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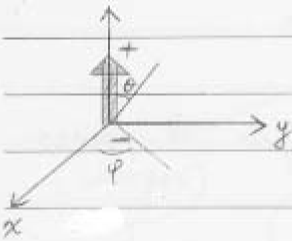
$(\hat{r} = \frac{\vec{r}}{r})$



$$\vec{E} = \vec{E_r}\,\hat{r} + \vec{E_\theta}\,\hat{\theta} + 0\,\hat{\varphi}$$

$$= \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

上週

$$V(r,r) = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\cdot\frac{\int r'\rho(r')\,dv'}{P} = \frac{1}{4\pi\epsilon_0}\frac{\vec{r}\cdot\vec{p}}{r^3}$$

P.S. The field of an electric dipole may be expressed in the following way.

$$\vec{E}_r = -\nabla_r V \text{ (符合上式) } = \frac{-1}{4\pi\epsilon_0}\nabla\left[\frac{\vec{r}\cdot\vec{p}}{r^3}\right]$$

$$= \frac{-1}{4\pi\epsilon_0}\left[\frac{1}{r^3}\nabla(\vec{p}\cdot\vec{r}) + (\vec{p}\cdot\vec{r})\nabla\frac{1}{r^3}\right]$$

Now, we know

$$\nabla_r(\vec{p}_{r'}\cdot\vec{r}) = \vec{p}\;\nabla\cdot\vec{r} = \vec{p}$$

$$\nabla\left(\frac{1}{r^3}\right) = \frac{-3\vec{r}}{r^5}$$

The Expression for $\vec{E}$ becomes

$$\boxed{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right] = \frac{1}{4\pi\epsilon_0 r^3}\left(\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right)$$

$\boxed{\text{Problem 3.42}}$ + $\boxed{\text{Problem 4.8}}$

E dipole   U dipole

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1. 自由电子 → ⊕ ⊖ 無交互作用 Chapter 2

2. Dipole → ⊕⟵⊖ 有交互作用 Chapter 3

3. Dipole in E-field , Chapter 4.  E

Chapter 4.

Isotropic system $\xrightarrow{\text{係数}}$ 常数, k, x 均匀

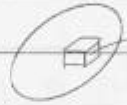
Anisotropic system $\xrightarrow{\text{係数}}$ 張量 (tensor) $[\ \ ]_{3\times3}$ 不均匀

Dielectric matter.

E-field ⟹ displacement by an electric field.

As a result of polarization each atom becomes a tiny dipole whose strength depends on $\vec{E}$, the vector electron cloud is displaced by a distance δ.

⟹ 

 $\vec{p} = \int r' \rho(r') dv'$

The P then can be written

as $\vec{p} = q\vec{\delta}$, $q = Ze$, Z is atomic number.

If we can define N atoms in unit volume. We can define a dipole moment per unit volume.

$$\vec{P} = Nq\vec{\delta} \ , \ \vec{p} = \vec{P}\cdot V .$$

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Comments 3:

$\boxed{1}$ If $\vec{E}$ is not too large the strength of each atomic dipole is $\vec{p} \propto \vec{E}$

$$\Rightarrow \vec{p} = \alpha \vec{E}, \quad \alpha \text{ is atomic polarizability.}$$

$\boxed{2}$ polarization $\vec{P}$ for isotropic $\vec{P} = N\alpha\vec{E}$

If in terms of a scalar quantity denoted as χ.

$$\vec{P} = N\alpha\vec{E} = \epsilon_0 \chi \vec{E}$$

If applied field $\vec{E} = \vec{E}_x \hat{x} + \vec{E}_y \hat{y} + \vec{E}_z \hat{z}$ (anisotropic E-field) the linear relation between $\vec{E} \propto \vec{P}$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \vdots & \ddots & \\ \alpha_{zx} & & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$\boxed{\alpha_{ij} \text{ is polarizability tensor}}$

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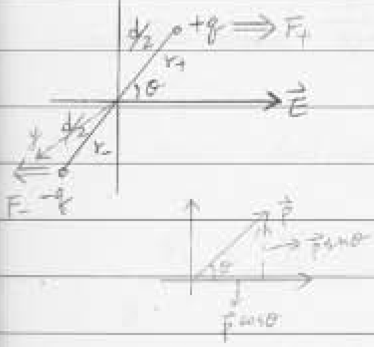
§4.1.3 Alignment of polar molecules

/ Electric dipole in an external Electric field.



① The force on the positive

$$\vec{F}_+ = q\vec{E}$$

negative

$$\vec{F}_- = -q\vec{E}$$

② Torque $\Rightarrow \vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$

$$= \frac{d}{2} \times q\vec{E} + \frac{d}{2}(-q\vec{E})$$

$$= q(\vec{d} \times \vec{E})$$

$$= \vec{p} \times \vec{E} \quad (\text{Eq } 4.4)$$

Math. Method for $\vec{\tau} = \vec{p} \times \vec{E}$

微分. 以 dT $\equiv \Delta\vec{E}$, ✱ $dT = \left(\frac{dT}{dx}\hat{x} + \frac{dT}{dy}\hat{y} + \frac{dT}{dz}\hat{z}\right) \cdot (dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z})$

$$\vec{F}_x = \vec{F}_+ + \vec{F}_- = q(\vec{E}_+ - \vec{E}_-) = q\Delta\vec{E}$$

应用上式， $\Delta E = \nabla\vec{E} \cdot \vec{d} = (\vec{d} \cdot \nabla)\vec{E}$

$\Longrightarrow$ Then we can get $\vec{F} = q(\vec{d} \cdot \nabla)\vec{E} = (\vec{p} \cdot \nabla)\vec{E}$

The total force at x-axis

$$\boxed{\begin{aligned} \vec{F}_x &= (\vec{p} \cdot \nabla)\vec{E}_x \\ \vec{\tau} &= \vec{p} \times \vec{E} = -pE\sin\theta\,\hat{y} \end{aligned}}$$

The energy / work

$$U_E = \int_{\pi/2}^{\phi} -pE\sin\phi\, d\phi$$

$$= pE\cos\phi$$

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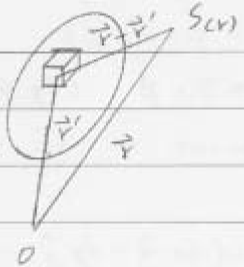
§ 4.2 Bound charge.

We can calculate the potential such a polarized dielectric system, we may ~~know~~ replace the dielectric itself by the volume polarization $\vec{P}$.

- We know potential $V = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{P}_{r'}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3},$$

where $\boxed{\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\nabla_r\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \nabla_{r'}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)}$

P.S. ① ∇_r operates only on $\vec{r}$

∇' operates on the primed coordinates r'

$$V(r) = \frac{1}{4\pi\epsilon_0}\vec{p}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

② $\vec{p} \Rightarrow \vec{P}\cdot V'$ to apply to the system by a volume polarization

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) d^3r'}$$

$$\nabla'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) = \vec{P}(r')\,\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) + \frac{\nabla'\cdot\vec{P}(r')}{|\vec{r}-\vec{r}'|} \quad \text{(Key method)}$$

We can find out the divergency of Polarization?

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$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \nabla'\left[\frac{\vec{P}(r')}{|\vec{r}-\vec{r}'|}\right] d^3r' - \frac{1}{4\pi\epsilon_0}\int \frac{\nabla'\cdot\vec{P}(r')}{|\vec{r}-\vec{r}'|} d^3r'$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}(r')\cdot d\vec{s}'}{|\vec{r}-\vec{r}'|} = (\vec{P}(r)\cdot\hat{n}\,da) \quad - \frac{1}{4\pi\epsilon_0}\int \frac{\nabla'\cdot\vec{P}(r')}{|\vec{r}-\vec{r}'|} d^3r'$$

We can define two

$\underline{\sigma_b = \vec{P}\cdot\hat{n}}$ surface density

&

$\underline{\rho_b = -\vec{\nabla}\cdot\vec{P}}$ volume density

Compare these results with.

(1) $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r)\,dv'}{|\vec{r}-\vec{r}'|}$ charge

(2) $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{p\cdot(r-r')}{|r-r'|^3}$ dipole moment

(3) $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(r)\,da'}{|\vec{r}-\vec{r}'|}$?

Bound charge / dielectric matter

$$V(r) = \underline{\frac{1}{4\pi\epsilon_0}\oint \frac{\sigma_b}{|\vec{r}-\vec{r}'|} da'} + \underline{\frac{1}{4\pi\epsilon_0}\int \frac{\rho_b}{|\vec{r}-\vec{r}'|} dv'}$$

The potential of polarization object is the same as that predured by a volume charge density $\boxed{\rho_b = -\vec{\nabla}\cdot\vec{P}}$

$\boxed{\text{PLUS}}$ a surface charge density $\boxed{\sigma_b = \vec{P}\cdot\hat{n}}$

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1. Surface $\boxed{\text{bound charge}}$ density

$\sigma_b = \vec{P} \cdot \hat{n}$, σ_P 〉 Polarization (極化)

2. Volume $\boxed{\text{bound charge}}$ density

$\rho_b = -\vec{\nabla} \cdot \vec{P}$, ρ_P

3. if Polarization is constant / uniform through out the medium, ρ_b is zero, because of the derivation in $\vec{\nabla} \cdot \vec{P}$ vanishes.

4. Total Polarization charge is always <u>Zero</u>.

$$\boxed{\begin{aligned} Q_b &= \int \sigma_b \cdot d\vec{s} + \int \rho_b \cdot dV \\ &= \underbrace{\int \vec{P} \cdot \hat{n}\, da}_{\Downarrow \text{ Divergency theorem}} + \int -\nabla \cdot P \, dv \\ &= \underline{\underline{\int \nabla \cdot \vec{P}\, dV}} + \int -\nabla \cdot P \, dV = \underline{\underline{0}} \end{aligned}}$$

Ex: A sphere of radius R carries a <u>polarization</u> $P(r) = kr$, where k is a constant, r is vector from center.

(A.) σ_b & ρ_b.

(B.) E-field ($r > R$, and $r < R$).

(C.) Total bound charges.

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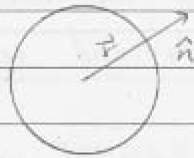
$* \quad \sigma_b = \vec{P} \cdot \hat{n} \quad \boxed{\text{at } r = R}$

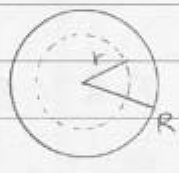


$= k\vec{r} \cdot \hat{n} = kR$ (surface)

$\rho_b = -\underline{\nabla} \cdot \underline{P} = -\left(\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \hat{r}\right) \cdot \vec{P}\,\hat{r}$

$= -\frac{1}{r^2}\frac{\partial}{\partial r} k r^3 = \frac{-3r^2 k}{r^2} = \underline{-3k}.$

(b) For $r < R$ inside the sphere



$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0} \cdot \frac{4}{3}\pi r^3 \rho$

$\Rightarrow \vec{E} = \frac{\rho}{3\epsilon_0}\hat{r}$, ρ_b for polarization.

$= \frac{-3k}{3\epsilon_0}\hat{r} = \frac{-k}{\epsilon_0}\hat{r}$

(c) $r > R$,

$\Rightarrow Q = 0 , E = 0$

Prove $E = 0$ for $r > R$

Total $Q = \sigma_b A + \rho_b \cdot V$

$= kR \cdot 4\pi R^2 + (-3k) \cdot \frac{4}{3}\pi R^3$

$= 0$

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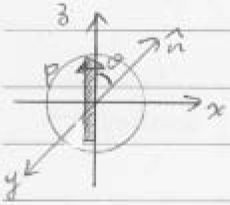
Ex 4.2, Find the electric field produced a $\boxed{\text{uniform}}$ $\boxed{\text{polarized sphere}}$ of radius R.



(1) IF uniform

$\rho_b = 0$, $\vec{\nabla} \cdot \vec{P} = 0$

There is no volume charge density.

(2) Surface charge density

$$\sigma_b = \vec{P} \cdot \hat{n} = P(\hat{z} \cdot \hat{n}) = P \cdot \cos\theta$$

From Ex 3.9

$$\sigma(\theta) = k \cos\theta$$

Potential $V(r,\theta) = \frac{k}{3\epsilon_0} r \cdot \cos\theta$, $r \le R$

$$V(r,\theta) = \frac{kR^3}{3\epsilon_0} \frac{1}{r^2} \cos\theta, \ (r \ge R)$$

(I) $\vec{E} = -\nabla V$

$r \le R$, $\vec{E} = -\left[\frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{1}{r\sin\theta} \frac{\partial}{\partial \phi} \right] V$

$$= \frac{-P}{3\epsilon_0} \cos\theta \, \underline{\hat{r}} + \frac{P}{3\epsilon_0 r} r\sin\theta \, \underline{\hat{\theta}}$$

* axis Transformation

$\hat{r} = \sin\theta\cos\phi \, \hat{x} + \sin\theta\sin\phi \, \hat{y} + \cos\theta \, \hat{z}$

$\hat{\theta} = \cos\theta\cos\phi \, \hat{x} + \cos\theta\sin\phi \, \hat{y} - \sin\theta \, \hat{z}$

$\hat{\phi} = -\sin\phi \, \hat{x} + \cos\phi \, \hat{y}$

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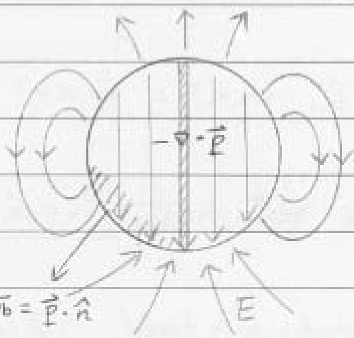
P.170・171 可去掉

$$\vec{E} = \frac{-P}{3\epsilon_0}\left[\cos^2\theta + \sin^2\theta\right]\hat{r} = \boxed{\frac{-P}{3\epsilon_0}\hat{r}}$$

$r > R$, $\quad V = \frac{P R^3}{3\epsilon_0 r}\cos\theta \qquad \vec{p} = \vec{P} V$, $\quad -\nabla V = E$

$$= \frac{p/V \cdot R^3}{3\epsilon_0 r^2}\cos\theta = \frac{\vec{p}\cdot\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{p}\cdot\hat{n}}{4\pi\epsilon_0 r^2} \quad (r \geq R)$$

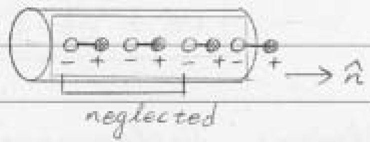
$$\underline{\vec{E} = \frac{p}{4\pi\epsilon_0}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$



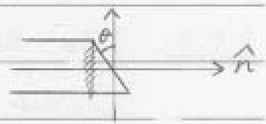
§ 1. uniform polarization

A. $\rho_b = 0$, $\underline{\sigma_b \neq 0}$, B. $\underline{Q = 0}$

$\sigma_b = \vec{P}\cdot\hat{n}$



if



2. Nonuniform polarization, if $P(r) = kr$.

A. $\rho_b \neq 0$, $\sigma_b \neq 0$, $\nabla\cdot\vec{P} \neq 0$

$Q = 0$.

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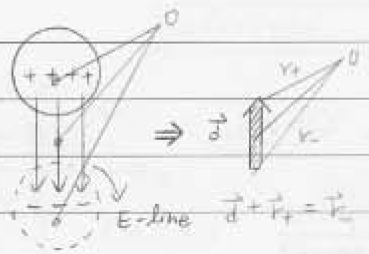
21th

bound charge $\leftrightarrow$ ion

19th (free electron , bound charge)

P.172 , Ex 4.3

A positive sphere & negative sphere without polarization , Slightly upward $\hat{z}$ - axis.



Gauss's law $\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$

(1) $\vec{E}_+ \cdot 4\pi r_+^2 = \frac{4}{3}\pi r_+^3 \cdot {}^{+\rho}/_{\epsilon_0} \Rightarrow \vec{E}_+ = \frac{\rho}{3\epsilon_0}\vec{r}_+$

$\Rightarrow$ (2) $\vec{E}_- \cdot 4\pi r_-^2 = \frac{4}{3}\pi r_-^3 \cdot ({}^{\ominus\rho}/_{\epsilon_0}) \Rightarrow \vec{E}_- = \frac{-\rho}{3\epsilon_0}\vec{r}_-$

(3) $\vec{E}_+ + \vec{E}_- = \frac{\rho}{3\epsilon_0}\underline{[\vec{r}_+ - \vec{r}_-]} = \frac{\rho}{3\epsilon_0}\underline{[-\vec{d}]} = \frac{-\rho d}{3\epsilon_0}$

$\therefore$ Total E - field $\vec{E}(r) = \frac{-\rho d}{3\epsilon_0} = \frac{-\vec{P}}{3\epsilon_0}$

P.S. $\vec{p} = V\vec{P}$, $q d = \vec{p}$, $\rho d = \vec{P}$

§ 4.3 The Electric displacement $\vec{D}$

Consider the free electron & bound charge.

Total charge containts of $\boxed{\text{electrons on a conductor}}_{\rho_f}$ + $\boxed{\text{ions embedded in the dielectric}}_{\rho_b}$ 離子

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Within the electric medium, Total charge density $\rho = \rho_f + \rho_b$

$\Rightarrow$ $\boxed{\nabla\cdot\vec{E} = \rho/\epsilon_0} = (\rho_f+\rho_b)/\epsilon_0$

$\Rightarrow$ Then $\epsilon_0\nabla\cdot\vec{E} = \rho_f + \rho_b = \rho_f + (-\nabla\cdot\underline{P})$

$\Rightarrow$ $\nabla\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_f$

$\underline{\vec{D} = \epsilon_0\vec{E} + \vec{P}}$, $\underline{\nabla\cdot\vec{D} = \rho_f}$

(1) $\vec{E}$ is a total electric field, not a part of polarization

(2) $\vec{D}$ is the electric displacement of free electrons in dielectric medium.

(3) Total free charge $\boxed{\oint\vec{D}\cdot d\vec{s} = Q_f}$

$$\oint\vec{D}\cdot\hat{n}\,da = \int\rho_f\cdot d^3r$$

Conclusion : "Maxwell introduced $\vec{D}$ & called it the electric displacement"

$\boxed{\text{IF In isotropic system}}$:

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon_0\vec{E} + \underbrace{\epsilon_0\chi\vec{E}}_{\vec{P}} = \epsilon_0(1+\chi)\vec{E}$$

$\epsilon = \epsilon_0(1+\chi) \Rightarrow \vec{D} = \epsilon\vec{E}$

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(4) ϵ is permitivity of the dielectric materials.

(5) In an anisotropic dielectric ϵ is tensor ϵ_{ij} of electric field.

$$\epsilon = \epsilon(\vec{E})$$

$\Longrightarrow$ (6) We introduce a dimensionless quantity K to charactrize the electric behavior of materials.

$$K = \frac{\epsilon}{\epsilon_0} = 1 + \chi \quad \text{(dielectric constant)}$$

介电常数

Then $\vec{D}$ can be written down as $\vec{D} = \epsilon_0 K \vec{E}$.

Let consider the electric field in $\boxed{\text{an isotropic dielectric}}$ surrounding a spherical charge Q in which the charge density is a function of distance from the center only.

$$\oint \vec{D} \cdot d\vec{s} = \oint \epsilon_0 K \vec{E} \cdot d\vec{s} = \Sigma q_i$$

$$\Rightarrow \int \vec{E} \cdot d\vec{s} = \frac{\Sigma q_i}{\epsilon_0 K} \qquad \left(\int \vec{E} \cdot d\vec{s} = \Sigma q_i / \epsilon_0 \right)$$

The differs from Gauss's Law for charge in empty space only in appearance of the factor K

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\left(\frac{1}{K}\right)$$

Electric field is reduced in the ratio of $\frac{1}{K}$ by the dielecetric. If we check the polorization P can be written

$$\vec{P} = \epsilon_0(K-1)\vec{E} = \epsilon_0 \frac{K-1}{K} \cdot \frac{Q}{4\pi\epsilon_0 r^2} = \boxed{\frac{K-1}{K}\frac{Q}{4\pi r^2}}$$

Ex: If a sphere of radius a with charge Q, polarization per unit area of the cavity in dielectric in which Q.

$$\sigma_P = \sigma_b = -\vec{P}\cdot\hat{n} = -\frac{K-1}{K}\frac{Q}{4\pi a^2}$$

$$Q_b = \sigma_b \cdot 4\pi a^2 = \boxed{-\frac{K-1}{K}Q}$$ 修正 Q (P.180)

* The effective charge produced the field is the sum of Q and Q_b.

Total $Q + Q_b = Q - \frac{K-1}{K}Q = \frac{Q}{K}$

consisted with E-field induced by $\frac{1}{K}$

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1/9 期末考, ch 3~4

$*$ Linear Dielectric, $D = \varepsilon E$

(1) $\oint \vec{D} \cdot d\vec{s} = Q$

(2) $\nabla \times \vec{E} = 0 \xrightarrow[\text{2nd law}]{} \boxed{\vec{E} = -\nabla V} \xrightarrow[\text{1st law}]{} \nabla \cdot \vec{E} = \rho/\varepsilon_0$, $-\nabla^2 V = \underset{=0}{\rho/\varepsilon_0}$

scalar

对 $\vec{D}$ 取 curl, $\nabla \times \vec{D} = ? = \nabla \times \vec{P}$

$= \nabla \times (\varepsilon_0 \vec{E} + \vec{P})$

$= \varepsilon_0 \nabla \times \vec{E} + \nabla \times \vec{P}$

$= 0 + \nabla \times \vec{P} = ?$

$\Rightarrow$ 無法找到一个純量的位能 V_D, 來表示

$\underbrace{\boxed{\vec{D} \neq -\nabla V_D}}_{\text{不存在}}$, 所以 $\nabla \times D \neq 0$, $\nabla \times P \neq 0$.

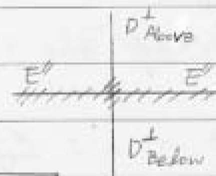
(3) 無 $\vec{D} = -\nabla V_D$ 存在

(4) $D^{\perp}_{Above} - D^{\perp}_{below} = \sigma_f$

(5) $D^{\parallel}_{Above} - D^{\parallel}_{below} = P^{\parallel}_{Above} - P^{\parallel}_{below}$

$\Downarrow$

$\nabla \cdot D = \rho_f$, $\nabla \times D \neq 0$



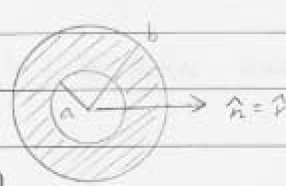
Problem 4.15:

A thick spherical shell inner radius a, outter 〃 b] is made of dielectric materials with a frozen in polarization $P = \frac{k}{r} \hat{r}$.

(A). Find σ_b, ρ_b.

(B). Find $\vec{D}$, $\vec{E}$.

$(r > b,\ r < a,\ a < r < b)$.



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$\boxed{\text{Hw } ※10. \text{ if } P = \frac{k}{r^2}\hat{r}}$

Ans: (1) $\rho_b = -\nabla \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r} r^2 P = -\frac{1}{r^2}\frac{\partial}{\partial r} kr\hat{r} = -\frac{k}{r^2}$

(2) $\sigma_b = \vec{P} \cdot \hat{n} = \frac{k}{r}\hat{r} \cdot \hat{n} = \frac{k}{r}$

(3) Total $Q = 0$, $Q = \int \sigma_b \cdot da + \int \rho_b \cdot dV = 0$

(4) Q at $r > b = 0$, $E = 0$, $D = 0$

(5) Q at $r < a = 0$, $E = 0$, $D = 0$

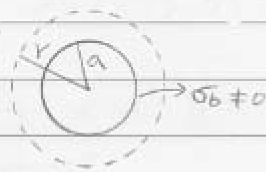
(6) at $a < r < b$, $D = 0 \Rightarrow \int D \cdot ds = Q = 0$

(7) $\boxed{D = \epsilon_0 \vec{E} + \vec{P} = 0}$

then $\vec{E} = -\frac{\vec{P}}{\epsilon_0}$ at $a < r < b$

$= \boxed{-\frac{k}{r\epsilon_0}\hat{r}}$

(8) Prove Q at $a < r < b$



$Q = \int \sigma_b \cdot da + \int_a^r \rho_b \cdot dV$

$= \frac{-k}{a} 4\pi a^2 + \int_a^r \frac{-k}{r^2} 4\pi r^2 dr = -4\pi kr$

(9) if apply Gauss's law, $\int E \cdot da = Q/\epsilon_0 = \frac{-4\pi kr}{\epsilon_0}$

$\Rightarrow E \cdot 4\pi r^2 = \frac{-4\pi kr}{\epsilon_0}$, $\boxed{E = \frac{-k}{r\epsilon_0}\hat{r}}$

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§4.4.3 Energy in Dielectric System

The expression obtained in Chapter 2 for the energy of the electric field needs to be modified when dielectric are present.

$$W = \frac{1}{2}\int \rho V \, d^3r' + \frac{1}{2}\int \sigma V \, ds$$

However, we must substitute $\nabla \cdot \vec{D} = \rho$

$$\Rightarrow \frac{1}{2}\int \rho V \, d^3r' = \frac{1}{2}\int (\nabla \cdot \vec{D}) V \, d^3r'$$

$$= \frac{1}{2}\int \nabla \cdot (V D) - \vec{D} \cdot (\nabla V) \, d^3r'$$

With the help of the divergence theorem, we can transfer the first integral.

$$\frac{1}{2}\int \nabla \cdot (VD) \, d^3r' = \frac{1}{2}\int V D \cdot d\vec{s}$$

$\nabla V = -\vec{E}$ 代入

$$\Rightarrow \frac{1}{2}\int \nabla \cdot (VD) - \vec{D} \cdot (\nabla V) \, d^3r'$$

$$= \frac{1}{2}\int V \vec{D} \cdot d\vec{s} + \frac{1}{2}\int \vec{D} \cdot (\vec{E}) \, d^3r'$$

代回 W 表示式

7.5mm × 28行

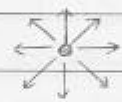
$$W = \frac{1}{2}\int \sigma V\, ds + \frac{1}{2}\int V \vec{D}\cdot d\vec{s} + \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d^3r'$$

$$\because \frac{1}{2}\int V D\cdot d\vec{s} = \frac{1}{2}\int V\cdot \vec{D}_{\hat{n}}\cdot dS_{\hat{n}} = \boxed{\frac{-1}{2}\int \sigma V ds}$$

$$\therefore\ \underline{W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d^3r'}$$

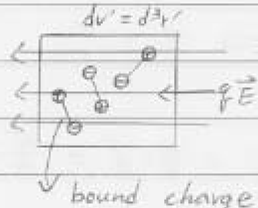
We may regard the electrostatic energy as being distributed are the field with density being gives by $\boxed{\frac{1}{2}\vec{D}\cdot\vec{E}}$

2008. 1/2 (三)

Work : $W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d^3r' \neq \frac{1}{2}\int \vec{E}\cdot\vec{E}\, d^3r$ 

$\Rightarrow$ replaced W as $qW = \frac{1}{2}\int \vec{D}\cdot \underbrace{q\vec{E}}_{\vec{F}}\, d^3r'$

$\vec{F}$: external field $q\vec{E}$

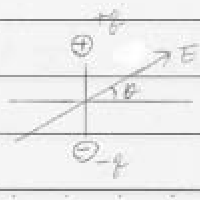


(4.4.3)

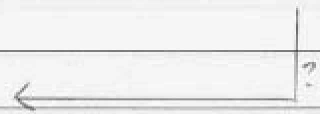
(P.165)

In dielecetric materials

One dipole : Force



$$\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$$



Force = ?

(4.1.3)

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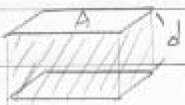
We may regard the electrostatic energy as being distributed over the field with density by $\boxed{\frac{1}{2}\vec{D}\cdot\vec{E}}$.

For isotropic dielectric, we write this as

$\vec{D} = \varepsilon\vec{E}$, $\vec{E} = \frac{\vec{D}}{\varepsilon}$

$\varepsilon\vec{E}/2$ or ? $D^2/2\epsilon$

* A parallel-plate capacity is filled with insulating materials of dielectric constant K, what effect does this has on its capacitor.



$\longrightarrow V = E\cdot d$

A. Capacitance : $c = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$

B. Gauss's law $\int D\cdot ds = Q_f \Rightarrow D\cdot A = \sigma A$

$\Rightarrow \boxed{D = \sigma}$

C. $D = \epsilon\vec{E} \Rightarrow \vec{E} = \sigma/\epsilon$

D. Potential $V = \vec{E}\cdot\vec{d} = \frac{\sigma}{\epsilon}d = \frac{Q}{A}\cdot\frac{d}{\epsilon}$

E. $\boxed{c = \frac{Q}{V} = \epsilon\frac{A}{d} = \epsilon_0 K \frac{A}{d}}$

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* The total work as

$$\frac{1}{2}CV^2 = \frac{1}{2}\epsilon_0 K \frac{A}{d} V^2$$

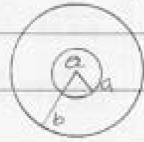
* Diving this by the volume of the capacitor $Ad \equiv$ volume.
We obtain the energy density

$$\frac{W}{\text{volume}} = \frac{\frac{1}{2}\epsilon_0 K Ad\left(\frac{V}{d}\right)^2}{Ad} = \frac{1}{2}\epsilon_0 K\left(\frac{V}{d}\right)^2 = \frac{1}{2}\epsilon_0 K E^2$$

$$= \frac{1}{2} D \cdot E$$

Ex 4.5 + Problem 4.26

A metal sphere of radius a carries a charge Q.
It's surrounded by linear dielectric material of
permitivity ϵ. Find the
A. potential at center.
B. bound charge density ρ_b. σ_b.
C. energy?

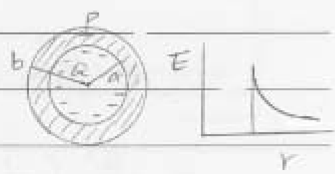


PS: "C" = ?

Ans: A. $r>b$, $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$.

$r<a$, $\vec{E} = 0$

$a<r<b$, $\vec{E} = \frac{Q}{4\pi\epsilon r^2}\hat{r}$



$r<a \Rightarrow D = \epsilon E = \frac{Q}{4\pi r^2}\hat{r}$

Potential $V = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\left[\int_{\infty}^{b} + \int_{b}^{a} + \int_{a}^{0}\right]\vec{E}\cdot d\vec{l}$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon_0 a} - \frac{1}{\epsilon b}\right]$$

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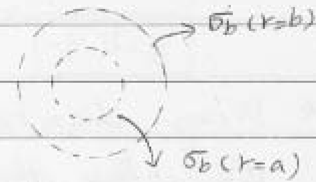
P.197 ~ (不考) (研究所)

Polarization at $a<r<b$.

$$\underline{P} = \epsilon_0 \chi E = \frac{\epsilon_0 \chi}{4\pi\epsilon} \frac{Q}{r^2} \hat{r}$$



B. $$\rho_b = -\nabla \cdot \underline{P} = -\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \cdot \frac{\epsilon_0 \chi Q}{4\pi\epsilon r^2} = 0$$

$$\sigma_b = \vec{P} \cdot \hat{n} = \frac{\epsilon_0 \chi Q}{4\pi\epsilon r^2} (\hat{r} \cdot \hat{n}) \quad \begin{matrix} r=b \\ r=a \end{matrix} \Longrightarrow \begin{matrix} \dfrac{\epsilon_0 \chi Q}{4\pi\epsilon b^2} \\ \dfrac{-\epsilon_0 \chi Q}{4\pi\epsilon a^2} \end{matrix}$$

C. Work: $\boxed{W = \frac{1}{2}\int \vec{D} \cdot \vec{E}\, d^3 r'}$

$$D = \begin{cases} 0 & , r<a \\ \dfrac{Q}{4\pi r^2}\hat{r} & , a<r<b \quad D=\epsilon E \\ \dfrac{Q}{4\pi r^2}\hat{r} & , r>b \quad D=\epsilon_0 E \end{cases} \qquad \vec{E} = \begin{cases} 0 \\ \dfrac{Q}{4\pi\epsilon r^2}\hat{r} \\ \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{r} \end{cases}$$

Energy $\frac{1}{2}\int \vec{E} \cdot \vec{D}\, d^3 r' = \frac{1}{2}\left[\int_0^a + \int_a^b + \int_b^\infty\right] \vec{E} \cdot \vec{D}\, d^3 r$

$$= \frac{1}{2}\frac{Q^2}{(4\pi)^2}\left[\frac{1}{\epsilon}\int_a^b \frac{1}{r^2} \cdot \frac{1}{r^2} 4\pi r^2 dr + \frac{1}{\epsilon_0}\int_b^\infty \frac{1}{r^2} \cdot \frac{1}{r^2} 4\pi r^2 dr\right]$$

$$= \frac{Q^2}{8\pi}\left[\frac{1}{\epsilon}\left(\frac{-1}{r}\right)\Big|_a^b + \frac{1}{\epsilon_0}\left(-\frac{1}{r}\right)\Big|_b^\infty\right]$$

$$= \frac{Q^2}{8\pi\epsilon_0}\left[\frac{1}{1+\chi}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{b}\right]$$

$$= \frac{Q^2}{8\pi(1+\chi)}\left(\frac{1}{a} + \frac{\chi}{b}\right).$$

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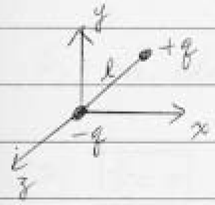
4.4.4 : A dielectric materials place in an electric field is subject to forces that aries from the interaction of electric field and dipoles in the dielectric.

$$\boxed{4.1.3 \quad \vec{F} = (\vec{p}\cdot\nabla)\vec{E}}$$



If electric field $\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$

dielectric

$$\boxed{F = (\vec{P}\cdot\nabla)\vec{E}}$$

$$F_x = (-q)E_x + (+q)\underbrace{(E_x + \vec{l}\,\nabla E_x)}_{4.1.3 \text{ prove}}$$

$$= q(x\hat{x} + y\hat{y} + z\hat{z})\cdot\left(\frac{\partial}{\partial x}E_x\hat{x} + \frac{\partial E_y}{\partial y}\hat{y} + \frac{\partial E_z}{\partial z}\hat{z}\right)$$

$$= \vec{P}\cdot\nabla E_x \quad , \quad F_y = \vec{P}\cdot\nabla E_y$$

$$F_z = \vec{P}\cdot\nabla E_z$$

The Force per unit valume of the dielectric materials.

$$\vec{F} = (\vec{p}\cdot\nabla)\vec{E} = ? \Rightarrow \text{The Force of the dipole moment}$$

$$\vec{F} = (\vec{P}\cdot\nabla)\vec{E} = ?$$

$$P = N\vec{p} \rightarrow \vec{F} = N(\vec{p}\cdot\nabla)\vec{E} = (\vec{P}\cdot\nabla)\vec{E}$$

$$\vec{P} + \epsilon_0\vec{E} = \vec{D} \Rightarrow \vec{P} = \vec{D} - \epsilon_0\vec{E} = \epsilon_0\chi\vec{E}$$

then $\boxed{\vec{F} = \epsilon_0\chi(\vec{E}\cdot\nabla)\vec{E}}$. This can be further simplified, Let us first examine.

NO

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$$F_x = \epsilon_0 \chi \left(E_x \frac{\partial E_x}{\partial x} + E_y \frac{\partial E_x}{\partial y} + E_z \frac{\partial E_x}{\partial z} \right)$$

$$= \epsilon_0 \chi \left(E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \right) \cdot \nabla_x \vec{E}$$

$$= \epsilon_0 \chi \vec{E} \cdot \nabla \vec{E} = \frac{1}{2} \epsilon_0 \chi \nabla E^2 = \frac{1}{2} \epsilon_0 \frac{\partial}{\partial x} E^2$$

$$F_y = \frac{1}{2} \epsilon_0 \frac{\partial}{\partial y} E^2 , \quad F_z = \frac{1}{2} \epsilon_0 \frac{\partial}{\partial z} E^2$$

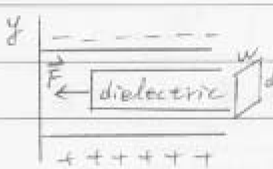
Finally we have

$$F = \frac{1}{2} \epsilon_0 \chi \nabla E^2 , \quad \text{if } K = \frac{\epsilon}{\epsilon_0} = 1 + \chi$$

$$= \frac{1}{2} \epsilon_0 (K - 1) \nabla E^2$$

The force is proportional to the gradient of E^2, therefore pulls the dielectric in the direction of incressing field strength. (E^2)

Ex: A slab of linear dielectric materials is portially inserted between the plates.



$$\vec{F} = \frac{1}{2} \epsilon_0 \chi \nabla E^2 \ / \text{per volume}$$

$$\vec{F}_{total} = \frac{1}{2} \epsilon_0 \chi \int_L^0 \nabla E^2 \, d^3x$$

$$= \frac{1}{2} \epsilon_0 \chi \int \frac{\partial}{\partial x} E^2 \, dx \, dy \, dz$$

$$= \frac{1}{2}\epsilon_0 \chi \,(\underset{\downarrow\, 0}{E_0^2} - \underset{\downarrow\, \frac{V}{d}}{E_L^2})\, W\cdot d \quad = \frac{-1}{2}\epsilon_0 \chi \left(\frac{V}{d}\right)^2 wd$$

Ex: Plasma Oscillations (电浆)

1. an electric field applied for short time.

2. $\boxed{\vec{D}=0}$, $\underline{\epsilon_0 \vec{E} + \vec{P} = 0}$, $\underline{Q = 0 + \int \rho_b \, dv + \int \sigma_b \, da}$

3. $\vec{P} = -ne\vec{l} = -ne\vec{x}$, $n = \frac{N}{V}$

$$\epsilon_0 \vec{E} + \vec{P} = 0 \Rightarrow -qE_x = m\ddot{x} = m_e \frac{d^2x}{dt^2}$$

$$\therefore E_x = \frac{-\vec{P}}{\epsilon_0} \Rightarrow m_e \frac{d^2x}{dt^2} = -q \cdot \frac{\vec{P}}{\epsilon_0} = \frac{-nq^2\vec{x}}{\epsilon_0}$$

$$\Rightarrow m_e \frac{d^2x}{dt^2} + \frac{nq^2}{\epsilon_0} x = 0$$

$\boxed{680\ nm}$

$\omega = \sqrt{\frac{nq^2}{\epsilon_0 m_e}}$, $300 \sim 400\ nm$

End