11/14 (三) 10~12:00 期中考
chapter 1-1 ~ 1.4
chapter 2

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## *3.1 Laplace's Eq. For chargeless, For spatial solution

1. From the continuous problem of spatial solution $V(x,y,z)$
   the superposition has no such problem.

2. From the continuous equation of $\vec{E}$ electric field and potential.

$$\vec{E}_{(\vec{r}-\vec{r}')} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv'$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|}dv' \Rightarrow \text{Integrated Form.}$$

Apply For $\nabla\cdot\vec{E} = \rho/\epsilon_0$ & $\nabla\times\vec{E} = 0 \Rightarrow \boxed{\nabla^2 V = -\rho/\epsilon_0}$

From $\nabla^2 V = -\rho/\epsilon_0$ to solve the spatial Form of

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|}dv' \quad \boxed{\text{Impossible}}$$

3. Then we consider chargeless, the poisson equation will reduce to Laplace's Equation.

$$\Rightarrow \nabla^2 V = 0 \quad \text{(Laplace's Eq)}$$

Example: One-dimension Laplace's eq. $\nabla_x^2 V_{(x)} = 0$
We propose the $V$ depends on variable $x$

$$\frac{d^2V_{(x)}}{dx^2} = 0, \quad \boxed{V_{(x)} = mx + b} \longrightarrow$$

* In a region where $\rho = 0$.

eq. $\nabla^2 V = 0$ , $\nabla \cdot (\nabla V) = 0$.

$\nabla^2$ is a scaler operator , different forms in different coordinate systems.

A. Rectangular coordinate $(x, y, z)$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

B. Spherical polar coordinate $(r, \theta, \phi)$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}.$$

C. Cylindrical polar coordinate $(r, \theta, z)$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}.$$

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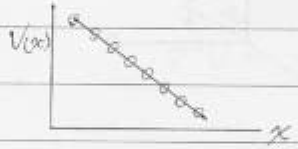
1 - Dimensional



continuous charge distribution

$\downarrow$ solution

* Average potential = Superposition

$$V_{AVE\,1D} = \frac{1}{N}\sum V_i \cdot V_{AVE\,2D} = \frac{1}{Area}\sum V \cdot V_{AVE\,3D}$$

$$= \frac{1}{Volume}\sum V.$$

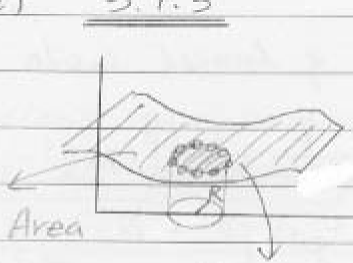
* potential has no local minimum or maximum.

$$\boxed{V_{n,m} = \sum_{n,m} X_n \cdot Y_m}$$

2 - Dimensional Case $V(x,y)$ 3.1.3

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} = 0$$

V for 2D, Average for a Area



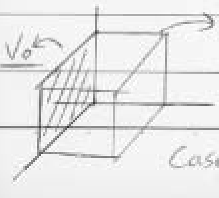
$V(x,y)$ solution

$$V(x,y) = \frac{1}{2\pi R}\int V\,dl$$

3.1.4 Laplace's eq for 3D

$\nabla^2 V = -\rho/\varepsilon_0$, if we find out a system without q?

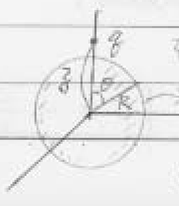
(Poisson) if a charge located at z.



$V_0$ $V(x,y,z) = ?$

at $y=0$ $V=V_0$ $\Longleftrightarrow$

Case II 相似



$\frac{q}{4\pi\varepsilon_0 r} = \underline{V_0(r=z)} = \frac{q}{4\pi\varepsilon_0 z}$

$\nabla^2 V = 0$

Case I

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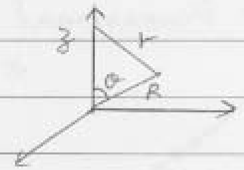
Case I.




$$V_{AVE} = \frac{1}{4\pi R^2}\int V\,dA$$

$$= \frac{1}{4\pi R^2}\int \frac{q \cdot R^2 \sin\theta\, d\theta\, d\phi}{4\pi\epsilon_0 \sqrt{z^2+R^2-2zR\cos\theta}}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{2zR}\sqrt{z^2+R^2-2zR\cos\theta}\,\Big|_0^{\pi}$$

Ex: 3.1.4 + Problem 3.1

if $z > R$, $\quad V_{AVE} = \frac{q}{4\pi\epsilon_0}\frac{1}{2zR}\left[(z+R)-(z-R)\right]$

PS. (if $z \to 0$, $V=0$) $\quad = \frac{q}{4\pi\epsilon_0 z}$, $\quad \nabla^2 V = 0$

if $z < R$, $q$ located inside the sphere

$$V_{AVE} = \frac{q}{4\pi\epsilon_0}\frac{1}{2zR}\left[(z+R)-(R-z)\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{R}, \quad \nabla^2 V = -\rho/\epsilon_0 \quad \text{(poisson)}$$

Problem 3.3, Laplace's eq (V depends on r)

* In spherical coordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dV}{dr}\right) = 0$$

solution: $\quad r^2\frac{dV}{dr} = c$ (constant)

$$\frac{dV}{dr} = \frac{c}{r^2}, \quad \boxed{V = -\frac{c}{r} + k}$$

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Ch3 物理觀念 1. Boundary Condition.
2. 唯一性定理

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* In cylindrical case

$$\nabla^2 V = \frac{1}{s}\frac{d}{ds}\left(s\frac{dV}{ds}\right) = 0$$

$$s\frac{dV}{ds} = c \Rightarrow \frac{dV}{ds} = \frac{c}{s}$$

$$\boxed{V = c \cdot \ln s + k}$$

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## 3.1.5 Boundary Condition & Uniqueness Theorem.

1D, 2D, 3D's solution of Laplacian eq. have two basic properties.

$\nabla^2 V = 0$

(A) The principle of Laplacian's superposition
if $V_1, V_2, \ldots, V_n$ are all solutions of Laplacian eq.
then total solution $\boxed{V = C_1 V_1 + C_2 V_2 + \ldots + C_n V_n}$

$C_n = ?$

where $c$ is arbitary constant & is also a solution because Laplacian eq is linear.

$C_n$ is dependent on Boundary Condition.

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## (B) Uniqueness Theorem

☆☆☆

Two solution of Laplacian eq. that satisfy the same boundary conditions differ at most by an additive constant. $C_n$ , $V = C_1 V_1 + C_2 V_2$

Prove:

consider the closed region $V_0$ ,

cubic, sphere, spherical shell

Suppose the solution is not unique & there are two solution $\Phi_1$ & $\Phi_2$ of Laplacian's eq. in $V_0$ with the same boundary condition.

$$V = \Phi_1 + \Phi_2 \quad (\text{the same B.C.})$$

* Dirichlet Condition.

if boundary condition may be specified by assiging, either $\Phi$ or the normal derivative $\frac{\partial \Phi}{\partial n}$ on the surface.

$\boxed{1.}$ if $\phi = \Phi_1 - \Phi_2$

because $\Phi_1$, and $\Phi_2$ on both solutions,

then

$$\nabla^2 \Phi_1 = \nabla^2 \Phi_2 = 0 \leftarrow \boxed{\text{Laplacian 特性}}$$

and so

$$\nabla^2 \phi = \nabla^2 (\Phi_1 - \Phi_2) = 0$$

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Further more, either $\phi$ or $\frac{\partial \phi}{\partial n}$ vanishes on the boundaries

$$\phi = \Phi_1 - \Phi_2$$

then,

$$\frac{\partial \phi}{\partial n} = \frac{\partial \Phi_1}{\partial n} - \frac{\partial \Phi_2}{\partial n} = 0 .$$

$$\nabla_n \phi = 0$$

We apply the divergency theorem

$$\int \underbrace{\nabla \cdot (\phi \nabla \phi)}_{\Downarrow} dv' = \int \phi \underbrace{\nabla^2 \phi}_{\Downarrow\, 0} + \underbrace{(\nabla \phi)^2}_{\Downarrow\, 0 \text{ at } \nabla_n} dv'$$

$$\int \nabla \cdot \vec{A} \, dv' = ?$$

$$\Rightarrow \int \nabla \cdot \vec{A} \, dv' = \int \vec{A} \cdot d\vec{s} = \int \vec{A} \cdot \hat{n} \, da = 0$$

solution: $\phi = \Phi_1 - \Phi_2 = C$ , $\boxed{C \text{ is constant}}$

A. if $\phi = 0$, at boundary $C = 0$ on the surface, then it is zero through the region & $\Phi_1 = \Phi_2$ is continuous on the surface.

B. Neumann Condition.

$\frac{\partial \Phi}{\partial n} = 0$ , $\Phi_1 - \Phi_2 = \phi = c$ , Because the contant in arbitary, we take it to be Zero.

$$V = V_1 C_1 + V_2 C_2 + \cdots + V_n C_n + \boxed{C}$$

take to Zero.

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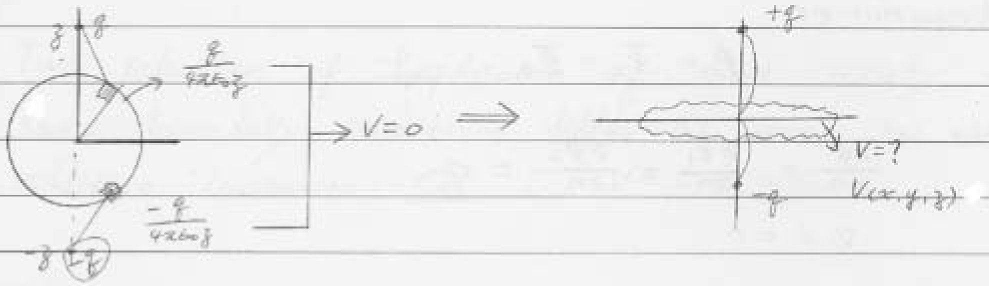
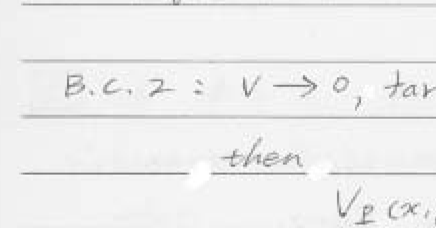
* Fig 3.3



### 3.2.1 Image Method.

Q: A point charge $q$ is located a distance $d$ above an infinite ground conducting plane.

Ans: What's the potential in the region above the plane?



1. In the region $z>0$, $q$ at $(0,0,d)$

B.C. 1: $V=0$, when $z=0$ (B.C.)

B.C. 2: $V \to 0$, far from the change $x^2+y^2+z^2 >> d^2$,

then

$$V_p(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{-q}{\sqrt{x^2+y^2+(z+d)^2}}\right]$$

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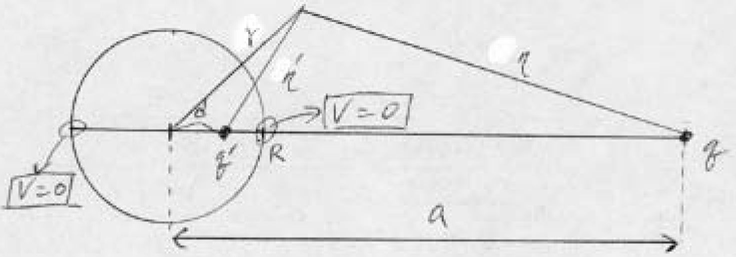
Example 3.2




$$\Phi = \frac{1}{4\pi\epsilon_0}\left[\frac{q'}{|\mathscr{r}'-b|} + \frac{q}{|\mathscr{r}-a|}\right]$$

$V=0 \Rightarrow \begin{cases} \dfrac{q'}{R-b} + \dfrac{q}{a-R} = 0 \\ \dfrac{q'}{R+b} + \dfrac{q}{a+R} = 0 \end{cases}$ (联立)

$\begin{cases} \dfrac{q'}{R-b} = \dfrac{-q}{a-R} \\ \dfrac{q'}{R+b} = \dfrac{-q}{a+R} \end{cases}$ , $\begin{cases} (a-R)\,q' = (R-b)(-q) \\ (a+R)\,q' = (R+b)(-q) \end{cases}$

$2a\,q' = (R-b)(-q) + (R+b)(-q) = -2Rq$ (相加)

$$q' = \frac{-R}{a}q \quad \#$$ (eq 3.15)

$\Rightarrow$

$$(a-R)\,\frac{-R}{a}q = (R-b)(-q)$$

$$= -Rq + bq$$

$$bq = (a-R)\frac{-R}{a}q + Rq = \cancel{-Rq} + \frac{R^2}{a}q + \cancel{Rq} = \frac{R^2}{a}q$$

$$\therefore\ b = \frac{R^2}{a} \quad \#$$ (eq 3.16)

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## 3.2.2 Induced Surface charge.

2. $\sigma = ?$ at $(z=0)$

$$\frac{\partial V}{\partial n} = -\frac{\sigma}{\epsilon_0}\Big|_{\text{at } z=0} \qquad \underline{Eq\,(2.49)}$$

$$\frac{\partial V}{\partial z} = \frac{1}{4\pi\epsilon_0}\left\{\frac{-q(z-d)}{[x^2+y^2+(z-d)^2]^{3/2}} + \frac{q(z+d)}{[x^2+y^2+(z+d)^2]^{3/2}}\right\}$$

So we can get the induced surface charge at $z=0$, $\sigma(x,y,0)$.

$$\sigma(x,y,0) = \frac{-qd}{2\pi(x^2+y^2+d^2)^{3/2}}$$

Induced charge is negative & greatest at $x=y=0$, $Q = \int \sigma\, da$.

$da = dx\,dy$ , $da = r\,dr\,d\phi = 2\pi r\,dr$

define $\underline{r}^2 = x^2+y^2$

at

$$Q = \int \sigma\, da = \int \sigma\, dx\,dy = 2\pi\int_0^\infty \sigma\, r\,dr = 2\pi\int_0^\infty \frac{-qd}{2\pi(r^2+d^2)^{3/2}}\, \underset{\frac{1}{2}dr^2}{r\,dr}$$

$$Q = \frac{qd}{\sqrt{r^2+d^2}}\Big|_0^\infty = 0 - \frac{qd}{\sqrt{0+d^2}} = \underline{-q}$$

prove the Image method is $\underline{\text{OK!}}$

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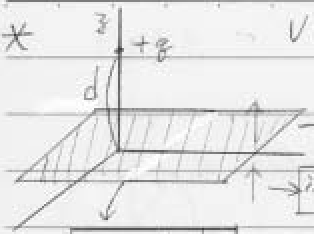
* 

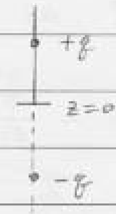

$V(x,y,z)$

$\to z=0,\ V=0$

$\to \boxed{\text{infinite } x^2+y^2 \gg d^2}$

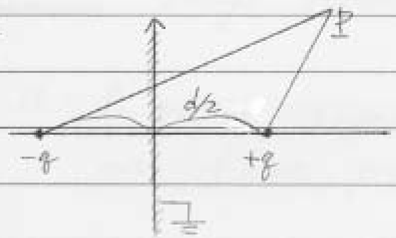
$r^2 \gg d^2$

$\boxed{\text{Ground}}$ $z=0, V=0$

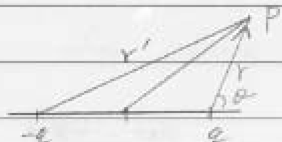



conducting $\to$ electron on surface $\Big] \to$ metal $\to$ charge density $\sigma$ $\to$ $\frac{\sigma}{\epsilon_0} = -\frac{\partial V}{\partial n}$

$n \Rightarrow z \quad = -\frac{\partial V}{\partial z}$

* 

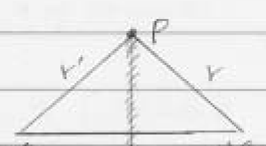

Point charge near an infinite ground conducting plane. The potential at P.




$$V_P = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r}\right) + \frac{-q}{4\pi\epsilon_0 r'}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{r} + \frac{-q}{4\pi\epsilon_0\sqrt{r^2+d^2+2rd\cos\theta}}$$

Notic that $V=0$ when $r=r'$




That's the middle plane AB is equi-potential surface of zero potential.

$\therefore$ Electric-field at P point the component of $-\nabla V$.

$$\checkmark\ E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r^2} - \frac{q(r+d\cos\theta)}{(r^2+d^2+2rd\cos\theta)^{3/2}}\right]$$

$$\checkmark\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\epsilon_0}\left[\frac{qd\sin\theta}{(r^2+d^2+2rd\cos\theta)^{3/2}}\right]$$

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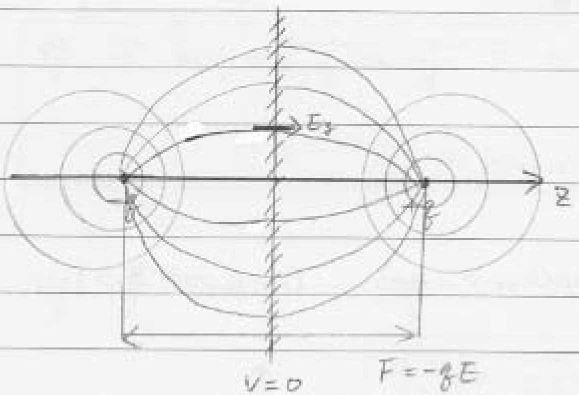
° surface density σ :  problem 3.7, p.126

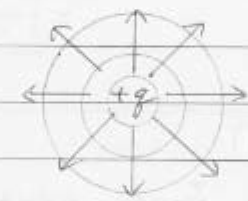
$$\frac{\sigma}{\epsilon_0} = -\frac{\partial V}{\partial n} = \underline{\underline{E}}_z = \left[E_r\cos\theta - E_\theta\sin\theta\right]_{r=r'}$$

$r' = \sqrt{r^2+d^2+2rd\cos\theta}$ , at $\theta = 90^\circ$

$$= \frac{-qd}{4\pi r^3}$$

$$Q = \int \sigma \cdot 2\pi r\,dr = \int \frac{-qd}{2}\left[\frac{2r\,dr}{(d^2+r^2)^{3/2}}\right] = -qd\left[\frac{-1}{\sqrt{r^2+d^2}}\right]_0^{\infty} = \boxed{-q}$$




° Force & Energy :  (3.14 最後一个)

* $\vec{F} = -q\vec{E} = \frac{-q^2}{4\pi\epsilon_0 d^2}\hat{z}$

* with the point charge & no conducting plane


$$W_{NC} = \frac{1}{4\pi\epsilon_0}\frac{1}{2}\frac{q(-q)}{d} = -\frac{q^2}{4\pi\epsilon_0 \cdot 2d}$$

* For a single charge & conducting plane, the energy is just half of the $W_{NC}$,
then the $W_C = \frac{1}{2}\left(-\frac{1}{4\pi\epsilon_0}\frac{q^2}{2d}\right)$

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* why half? Think of the energy stored in the fields?

$$W = \frac{\epsilon_0}{2}\int \underline{\underline{E}}^2\, dv' \longrightarrow \text{stored in } E\text{-field.}$$

* Energy: calcate the work

$$W = \int_\infty^d \vec{F}\cdot d\vec{l} = \frac{1}{4\pi\epsilon_0}\int_\infty^d \frac{q^2}{4z^2}\, dz$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{-q^2}{4z}\right)\Bigg|_\infty^d$$

$$= \boxed{\frac{-1}{4\pi\epsilon_0}\frac{q^2}{4d}}$$


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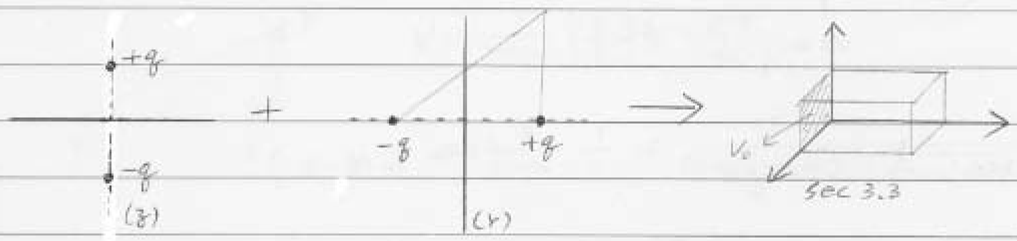
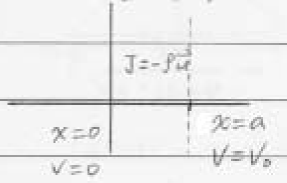
* From Image method to Laplace's eq. (3.2 → 3.3)




Ex: The potential distribution in a space charge limited diode.




1. If large enough so that the edge effect may be neglected.

2. $V=0,\ x=0$
$V=V_0,\ x=a$

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3. $\boxed{\text{The region between the electrodes \& limits the flow of the current.}}$

We can write down the eq. of Laplace or Poisson

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon_0}, 0$$

4. $\boxed{\text{if a current density } \vec{J} \text{ is related to electron velocity} \quad \vec{J} = -\rho\vec{u} \rightarrow \text{velocity}}$

eq of motion, $\frac{1}{2}mu^2 = eV$ —— B.C.

$$\boxed{\frac{d^2V}{dx^2} = \frac{-J}{u\epsilon_0} = \frac{J}{u}\sqrt{\frac{m}{2eV}}}$$

Mathe. Tech. ques.

$$\frac{d^2V}{dx^2} = AV^{-1/2} \quad \left(\boxed{\frac{d^2V}{dx^2}} \sim V^{-\frac{1}{2}} = \boxed{AV^{-1/2}}\right)$$

Integration $\frac{1}{2}\left(\frac{dV}{dx}\right)^2 = \frac{2J}{\epsilon_0}\left(\frac{m}{2e}\right)^{1/2} V^{1/2}$

or

$$V = \left(\frac{3}{2}\right)^{4/3}\left(\frac{J}{\epsilon_0}\right)^{2/3}\left(\frac{m}{2e}\right)^{1/3} x^{4/3}$$

$$V = \sim x^{4/3}, \quad V_0 = \sim a^{4/3}$$

1-D laplace $V = ax + b$ (3.1), $V = x^{4/3} = x^P$

$$V'' \Rightarrow (Px^{P-1})' \Rightarrow P\cdot(P-1)x^{P-2} \Rightarrow V^{-1/2} = \frac{1}{\sqrt{V}}$$

$P = 4/3 \Rightarrow V \sim x^{4/3}$, $V^{-1/2} \sim x^{-2/3}$, $x^{P-2} = x^{4/3-2} = x^{-2/3}$

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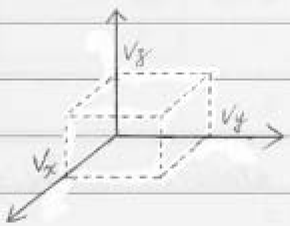
★ Method of variable Separation.

In rectangular coordinates

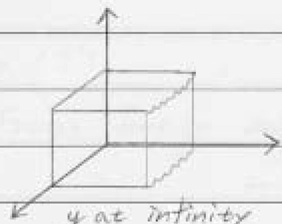
$$\nabla^2 V = \nabla^2 V(x,y,z) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) V = 0.$$




$V_x, V_y, V_z$ independent.

$V = V_x \cdot V_y \, V_z$

if $x, y, z$ at infinity.




if the solution can be expressed as the product of $X(x)$, $Y(y)$, $Z(z)$.

$$V(x,y,z) = X(x)\,Y(y)\,Z(z)$$

then, the Laplace's equation can be written as

$$\frac{1}{X(x)}\frac{d^2}{dx^2}X(x) + \frac{1}{Y(y)}\frac{d^2}{dy^2}Y(y) + \frac{1}{Z(z)}\frac{d^2}{dz^2}Z(z) = 0$$

$$\frac{1}{X}\frac{d^2}{dx^2}X + \frac{1}{Y}\frac{d^2}{dy^2}Y = -\frac{1}{Z}\frac{d^2}{dz^2}Z$$

to be valid for arbitrary values of independent coordinates.

Then, we can let

$$\frac{1}{Z}\frac{d^2}{dz^2}Z = \boxed{c_3} > 0$$

$$\frac{1}{X}\frac{d^2}{dx^2}X + \frac{1}{Y}\frac{d^2}{dy^2}Y = \boxed{-c_3} < 0$$

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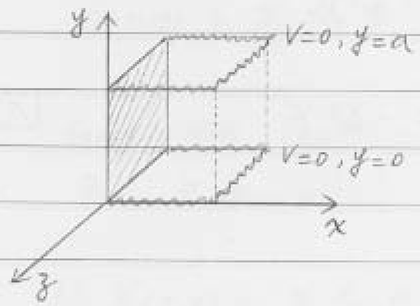
similary, $\frac{1}{X}\frac{d^2}{dx^2}X = c_1 = \frac{-1}{Y}\frac{d^2}{dy^2}Y - c_3$

then, $\frac{1}{Y}\frac{d^2}{dy^2}Y = c_2 \implies c_1 + c_2 + c_3 = 0.$

Ex 3.3




i) $V=0$, when $y=0$

ii) $V=0$, $y=a$

iii) $V=0$, when $z \to \infty, -\infty$

iv) $V_0$, when $x=0$

v) $V=0$, as $\underline{x \to \infty}$

1. if $x, y$ have B.C., then Laplace's eq's solution is $\boxed{V(x,y) = X(x)Y(y)}$

2. Due to the X.Y, 正交 (independent)

3. eq. can be written as

$$\frac{1}{X}\frac{d^2}{dx^2}X + \frac{1}{Y}\frac{d^2}{dy^2}Y = 0$$

then, let $c_1 = k^2$, positive

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For $\frac{1}{X}\frac{d^2X}{dx^2} = k^2$

$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k^2$

eq $= 0$

Laplace's eq can be solved as

$$\frac{1}{X}\frac{d^2}{dx^2}X = k^2 \implies \frac{d^2}{dx^2}X = k^2X$$

$$\frac{1}{Y}\frac{d^2}{dy^2}Y = -k^2 \implies \frac{d^2}{dy^2}Y = -k^2Y$$

then, we can get the formula

$$X_{(x)} = Ae^{kx} + Be^{-kx}, \quad Y_{(y)} = C\sin ky + D\cdot\cos ky$$

at $x=\infty$, $V=0$, then

$$x_{(\infty)} = \underset{\substack{\downarrow\\0}}{A}\,\underset{\substack{\neq\\0}}{e^{k\infty}} + \underset{\substack{\neq\\0}}{B}\cdot\underset{\substack{\downarrow\\0}}{e^{-k\infty}}, \quad \text{so } A=0,\ B\neq 0$$

at $y=0$, $y_{(0)} = \underset{\substack{\neq\\0}}{C}\cdot\underset{\substack{\downarrow\\0}}{\sin(k\cdot 0)} + \underset{\substack{\|\\0}}{D}\cdot\underset{\substack{\|\\1}}{\cos(k\cdot 0)}$

$$\Rightarrow \left.\begin{array}{l} X_{(x)} = Be^{-kx} \\ Y_{(y)} = C\cdot\sin(ky) \end{array}\right\}$$

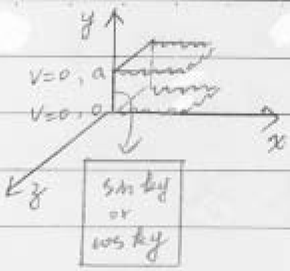
So, the total solution of $X_{(x)}$ product $Y_{(y)}$ is $V_{(x,y)} = Be^{-kx}\cdot C\sin(ky)$

$\therefore$ the superposition of eq is

$$\boxed{V_{(x,y)} = \sum_{n=1}^{\infty} C_n\cdot e^{-kx}\cdot\sin(ky).}$$

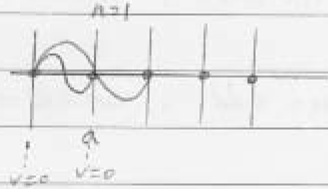
$k = \frac{n\pi}{a}$

敘述：P129




$\boxed{1.}$ $V(x,y) = c \cdot e^{-kx} \cdot \sin ky$

• For B.C. $\boxed{\begin{array}{ll} V=0, & y=a \\ V=0, & y=0 \end{array}} \Rightarrow$




$\boxed{2}$

• Superposition of linear for $k = n\pi/a$.

$$V(x,y) = \sum_{n=1}^{\infty} C_n \, e^{-\frac{n\pi x}{a}} \cdot \sin\left(\frac{n\pi}{a} y\right) \qquad \boxed{k = \frac{n\pi}{a}}$$

$\boxed{3.}$ Solve the coefficient of $\underline{C_n}$

B.C. $x=0$, $V=V_0$

(1) $V(0,y) = \sum_n C_n \underset{1}{\underset{\shortparallel}{e^{-k\cdot 0}}} \cdot \underline{\sin\left(\frac{n\pi}{a} y\right)} = V_0$

(2) $V_0 = \sum_n C_n \boxed{\sin\left(\frac{n\pi}{a} y\right)}$ $\boxed{\text{Fourier}}$

$$\int V_0 \boxed{\sin\left(\frac{n'\pi}{a} y\right) dy} = \sum_n C_n \int \boxed{\sin\left(\frac{n'\pi}{a} y\right)} \cdot \boxed{\sin\left(\frac{n\pi}{a} y\right)} dy$$

We can work out the integral on

$$\int_0^a \sin\left(\frac{n'\pi}{a} y\right) \cdot \sin\left(\frac{n\pi}{a} y\right) dy \Rightarrow \begin{array}{l} n \neq n' \Rightarrow 0 \\ n = n' \Rightarrow \frac{a}{2} \end{array}$$

$$C_n \cdot \frac{a}{2} = \int V_0 \cdot \sin\left(\frac{n\pi}{a} y\right) dy$$

$$C_n = \frac{2V_0}{a} \int_0^a \sin\left(\frac{n\pi y}{a}\right) dy = \frac{2V_0}{a} \cdot \frac{-a}{n\pi} \cos\left(\frac{n\pi y}{a}\right)\Big|_0^a$$

$$= \underline{\frac{2V_0}{n\pi}\left(1 - \cos n\pi\right)}$$

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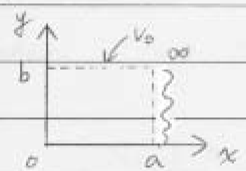
if $n =$ even , $\cos n\pi = 1$ , $C_n = 0$

if $n =$ odd , $\cos n\pi = 1$ , $C_n = \frac{2V_0}{n\pi} \cdot 2 = \frac{4V_0}{n\pi}$

So, the total solution of 2-D Laplace

$$V(x,y) = \frac{4V_0}{\pi} \sum_{\substack{n=1,3,5 \\ (odd)}}^{\infty} \frac{1}{n} e^{-\frac{n\pi}{a} \cdot x} \cdot \sin\left(\frac{n\pi}{a} y\right) \quad *$$

*




$0 \le x \le a$ i) $y = b$ , $V = V_0$ → coefficient

$0 \le y \le b$ ii) $x = 0$ , $V = 0$ ; $x = a$ , $V = 0$ ] $e^{-kx}$ wave function

if $y \to \infty$ , $V = 0$ iii) $y = 0$ , $V = 0$ ⇒ $\frac{e^{-x} + e^{x}}{2}$ $-$

if An infinitely long rectangular conductive cylinder has three sides that are grounded & the fourth side is held at potential $V_0$ .

$0 \le x \le a$ , $0 \le y \le b$ , Laplace's eq $\nabla^2 V = \frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} = 0$ .

then, $V(x,y) = X(x) + Y(y)$

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

if $x=0$, $V=0$
$x=a$, $V=0$ → then we can get a wave-function.

二次微分呈負

$$\frac{X''}{X} = \boxed{-k^2} \quad , \text{ others } \quad \frac{Y''}{Y} = k^2$$

二次微分 $>0$ ← $\boxed{\sinh(kx) \quad \text{or} \quad \cosh(kx)}$

we can solve the $x$ eq first, has the form of

$$X = A \sin kx + B \cos kx \quad , \quad k = \frac{n\pi}{a}$$

at $x=0$, $\sin k \cdot 0 = 0$, $A \neq 0$

$x=a$, $\cos ka = \cos n\pi \neq 0$, $B=0$
$(-1, 1)$

then, $X = A \sin kx$.

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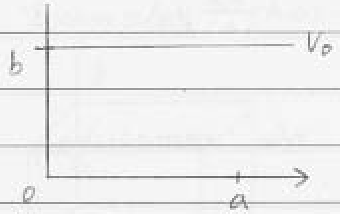
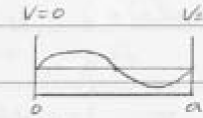



(1) $x=0$ , $x=a$ , $V=0$




解: $\sin kx$ , $\cos kx$

(2) if $x \rightarrow \infty$ , $V=0$

decay $\infty$, $V=0$

解: $e^{-kx}$

(3) $y=0$ , $V=0$
$y=b$ , $V=V_0$ . if $b \rightarrow \infty$ $\Big] \rightarrow$ sum of $e^{-ky}$ & $e^{ky}$

解: $\sinh ky$ , $\cosh ky$

(4) Solution for $X(x) = A \sin kx + B \cos kx$

if $x=0$, $V=0$ $\quad A \neq 0$, $\sin kx = 0$ ; $B=0$, $\cos kx = 1$

$$X(x) = A \sin kx .$$

(5) Solution for $Y(y) = C \cdot \sinh ky + D \cdot \cosh ky$ .

代入 Boundory Condition .

$y=0$ , $Y(y)=0$ $\qquad Y = C \left[ \frac{e^{ky}-e^{-ky}}{2} \right] + D \cdot \left[ \frac{e^{ky}+e^{-ky}}{2} \right]$

$C \neq 0$ , $\frac{e^{ky}-e^{-ky}}{2} = 0$ ; $D=0$ , $\frac{e^{ky}+e^{-ky}}{2} = 1$

$$Y = C \cdot \sinh ky .$$

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Laplace's solution , $V(x,y) = \sum_n A_n \sin \frac{n\pi}{a}x \cdot \sinh \frac{n\pi}{a} y$

(6) The coefficient $A_n$ are determined by the remaining boundary condition. ($y=b$, $V=V_0$)

+

Fourier series method.

$\cdot \boxed{\sin \frac{n'\pi x}{a}}$

A. $y=b$, $V=V_0$ , $V_0 = \sum A_n \boxed{\sin \frac{n\pi x}{a}} \cdot \sinh \frac{n\pi b^{=y}}{a}$

(using Fourier)

$$\int_0^a V_0 \sin \frac{m\pi x}{a}\, dx = \sum_n A_n \sinh \frac{n\pi b}{a} \int_0^a \sin \frac{n\pi x}{a} \sin \frac{m\pi x}{a}\, dx$$

$$= \sum_n A_n \sinh \frac{n\pi b}{a} \left(\frac{1}{2}\delta_{nm}\right) \begin{cases} n \neq m, & \delta_{nm} = 0 \\ n = m, & \delta_{nm} = 1 \end{cases}$$

For which we can get

$$\frac{1}{2} A_m \sin \frac{m\pi b}{a} = V_0 \int_0^a \sin \frac{m\pi x}{a}\, dx = \frac{aV_0}{m\pi}(1-\cos m\pi)$$

$$\Longrightarrow A_m = \frac{2aV_0}{m\pi \sin \frac{m\pi b}{a}} (1-\cos m\pi)$$

(7) if $m = \text{odd} \longrightarrow \frac{2aV_0}{m\pi \sin \frac{m\pi b}{a}} \times 2$

$m = \text{even} \longrightarrow 0$

& $A_m = \frac{\overset{4V_0 ?}{4aV_0}}{m\pi \sin\left(\frac{m\pi b}{a}\right)}$

The Total solution :

for $m = \text{odd}$ , $V(x,y) = \frac{\overset{4V_0 ?}{4aV_0}}{\pi} \sum_m \frac{1}{m} \left(\sin \frac{m\pi x}{a}\right) \frac{\sinh\left(\frac{m\pi y}{a}\right)}{\sinh\left(\frac{m\pi b}{a}\right)}$ #

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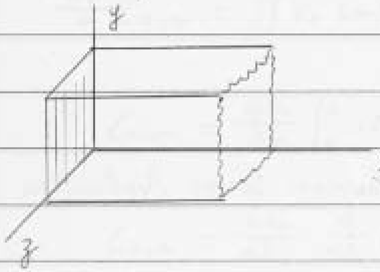
Example 3.5




(1) $x \to \infty$ , $V=0$ $\longrightarrow e^{-k'x}$ or $e^{k'x}$ , $c_1 > 0$

$x=0$ , $V=V_0$

(2) $y=0, y=a$, $V=0$ $\to \sin ky, \cos ky$ ; $k=\frac{n\pi}{a}$ , $c_2<0$

(3) $z=0$ , $z=b$, $V=0 \to \sin lz, \cos lz$ ; $l=\frac{m\pi}{b}$ , $c_3<0$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{c_1}+\underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{c_2}+\underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{c_3}=0$$

$c_1+c_2+c_3=0$ , $c_1>0$ , $c_2+c_3<0$

$\frac{d^2Y}{dy^2}=-k^2Y$ , $\frac{d^2Z}{dz^2}=-l^2Z$ , $k'^2=\underline{k^2+l^2}$ , $\boxed{解\ I}$ 條件

$$X(x) = A e^{\sqrt{k^2+l^2}\cdot x} + B e^{-\sqrt{k^2+l^2}\cdot x}$$

$x\to\infty$ $A\ \infty + B\cdot 0 \Rightarrow \left.\begin{matrix}A=0\\B\neq 0\end{matrix}\right] \to X(x) = Be^{-\sqrt{k^2+l^2}\cdot x}$


$Y(y) = C\cdot\sin ky + D\cdot\cos ky$

$y=0$ $C\neq 0$ $\overset{\shortparallel}{0}$ $D=0$ $\shortparallel$ $\Rightarrow Y(y) = C\cdot\sin ky$ , $k=\frac{n\pi}{a}$

$Z(z) = E\cdot\sin lz + F\cdot\cos lz \Rightarrow Z(z) = E\cdot\sin lz$ , $l=\frac{ml}{b}$

$$\Rightarrow V(x,y,z) = \sum_n\sum_m C_{n,m}\, e^{-\pi\sqrt{\frac{n^2}{a^2}+\frac{m^2}{b^2}}\cdot x}\cdot\sin\left(\frac{n\pi y}{a}\right)\cdot\sin\left(\frac{m\pi z}{b}\right)$$

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$$V(x,y,z) = \sum_n \sum_m C_{n,m} e^{-\pi\sqrt{\frac{n^2}{a^2}+\frac{m^2}{b^2}}\cdot x} \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{m\pi z}{b}\right)$$

at $x=0$, $V=V_0$

$$V(0,y,z) = \sum_n \sum_m C_{n,m} \sin\frac{n\pi \boxed{y}}{a} \sin\frac{m\pi \boxed{z}}{b} = V_0$$ ← Fourier Series Application

$$\sum_n \sum_m C_{n,m} \int_0^a \left(\sin\frac{n\pi y}{a}\cdot \sin\frac{n'\pi y}{a}\right) dy \cdot \int_0^b \sin\frac{m\pi z}{b}\cdot \sin\frac{m'\pi z}{b}\, dz$$
$$= V_0 \int \sin\left(\frac{n'\pi}{a}y\right) dy \cdot \int \sin\left(\frac{m'\pi}{b}z\right) dz$$

$$\int \sin\frac{n\pi y}{a} \sin\frac{n'\pi y}{a}\, dy = \frac{a}{2}\ (n=n') \text{ or } \frac{b}{2}\ (m=m')$$
$$0\quad (n\neq n')\quad (m\neq m')$$

左邊 $\Rightarrow \frac{ab}{4} C_{n,m}$ for $\begin{matrix} n=n' \\ m=m' \end{matrix}$

代入右邊, $C_{nm} = \frac{4}{ab}\int_0^a\int_0^b V_0(0,y,z)\cdot\sin\left(\frac{n\pi y}{a}\right)\cdot\sin\left(\frac{m\pi z}{b}\right) dz$ ※

$n=\ ,\ m=$ odd

& even

conditions

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$$\frac{ab}{4} C_{n,m} = \iint V_0 \sin\left(\frac{n\pi}{a} y\right) \sin\left(\frac{n\pi}{b} z\right) dy\, dz$$

$$C_{n,m} = \frac{4V_0}{ab} \int_0^a \sin\left(\frac{n\pi}{a} y\right) dy \cdot \int_0^b \sin\left(\frac{m\pi}{b} z\right) dz$$

$$C_{n,m} = \frac{4V_0}{ab} \cdot \frac{a}{n\pi} (1-\cos n\pi) \cdot \frac{b}{m\pi} (1-\cos m\pi)$$

if n, m is even,

Case I, $C_{n,m} = 0$

Case II, $C_{n,m} = \frac{16 V_0}{\pi^2 nm}$, for n.m is odd.

---

Total solution.

$$V(x,y,z) = \frac{16V_0}{\pi^2} \sum_{n,m \text{ is odd}} \frac{1}{nm} e^{-\pi\sqrt{(\frac{n}{a})^2 + (\frac{m}{b})^2}\cdot x} \cdot \sin\left(\frac{n\pi}{a} y\right) \cdot \sin\left(\frac{m\pi}{b} z\right)$$

Laplace's eq in spherical polar coordinates.

where V is a function of $\boxed{r, \theta \text{ only}}$

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] V = 0$$

then, we drop out $\mu = \cos\theta$, $d\mu = \sin\theta\, d\theta$

$$d\theta = \frac{1}{\sin\theta} d\mu, \quad 1-\mu^2 = \sin^2\theta$$

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$$\frac{\partial}{\partial r} r^2 \frac{\partial V}{\partial r} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\frac{1}{\sin\theta}\partial\mu}\right) + 0 = 0$$

$$\boxed{\frac{\partial}{\partial r} r^2 \frac{\partial V}{\partial r} + \frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial V}{\partial\mu} = 0}$$

$V(r,\theta)$ is $(r,\theta)$ dependent.

Case I.

$V_r = ?$ , $V_\theta = ?$

Case II.

$V(r,\theta) = V_r \cdot V_\theta$ , $V_r, V_\theta$ is independent.

Then we can get what is the form of $V_r$.

if $\boxed{V_r = r^n}$ ならば,

$$\left(\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} r^n\right) = \frac{\partial}{\partial r} r^2 \cdot n \cdot r^{n-1} = n\cdot(n+1) r^n = n(n+1) V_r.$$

if $\boxed{V_r = r^{-(n+1)}}$

$$\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} r^{-(n+1)} = \frac{\partial}{\partial r} r^2 \left[-(n+1)\right]\cdot r^{-(n+1)-1}$$

$$= -(n+1)\frac{\partial}{\partial r} r^{-(n+1)+1}$$

$$= -(n+1)\cdot(-n)\, r^{-(n+1)} = \boxed{n(n+1) V_r}$$

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The solution of $V_r$ is

$$V = \underline{P_\theta^n \cdot r^n} \quad \text{or} \quad \underline{\frac{P_\theta^n}{r^{n+1}}}$$

or the linear superposition.

The consideration of $\theta$.

Case I. $V = P^n r^n$ , $\boxed{\frac{d}{d\mu}(1-\mu^2)\frac{dP_n}{d\mu} + n(n+1)P_n = 0}$

$n$ is an integer , it's solution are polynomials in $\cos\theta$


$n=0$ , $P_n = 1$ .

$n=1$ . $P_1 = \cos\theta = x$ .

$n=2$ , $P_2 = \frac{3}{2}\cos^2\theta - \frac{1}{2} = \frac{3}{2}x^2 - \frac{1}{2}$ .

$n=3$ . $P_3 = \frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta = \frac{5}{2}x^3 - \frac{3}{2}x$ . (P.138)

The Legendre' polynomials.

$\Theta(\theta) = P_l(\cos\theta)$ , $l = n$

$$P_n(\cos\theta) = \frac{1}{2^n \cdot n!}\left(\frac{d}{d\cos\theta}\right)^n (\cos^2\theta - 1)^n$$

Total solution for spherical Laplace's eq.

$$V(r,\theta) = \sum_{n=0}^{\infty}\left(Ar^n + \frac{B}{r^{n+1}}\right)P_n(\cos\theta)$$

n is For linear combination.

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Spherical Laplace's eq.

Review $V(r,\theta) = r^n \cdot P_n(\cos\theta)$

or $V(r,\theta) = r^{-(n+1)} \cdot P_n(\cos\theta)$ for $n=0$, $\boxed{V(r,\theta) = \frac{a}{r} + b}$

Legendre's eq $P_n(\cos\theta)$

Cylindrical Laplace's equation $V(r,\phi,z)$ solution.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

we use the method of separation of variables and write

$$V(r,\phi,z) = R(r)\,\Phi(\phi)\,Z(z).$$

* If we reduce to 2D, then

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} = 0 \quad \text{, then } V = R(r)\,\Phi(\phi).$$

$V = R(r)\,\Phi(\phi)$ 代入 Eq.

$$\underline{\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right)} = -\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial \phi^2} = \underline{k^2} \quad , \ k \text{ is constant}$$

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Then we can calculate $\Phi$ function is very simple of $\boxed{\cos k\phi}$ or $\boxed{\sin k\phi}$ $\Rightarrow$ 與直角座标不同

於 $\left.\begin{aligned} \cos k(\phi+2\pi) &= \cos k\phi \\ \sin k(\phi+2\pi) &= \sin k\phi \end{aligned}\right]$ $k$ is integer, $\boxed{\Phi = A\cos k\phi + B\sin k\phi}$.


The $R(r)$ function can be written as $e^t$ ($t$ is 代換).

We can find the eq. $\boxed{\frac{d^2R}{dt^2} - k^2 R = 0}$

where $D \equiv \frac{d}{dt}$, then we can rewrite

$$(D-k)(D+k)R = 0$$

$\Rightarrow$ We let $Y = (D+k)R$

$\Rightarrow$ $(D-k)Y = 0$

We can get $Y = e^{kt}$

$(D+k)R = e^{kt}$ , $(\frac{d}{dt}+k)R = e^{kt}$ (兩边積分)

$\Rightarrow R = e^{-kt}\left[\int e^{2kt}dt + c\right]$

$R(r) = \frac{1}{2k}e^{kt} + ce^{-kt} = \boxed{Dr^k + Er^{-k}}$

$\Rightarrow$ For $r$ dependent $\rightarrow$ $R = C\ln r + D$.

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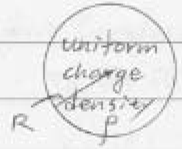
problem :




$\nabla^2 V = -\rho/\varepsilon_0$ ，解 $r>R$ & $r<R$ 之

Poisson's eq.

Ans: $V = \frac{\rho}{6\varepsilon_0}(R^2-r^2) + \frac{\rho R^2}{3\varepsilon_0} \quad (r<R)$

§ 3.4 Approximate Potential at large distance.

J: coupling constant

* assumption: if the electric flux vector of a point charge is directed radially outward, and its magnitude can be found by Gauss's law.

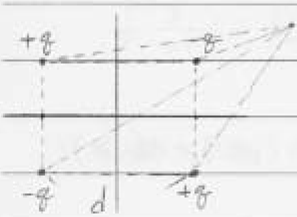

$$r_\pm^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp r \cdot d \cdot \cos\theta$$

Potential $V = V_+ + V_- \sim \frac{1}{r^2}$

A. Mono (single) $V \sim \frac{1}{r}$

B. Dipole $V \sim \frac{1}{r^2}$

C. Quadrupole. $V \sim \frac{1}{r^3}$




Ex 3.10 , Systematic Expansion for the potential of an arbitrary localized charge distribution.

$$|\vec{r}-\vec{r}'|^2 = r^2 + r'^2 - 2rr'\cos\theta'$$

$\Rightarrow$ Expansion as $\frac{r'}{r}$


where ,

potential $V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dv'$

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$$|r-r'|^2 = r^2\left[1+\left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\theta'\right]$$

$$= r^2\left\{1+\left(\frac{r'}{r}\right)\left[\frac{r'}{r}-2\cos\theta'\right]\right\}$$

Let $\epsilon = \frac{r'}{r}\left[\frac{r'}{r}-2\cos\theta'\right]$ ⇒ So we can get

代入展開

$$|\vec{r}-\vec{r}'| = r\sqrt{1+\epsilon}$$

⇒ 查 Taylor Expansion :

$$\frac{1}{|r-r'|} = r^{-1}(1+\epsilon)^{-1/2} = r^{-1}\underbrace{\left[\frac{1}{\sqrt{1+\epsilon}}\right]}_{\text{查表}} = r^{-1}\left[1-\frac{1}{2}\epsilon+\frac{3}{8}\epsilon^2-\frac{5}{16}\epsilon^3+\dots\right.$$

Potential for $\frac{1}{|r-r'|}$

$$= \frac{1}{r}\left[1+\frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\frac{3\cos^2\theta'-1}{2} + \left(\frac{r'}{r}\right)^3\frac{5\cos^3\theta'-3\cos\theta'}{2}+\dots\right.$$

⇒ where we can conclude the form of potential as

$$\frac{1}{|r-r'|} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)P_n(\cos\theta')$$ Total solution

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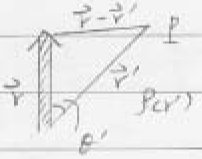
## § 3.4.2 Dipole moment terms

For $V \sim \frac{1}{r^2}$, then we can write the term of

$$V = \frac{q l \cos\theta}{4\pi\epsilon_0 r}, \text{ where } q = \int \rho(r') dv', \quad l' = r'\cos\theta$$

So we can rewrite as

$$V_{Dipole} \equiv \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\int \rho(r')\, r'\cos\theta'\, dv'$$




$r'\cos\theta' = \vec{r}'\cdot\hat{r}$ （兩个向量）

then, $V_{Dipole} \equiv \underbrace{\frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}}_{向量} \cdot \underbrace{\int \vec{r}'\rho(r')\,dv'}_{向量}$

| | |
|---|---|
| 1. 觀測点 $\frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$ <br> 2. Dipole moment <br> $p \equiv \int \vec{r}'\rho(r')\,dv'$ | $\Rightarrow$ The dipole distribution to the potential simplifies to <br> $$V \equiv \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$ |

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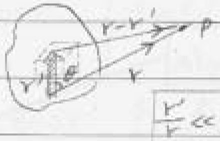
## Review


flux vector

$\boxed{\frac{r'}{r} \ll 1} \Rightarrow p = \int r' \rho(r')\, dv'$ ($\hat{r}$)

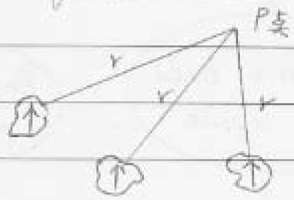
近似 We define as dipole moment.

summary: $V_{Dipole} \equiv \frac{1}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \cdot \int \vec{r}' \rho(r')\, dv'$

$$= \frac{1}{4\pi\epsilon_0} \frac{p \cdot \hat{r}}{r^2}$$

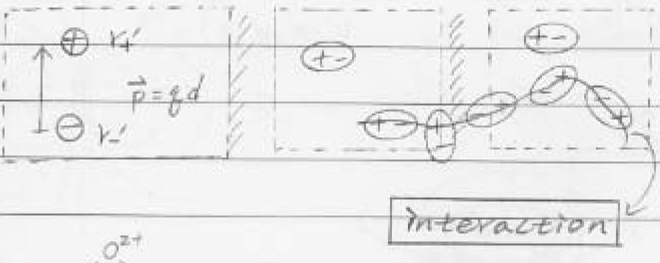
⇒ P.S. For many continuous bodys of charge,
P can be rewritten as

$p = \sum_{i=1} q_i' r_i'$




For physical dipole:
we really define dipole moment.

$$p = q r_+' - q r_-' = q(r_+' - r_-') = q\vec{r}'$$
$$= ql$$





⇒ $OH^-$

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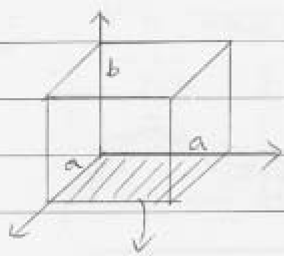
Electric dipole

- charges → Dipole moment → Electric field of a dipole → Potential of a dipole
- Electric Field → Torque on dipole → Energy of Dipole
- Molecular Dipole moment ($H_2O$) → Water
- Chapter 4: Polarization or Dielectric 介电 → Capacitors

homework: (12/19 交)

Laplace's problem

$V(x, y, z) = ?$

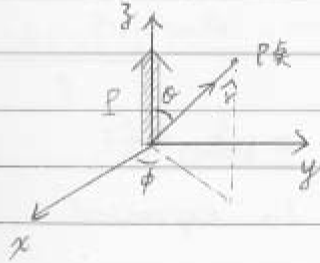



$$\begin{cases} V_0, & y = 0 \\ 0, & y = b \end{cases}$$


$\Rightarrow OH^-$

7.5mm×28行

§ 3.4.4 , The electric field of a dipole ,




that P lies at the origin & points in the z-direction .

$$\hat{r} \cdot \vec{p} = p\cos\theta \quad , \quad V_{Dipole} \equiv \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

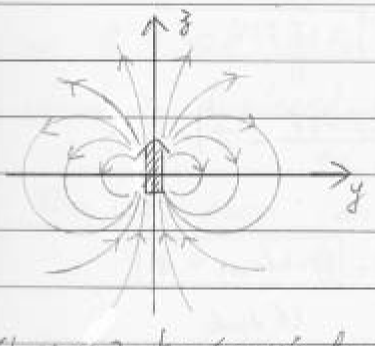
then , we can get the negative gradient of V as

$$E_r = \frac{-\partial V}{\partial r} \quad , \quad E_\theta = \frac{-1}{r}\frac{\partial V}{\partial \theta} \quad , \quad E_\phi = \frac{-\partial V}{\partial \phi}$$

$$= \frac{2p\cos\theta}{4\pi\epsilon_0 r^3} \qquad = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \qquad = 0$$

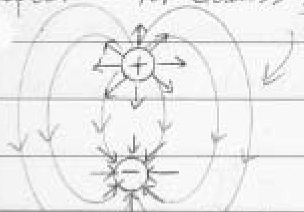
The total electric field

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

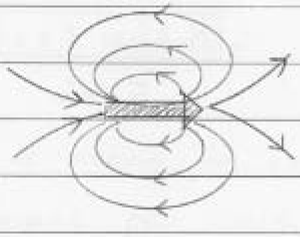



The equation for the lines of force can be found by considering the fig. below .

Chapter 2 for Gauss's law



physical view



(Note: eq 3.104)

the slope of the tangent to the line of force at

$$\frac{E_r}{E_\theta} = \frac{\sin\theta}{2\cos\theta}$$