$, then $I \equiv 1$ turn current

NI = Enclosed current

$= I_{enclosed} = NI$

(a) if s<R . then the rule of $B = \nabla \times A$ & the magnetic flux.

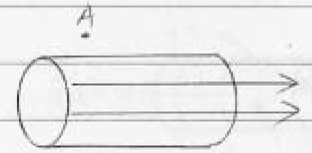
$$\underline{\int \vec{B} \cdot d\vec{a}}_{\uparrow \pi s^2} = \int \nabla \times \vec{A} \cdot d\vec{a} = \underline{\int \vec{A} \cdot d\vec{l}}_{\downarrow 2\pi s}$$

$$\boxed{\begin{aligned} \int \vec{B} \cdot d\vec{l} &= \mu_0 \cdot I_{enclosed} \\ B \cdot L &= \mu_0 N I \end{aligned}} \Rightarrow B = \mu_0 I n .$$

7.5mm×28行

Then, $\int B \cdot da = \int \mu_0 n I \cdot da = \mu_0 n I \cdot \pi s^2$

$= \int A \cdot dl = A \cdot 2\pi s$



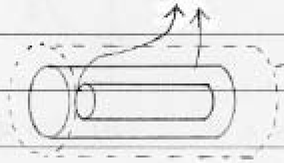
Then we can get $\Longrightarrow A = \frac{\mu_0 n I}{2} s \hat{\phi}$

<if $s > R$> $\int B \cdot dl = \mu_0 \cdot I_{enclosed}$

$B = \mu_0 n I$

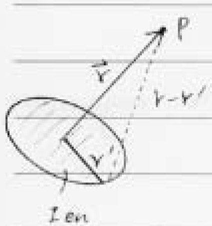
Then, $\underline{\mu_0 I n \cdot \pi R^2} = A \cdot 2\pi s$

$\int B \cdot da \equiv$ fixed



$\Longrightarrow A = \frac{\mu_0 I n}{2\pi s} \pi R^2 \hat{\phi}$.

§5.4.3 Find the small current loop: the magnetic dipole.



The vector potential $\vec{A}$

$$\vec{A} = \frac{\mu_0 I_{en}}{4\pi} \oint \frac{d\vec{r}'}{|\vec{r} - \vec{r}'|} \hat{\phi}$$

where, $\vec{r}$ is far away from r'

(不同於 P.243)

if $r \gg r'$, Then we can expand $|\vec{r}-\vec{r}'|$ in power of $r'/r \ll 1$.

$$|\vec{r}-\vec{r}'|^{-1} = \frac{1}{r}\left[1+\frac{\vec{r}'\cdot\vec{r}}{r^2}+\cdots\cdots\right]$$

The vector potential can be re-written as

$$\vec{A} = \frac{\mu_0 I_{en}}{4\pi}\left[\underbrace{\oint \frac{1}{r}\,dr'}_{\boxed{\frac{1}{r}\oint dr' = 0}} + \frac{1}{r^3}\oint(\vec{r}'\cdot\vec{r})\,dr'\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\frac{1}{r^3}\oint(\vec{r}'\cdot\vec{r})\,dr'$$

We use the math. method

$$(\vec{r}'\times d\vec{r}')\times\vec{r} = \underline{d\vec{r}'(\vec{r}\cdot\vec{r}') - \vec{r}(\vec{r}'\cdot d\vec{r}')}$$

$$\vec{r}(\vec{r}'\cdot dr') = -(\vec{r}'\cdot\vec{r})\,d\vec{r}'$$

$\Rightarrow$ So, the $(\vec{r}'\times d\vec{r}')\times\vec{r} = \underline{2\,d\vec{r}'(\vec{r}\cdot\vec{r}')}$

Then the vector potential can be obtain as

被观察物

$$\vec{A} = \frac{\mu_0 I_{en}}{4\pi}\frac{1}{r^3}\boxed{\oint_{r'}\frac{1}{2}(\vec{r}'\times d\vec{r}')}\times\vec{r}$$

We can define $\left[\frac{1}{2}\oint(\vec{r}\times d\vec{r}')\right]\times\vec{r} = \underline{\vec{m}}\times\vec{r}$.

next weak (四)，演習課

(三)，小考 ；小考 5.8?

P.234 ~5.4.1 (P.240)

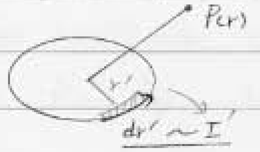
NO.

DATE 4/2 (三)

($\vec{B} \rightarrow \vec{A} \rightarrow \vec{m}$)

4/2 (三)

$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{r}'}{|\vec{r} - \vec{r}'|}$



($\frac{I}{2} \oint \vec{r}' \times d\vec{r}'$)

, $\boxed{m = \frac{1}{2} \oint \vec{r}' \times d\vec{r}'}$

↑ r'-axis part

grill

$= \frac{\mu_0 \vec{r}'}{4\pi r^3} \vec{m}' \times \vec{r}$

$= \vec{m}' \times \left(\frac{\mu_0 \vec{r}}{4\pi r^3} \right)$

P.244, 5.8 $\Rightarrow$ $m = I \int da = IA$ = magnetic dipole moment!

(被觀測物)

The magnetic field $\vec{B}$ is determied by calculating $\vec{\nabla} \times \vec{A}$.

(1) $\vec{B} = \vec{\nabla} \times \vec{A} = \nabla_r \times \left[\frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3} \right]$,

$\nabla \times (A \times B) = A(\nabla \cdot B) - (A \cdot \nabla) B$

$= \frac{\mu_0}{4\pi} \left[\vec{m} \cdot \left(\nabla \left[\frac{\vec{r}}{r^3} \right] \right) - (\vec{m} \cdot \nabla) \frac{\vec{r}}{r^3} \right]$

→ why? ~~hard~~ (some people have problem)

$= \frac{-\mu_0}{4\pi} (m \cdot \nabla) \frac{\vec{r}}{r^3}$

$(m \cdot \nabla) \frac{\vec{r}}{r^3} = \left(m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z} \right) \left[\frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}} \right]$

$m_x \frac{\partial}{\partial x} \left[\frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}} \right]$ $\boxed{\text{X-axis}}$

$= \frac{m_x}{(x^2+y^2+z^2)^{3/2}} + \boxed{m_x \cdot x} \left(\frac{-3}{2} \right) \cdot 2x \cdot \frac{1}{(x^2+y^2+z^2)^{3/2}}$

Sun over $x \cdot y \cdot z$ $\quad \frac{\vec{m}}{r^3} - \frac{3\vec{m} \cdot \vec{r}}{r^5} \vec{r}$

7.5mm×28行

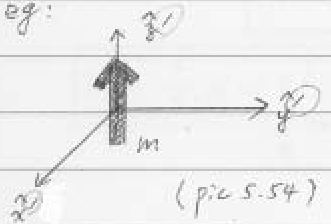
So we can obtain the magnetic field

$$(m \cdot \nabla)\frac{\vec{r}}{r^3} = \frac{\vec{m}}{r^3} - \frac{3\vec{m}\cdot\vec{r}}{r^5}\vec{r}$$

$$\vec{B} = \frac{+\mu_0}{4\pi}\left[\frac{3\vec{m}\cdot\vec{r}}{r^5}\vec{r} - \frac{\vec{m}}{r^3}\right] \#$$

$$= \frac{\mu_0}{4\pi r^3}\left[\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^2} - \vec{m}\right] = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

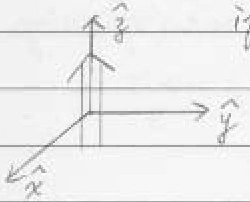
eg:



(pic 5.54)

$$\Rightarrow \frac{\mu_0 m r \sin\theta}{4\pi r^3}\hat{\phi} = \boxed{\frac{\mu_0 m \sin\theta}{4\pi r^2}\hat{\phi}}$$

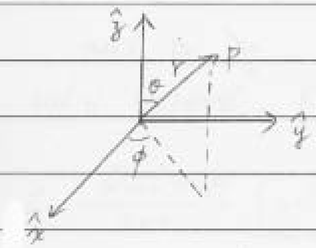
Example: Find the magnetic dipole moment $A_{dip}(r)$, if $m = m_0\hat{z}$. $(= I\int da = I\vec{a}\hat{n})$



1. Then the magnetic field of a pure dipole is easiest to calculate if we put m at the origin.

$$\vec{A} = \frac{\mu_0}{4\pi r^3}(\vec{m}\times\vec{r}), \quad \text{if } \vec{m} = m_0\hat{z}$$

$$\vec{r} = r\cos\phi\sin\theta\,\hat{x} + r\sin\phi\sin\theta\,\hat{y} + r\cos\theta\,\hat{z}$$



$$\vec{m}\times\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & m_0 \\ r\cos\phi\sin\theta & r\sin\phi\sin\theta & r\cos\theta \end{vmatrix}$$

$$= r m \sin\theta\left[-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}\right]$$

(直角座标)

NO.

DATE 4/2

先5.58 → 再 5-57

$(\hat{r}, \hat{\theta}, \hat{\phi}) \longleftrightarrow (x, y, z)$

Transformation

(a) $\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

(b) $\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\theta\,\hat{y} - \sin\theta\,\hat{z}$

(c) $\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

$\Rightarrow \vec{m}\times\vec{r} = r m \sin\theta\,\hat{\phi}$, $\vec{A} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$

Pro 5.58, A uniform charged $\boxed{\text{solid sphere}}$ of radius R carries a total charge Q and its set spinning with angular velocity ω. about the z-axis.

Ex 5.11 Vector potential

$$A(r,\theta,\phi) = \begin{cases} \frac{\mu_0 R\omega\sigma}{3} r\sin\theta\,\hat{\phi} & (r\le R) \\ \frac{\mu_0 R^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\hat{\phi} & (r>R) \end{cases}$$

$\boxed{\text{skill}}$ if we know the dipole moment of Δ can be rewritten as

$A = \frac{\mu_0 m}{4\pi r^2}\sin\theta\,\hat{\phi}$. $\vec{m} = ?$

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$(r>R)$

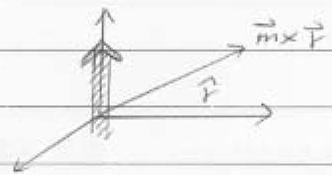
$$\boxed{\frac{\mu_0 \sin\theta}{4\pi r^2}\, m} = \boxed{\frac{\mu_0 R^4 \omega \sigma}{3}\ \frac{\sin\theta}{r^2}} \quad , \quad m = \frac{4\pi}{3} R^4 \omega \sigma . \quad \text{(shell)}$$

In the case of solid sphere, let the parameters of $\boxed{R \to r}$ $\boxed{\sigma \to \rho\, dr}$

$$m = \frac{4}{3}\pi \omega \int_0^R \rho\, r^4\, dr$$

$$= \frac{4\pi}{3}\omega \rho\, \frac{1}{5} R^5\, \hat{z}$$

$$\boxed{m = \frac{1}{5} Q \omega R^2\, \hat{z}}$$



(b) Find the average magnetic field within the sphere.

$$\langle B_{avg} \rangle = \frac{1}{V}\int B\, dv = \frac{1}{4/3\, \pi R^3}\int B\, dv$$

if shell

$$B = \frac{\mu_0 \cdot 2m}{4\pi R^3} \quad , \quad \langle B \rangle = \frac{\mu_0 \cdot 2 Q \omega}{4\pi \cdot 5 R}\, \hat{z} \qquad \langle B \rangle = \frac{1}{A}\int B\, da$$

(c) Find the $\vec{A}$ at a point of (r, θ) where $r \geq R$.

$$\vec{A} = \frac{\mu_0 R^4 \omega \sigma}{3}\ \frac{\sin\theta}{r^2}\, \hat{\phi} \quad \text{(spheric shell)}$$

solid sphere $R \Rightarrow r'$, $\sigma \Longrightarrow \rho\, dr'$

$$\vec{A} = \frac{\omega \mu_0 \sin\theta}{3 r^2} \underbrace{\int r'^4 \rho\, dr'}_{R^5/5} = \frac{\mu_0 Q \omega R^2 \sin\theta}{4\pi \cdot 5 \cdot r^2}\, \hat{\phi} \quad \text{(solid sphere)}$$

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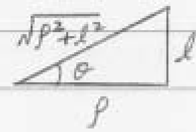
P.S. (I)

$$\int_0^L \frac{dl}{\sqrt{\rho^2+l^2}}$$ ， 三角代换法



let $l = \rho \tan\theta$ ， $dl = \rho \sec^2\theta \, d\theta$

$$= \int \frac{\rho}{\sqrt{\rho^2+l^2}} \sec^2\theta \, d\theta$$

$$= \int \cos\theta \cdot \sec^2\theta \, d\theta$$

$$= \int \sec\theta \, d\theta$$

※ P.S. (II)

$$= \ln\left|\sec\theta + \tan\theta\right| \Big|_{\bigcirc}^{\bigcirc}$$

$$= \ln\left|\frac{\sqrt{l^2+\rho^2}}{\rho} + \frac{l}{\rho}\right| \Bigg|_0^L$$

$$= \ln\left(\frac{\sqrt{L^2+\rho^2}}{\rho} + \frac{L}{\rho} - \frac{\rho}{\rho} - 0\right)$$

$$= \ln\left(\frac{\sqrt{L^2+\rho^2}+L}{\rho} - 1\right)$$ ※

NO.

DATE

P.S. (II)

$$\int \sec\theta \, d\theta = \int \sec\theta \left(\frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \right) d\theta$$

$$= \int \frac{\sec^2\theta + \sec\theta \tan\theta}{\sec\theta + \tan\theta} \, d\theta$$

考慮分母.

$$\Longrightarrow \frac{d}{d\theta}(\sec\theta + \tan\theta) = \frac{d}{d\theta}\left(\frac{1+\sin\theta}{\cos\theta} \right)$$

$$= \frac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{1+\sin\theta}{\cos^2\theta}$$

$$= \sec^2\theta + \sec\theta \tan\theta \quad (\text{等於分子})$$

$$\Longrightarrow \int \frac{\sec^2\theta + \sec\theta \tan\theta}{\sec\theta + \tan\theta} \, d\theta = \int \frac{d(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta}$$

let $(\sec\theta + \tan\theta) = y$

$$\Longrightarrow \int \frac{1}{y} \, dy = \ln|y| + c$$

$$= \ln|\sec\theta + \tan\theta| + c \quad \#$$

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