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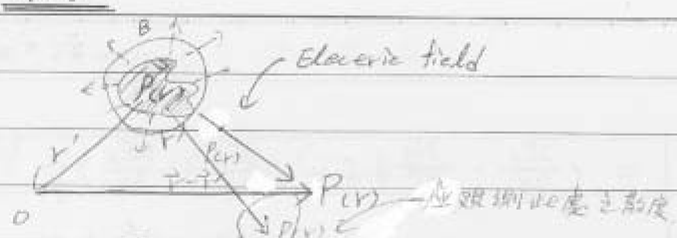
ch 2



$E(r, r')$ (change with r and r')

$B(r)$ (change with r only)

(1) $dE(r-r') = \dfrac{1}{4\pi\epsilon_0} \dfrac{\rho(r')}{|\vec{r}-\vec{r}'|^3} (\vec{r}-\vec{r}') \cdot dr'$

(因為电場可分割)

$\rho(r') \to Q$

$\rho(r')\, dr' = dQ$

(2) $E(r-r') = \dfrac{1}{4\pi\epsilon_0} \displaystyle\int \dfrac{\rho(r')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} dr'$

(3) $\nabla_r \cdot \vec{E}(\vec{r}-\vec{r}') = \dfrac{1}{4\pi\epsilon_0} \displaystyle\int \rho(r') \boxed{\nabla_r \cdot \dfrac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}\, dr'$ (因 r 与 dr' 無関)

$(\nabla_{r'})$

① $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \longleftrightarrow$ 电場散度 $\longleftrightarrow$ 电荷分佈

↓ 位能

$\nabla^2 u$

Gauss's law

↓

1. 証明电荷的存在 是
2. 电場散度不為零
3. 且与 $\vec{\nabla} \cdot \vec{E}$ 成正比

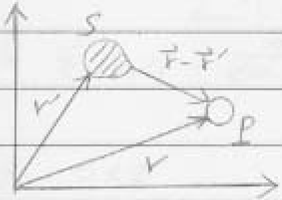
$\to \rho(r) \neq 0$ under electric field

$\rho > 0$

$\rho < 0$

2.1.2 Columb

→ 可疊加



① 在P点上量測S对点的電場.

$$\vec{E}(r-r') = \frac{q'}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho'(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dv'$$

⇒ 座标有関.

⇒ 被观测电荷有関.

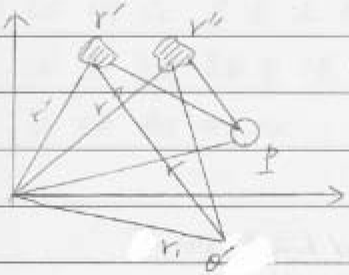
② 作用力在P点上的电荷為q.

⇒ 与观测物的电荷有関.

③ $F = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2} = q\vec{E}$

$$\boxed{\epsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}}$$

2.1.3 Electric field.



① $\left(\vec{E}_{r'} = \frac{q'}{4\pi\epsilon_0|\vec{r}-\vec{r}'|^2}\right) + \left(\vec{E}_{r''} = \frac{q''}{4\pi\epsilon_0|\vec{r}-\vec{r}''|^2}\right)$

$+ \cdots = \vec{E}_{total} = \sum_i \frac{q_i}{4\pi\epsilon_0|\vec{r}-\vec{r}_i|^2}$

⇒ 電場的疊加 ≡ Superposition property

② Total Force, $\vec{F} = Q\,\vec{E}_{total}$.

③ 普物: point change, massless, thickness $= \frac{1}{r} = \frac{1}{\infty} = 0$

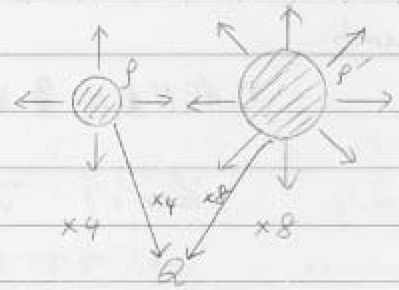
电磁学: 实体描述.

2.2 Divergency of $\vec{E}$

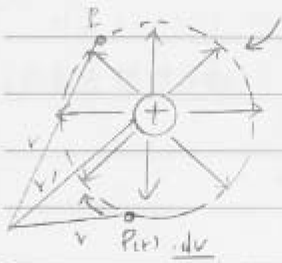
$\rho' > \rho$ $\Leftarrow$

$\downarrow$

可用來表示 $\nabla\cdot\vec{E}$



① Gauss Theorem



我們若對 Gauss 面考慮 $\boxed{\text{積分形式}}$，
我們可以計算

$\Rightarrow \int_V \nabla\cdot\vec{E}\,dv$

电荷密度的体積積分，半徑 $=a$

$$\underline{\frac{1}{\varepsilon_0}\int \rho(r')\,dv'} \quad \text{or} \quad \boxed{\int_0^a \rho(r')\,dv'}$$

$$= \int_0^a \rho(r')\,dv' + \int_a^b \underbrace{(\rho(r')=0)}_{0}\,dv$$

$$= \frac{1}{\varepsilon_0}\int_0^a \rho(r')\,dv$$

(1) 故，散度的積分式

$$\int_{V=\text{Gauss surface}} \nabla\cdot\vec{E}\,dv = \frac{1}{\varepsilon_0}\int_{V=\text{change valume}} \rho(r')\cdot dv'$$

(Gauss law)
(第一方程式)

(2) 積分的意义即是在一个已知的"<u>电荷的分佈</u>"求<u>場的分佈</u>

*若是具有一定对稱性的問題，

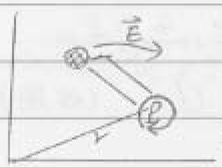
球型、柱狀、立浅，才能使用 Gauss surface.

静电場第二方程式

① $\vec{E}_{(r-r')} = \frac{1}{4\pi\epsilon_0}\int \rho(r') \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dv$] 已知电場 ← from $(\vec{r}-\vec{r}')$

② $\nabla\times\vec{E}$

$= \vec{\nabla}_r \times \vec{E}(\vec{r}-\vec{r}')$



⇒ $\boxed{\text{在P点上的旋度}}$

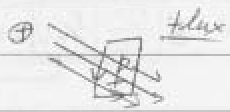
$$\vec{\nabla}\times\vec{E} = \underbrace{\nabla_r \times \frac{1}{4\pi\epsilon_0}\int \rho(r')}_{\text{在P点，故可放入積分}} \cdot \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dv'$$

$$= \frac{1}{4\pi\epsilon_0}\int \rho(r') \cdot \boxed{\nabla_r \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}} \cdot dv'$$

⇓

計算並証明 $=0$.

*証明在P点上的静电場是一种"無旋度場"，
或者説静电場中任何一處不会出現电力线
呈旋転形狀、



$\nabla_{r=P点} \times \vec{E}(r-r') = 0$

*向量場$\vec{E}$的旋度 $\nabla_r\times\vec{E}$，即代表在任一个面上（P点）
的通量等於
這个場沿該面所圍閉合曲线上的
環流 $\oint \vec{E}\cdot dl = 0$.

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$\nabla_r \times \vec{E} = 0 \longrightarrow$ 計算 $\nabla_r \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = 0$ (之後再談)

(1) 在觀測的位置 r 座標上，量測电場的旋度.

(2) 在一点上旋度為零、$\nabla_r \times \vec{E}(r-r') = 0$.
物理上，可引進一个純量 U. (位能) ≡ 純量場.

(3) 純量場 ≡ 純量的梯度，是一向量，正比於电場.

(4) 若定义無窮遠處為零. $\boxed{-\nabla U = \vec{E}}$.

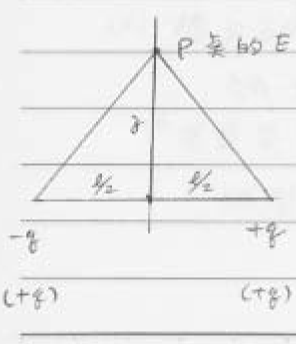
(5) 其比值為"-1"，滿足在电場中某處的电位等於
"外力克服电場力，將單位正电荷從零电位面
移至場中，該点所作的功".

(6) $\nabla \times (\nabla U) = 0$
$\Rightarrow \nabla \times \vec{E} \Longleftrightarrow \nabla \times (\nabla U) = 0$

(7) $\nabla \cdot \vec{E} = \boxed{\rho/\epsilon_0}$ 　　 證明 U 的存在

$\longrightarrow$ Gauss 已知 $\longrightarrow$ 解 $\vec{E} \xrightarrow[2^{nd}\ law]{} \nabla \times \vec{E} \longrightarrow \vec{E} = -\nabla U$

* Problem 2.2



$\vec{E} = \frac{q(\vec{r}-\vec{r}')}{4\pi\epsilon_0 |\vec{r}-\vec{r}'|^3}$

$+q = (\frac{d}{2}\hat{x}, 0\hat{z})$
$-q = (\frac{-d}{2}\hat{x}, 0\hat{z})$
$\vec{r} = (0\hat{x}, z\hat{z})$

$\rightarrow$

$\vec{r}-\vec{r}_+ = (\frac{-d}{2}\hat{x}, z\hat{z})$
$\vec{r}-\vec{r}_- = (\frac{d}{2}\hat{x}, z\hat{z})$

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superposition
$$\begin{cases} \vec{E}_+ = \dfrac{+q\,(\vec{r}-\vec{r}_+)}{4\pi\epsilon_0 |\vec{r}-\vec{r}_+|^3} = \dfrac{+q\,(-\frac{l}{2}\hat{x}+z\hat{z})}{4\pi\epsilon_0 (\frac{l^2}{4}+z^2)^{3/2}} \\ \vec{E}_- = \dfrac{-q\,(\frac{l}{2}\hat{x}+z\hat{z})}{4\pi\epsilon_0 (\frac{l^2}{4}+z^2)^{3/2}} \end{cases}$$

$$\vec{E}_+ + \vec{E}_- = \vec{E} = \frac{q\,l\,\hat{x}}{4\pi\epsilon_0 (\frac{l^2}{4}+z^2)^{3/2}}$$

check:

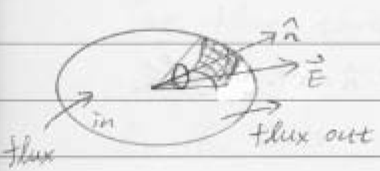
if $z \gg l$, $E = \dfrac{q\,l\,\hat{x}}{4\pi\epsilon_0 z^3}$

Gauss' law : $\nabla\cdot\vec{E}=0 \Rightarrow$ Intergral & Differential Forms

Case 1. $\rho > 0$

flux out of surface.

$$\oint \nabla\cdot\vec{E}\,dV \equiv \oint \vec{E}\cdot d\vec{s} = \underline{flux}$$



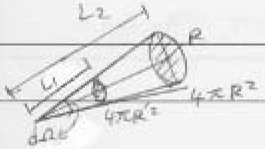
$$d\vec{s} \equiv \hat{n}\cdot da \Rightarrow \therefore \vec{E}\cdot\hat{n}\,ds = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}\cdot\hat{n}}{r^2}\,da$$

① $\hat{r}\cdot\hat{n}\,da = dA \equiv$ 總面積

② $r^2 d\Omega = dA$, $d\Omega \equiv$ Solid Anger

③ 所以 $\boxed{d\Omega = \dfrac{dA}{r^2}}$

$\Rightarrow \int d\Omega = 4\pi$



$$d\Omega = \frac{dA_1}{L_1^2} = \frac{dA_2}{L_2^2}$$
$$\Downarrow \qquad \Downarrow$$
$$\frac{4\pi R'^2}{L_1^2} \qquad \frac{4\pi R^2}{L_2^2}$$

④ then E - field , $\vec{E}\cdot\hat{n}\,da = \dfrac{q}{4\pi\epsilon_0}\,d\Omega$

$$\oint \vec{E}\cdot d\vec{s} = \int \frac{q}{4\pi\epsilon_0}\,\underline{\underline{d\Omega}} = \frac{q}{\epsilon_0}$$ $\boxed{\text{prove it}}$

Q.E.D.

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小考 2.2.3前 $\frac{10}{3}$

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(P.50) prove 1.100 ?

① Differential form : $\nabla_r \cdot \vec{E} = \rho/\epsilon_0$ $\Big] \rightarrow 1^{st}$ law $\rightarrow$ at point r's divergency of electric field.

↑ 証明為点电荷 $\Rightarrow \boxed{\nabla_r \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta(\vec{r})}$

② $\oint \vec{E}_i \cdot \underset{\underset{d\vec{s}}{\parallel}}{d\vec{a}} = \oint \vec{E} \cdot \hat{n}\, \underline{da} = q_i/\epsilon_0 \rightarrow$ flux $\underline{(2.12)}$ at point of $\underline{q_i}$

if $i = 1$ to n, then the total changes $Q = \sum_{i=1}^{n} q_i$,

the total flux of superposition $\oint \vec{E}_i \cdot d\vec{a} = Q/\epsilon_0 = \frac{1}{\epsilon_0}\sum_{i=1}^{n} q_i$.

③ Differential Form $\rightarrow$ well known of change, calculate $\vec{E}$.

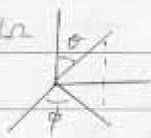
Integrated Form $\rightarrow$ 〃 of $\vec{E}$, calculate Q.

$\rightarrow$ Electrical Form.

$\oint \vec{E} \cdot d\vec{s} = \oint \boxed{\frac{1}{4\pi\epsilon_0} \frac{\rho\hat{r}}{r^2}} \cdot \hat{n}\, da$ at point charge

$\equiv \oint \frac{1}{4\pi\epsilon_0} \frac{\rho}{r^2} \hat{r} \cdot \hat{n} \cdot r^2 \cdot 4\pi = \rho/\epsilon_0$

$da = \underline{r^2 \sin\theta\, d\theta\, d\phi} = r^2 d\Omega$



⇓

space phase

$= \boxed{\text{solid angle}} = 4\pi$

Divergency Theorem

$\int \nabla \cdot \vec{A}\, d\tau = \int \vec{A} \cdot d\vec{a} = \oint \vec{A} \cdot \hat{n} \cdot da$

$-\nabla U = \vec{E}$, Gradient Theorem

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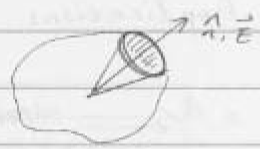
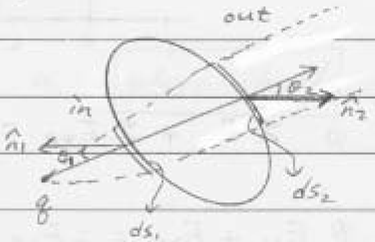
Case 1 : $\rho > 0$, flux $\oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$



The flux of $\vec{E}$ out of any closed surface containing a charge of q, is equal to $q/\underline{\underline{\epsilon_0}}$.

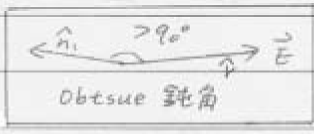
Case 2 : $\rho = 0$, flux in and out of surface



$$d\Omega = \frac{ds_1 \cos\theta_1}{r_1^2} = \frac{ds_2 \cos\theta_2}{r_2^2}$$

$$\equiv \frac{\hat{n}_1 \cdot \hat{r}_1 \, ds_1}{r_1^2} = \frac{\hat{n}_2 \cdot \hat{r}_2 \, ds_2}{r_2^2}$$

The flux through ds_1 is negative.



$\hat{r} \cdot \hat{n} = \cos(\pi - \theta_1)$

<u>A.</u> $\vec{E}_1 \cdot \hat{n}_1 ds_1 = E \hat{r}_1 \cdot \hat{n}_1 ds_1 = \frac{q}{4\pi r^2} (\hat{r}_1 \cdot \hat{n}_1) ds_1$

$\hat{r} \cdot \hat{n} = \cos(\pi - \theta_1) = -\cos\theta_1$

<u>B.</u> $\vec{E}_2 \cdot \hat{n}_2 ds_2 = \frac{q}{4\pi r^2} (\hat{r}_2 \cdot \hat{n}_2) ds_2$

<u>A. + B.</u> $= \frac{q}{4\pi} \boxed{\frac{(-\cos\theta_1)}{r_1^2} ds_1} + \frac{q}{4\pi} \boxed{\frac{\cos\theta_2}{r_2^2} ds_2} = 0$

$\Rightarrow \oint \vec{E} \cdot d\vec{s} = 0$, $\boxed{\because \rho = 0}$

$\nabla \cdot \vec{E} = 0 \equiv$ The sum of flux in & out $= 0$

\#

No flux.

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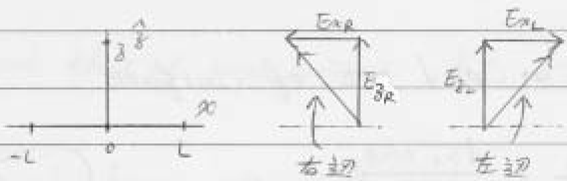
2.2.3 Applications of Gauss's law

A. $\Sigma\, q/\epsilon_0 = Q/\epsilon_0$. superposition of Electric field.

$\int dE_i = E$ Example 2.1 ,

$\sum_{i=1}^{n} E_i = E$ Find the electric field at a distance of z.



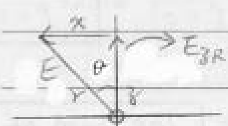
① $\Sigma E_{xL} + \Sigma E_{xR} = 0$ ③ $\int d\vec{E} = \int \frac{\rho}{4\pi\epsilon_0 r^2}\, \underline{dv} \Rightarrow$ 体積分

② $E_{zL} + E_{zR} = 2\,\underline{E_{zR}}$ $\int \frac{\sigma}{4\pi\epsilon_0 r^2}\, \underline{da} \Rightarrow$ 面積分

$\int \frac{\lambda}{4\pi\epsilon_0 r^2}\, \underline{dx} \Rightarrow$ 線積分

$dE = \frac{dq}{4\pi\epsilon_0 r^2}$, $dq = \lambda\, dx$

line density $\lambda = \frac{Q}{2L}$



$\to E_{zR} = E\cos\theta\, \hat{z} \Rightarrow dE_{zR} = dE\cos\theta\, \hat{z} = \frac{\lambda\, dx \cdot \cos\theta}{4\pi\epsilon_0\, r^2}\, \hat{z}$

$\boxed{\cos\theta = \frac{z}{r}} = \frac{z}{\sqrt{z^2+x^2}}$, $\frac{x}{z} = \tan\theta$

$dx = d(z\tan\theta) = z\sec^2\theta\, d\theta$

$\int dE_{zR} = \int \frac{\lambda\, dx \cdot \cos\theta}{4\pi\epsilon_0\, r^2}\, \hat{z} = \int \frac{\lambda \cdot z\sec^2\theta\, d\theta \cdot \cos\theta}{4\pi\epsilon_0 \cdot z^2/\cos^2\theta}$

$= \frac{\lambda}{4\pi\epsilon_0 z} \int \cos\theta\, d\theta = \boxed{\frac{\lambda}{4\pi\epsilon_0} \sin\theta} \times 2 \equiv E_{total}\, \hat{z}$.

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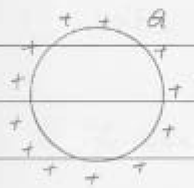
$\sin\theta = \frac{L}{\sqrt{z^2+L^2}}$

then, the total Electric field $\boxed{E = \frac{2\lambda L}{4\pi\epsilon_0 z\sqrt{z^2+L^2}}}$

A. if $z \gg L$, $E = \frac{2\lambda L}{4\pi\epsilon_0 z^2} = \boxed{\frac{Q}{4\pi\epsilon_0 z^2}}$ $\boxed{\text{point charge}}$

B. if $L \to \infty$, $E = \frac{2\lambda}{4\pi\epsilon_0}\left(\frac{1}{z}\right)\left(\frac{1}{\sqrt{z^2/L^2+1}}\right) = \boxed{\frac{2\lambda}{4\pi\epsilon_0 z}}$

Example: The electric field $\left(\begin{array}{l}\text{inside}\\ \text{outside}\end{array}\right.$ of a $\boxed{\text{sphere shell}}$ of surface charge density σ.



$\sigma = \frac{Q}{4\pi a^2}$, if $r > a$

then, $\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\epsilon_0} = \int \frac{\sigma \cdot da}{\epsilon_0}$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r^2}\right)$, if $r > a$

if $r < a$, then $\Rightarrow \rho = 0$, then $E = 0$

if a solid sphere with volume charge density

$\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3}a^3}$

(a). if $r > a$, then $\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}}$

(b). if $r < a$,

电量 $= ? \Rightarrow \frac{\rho\cdot\frac{4\pi}{3}a^3}{\rho\cdot\frac{4\pi}{3}r^3} = \frac{Q}{Q'}$

Q'

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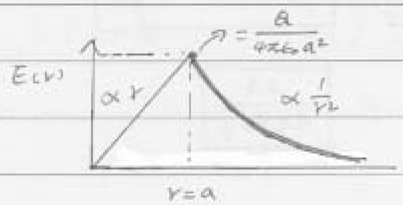
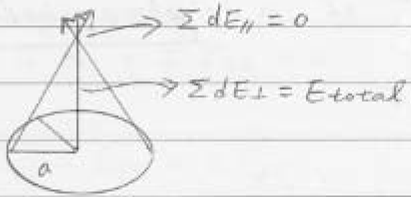
$r<a$, $Q' = Q \, r^3/a^3$

then, the obtained electric field is

equal to $\vec{E} = \dfrac{Q'}{4\pi\epsilon_0 r^2} = \dfrac{Q \, r^3/a^3}{4\pi\epsilon_0 r^2} = \dfrac{Q}{4\pi\epsilon_0} \dfrac{\boxed{r}}{a^3}$

$\boxed{\text{linear with } r}$





$F = m\ddot{x} = m \dfrac{d^2x}{dt^2}$

$F = -q\vec{E}$. if $E \sim x$

equation of motion $\rightarrow$ $\boxed{m\ddot{x} = -qE}$

$\ddot{x} + \dfrac{q}{m}E = 0$.

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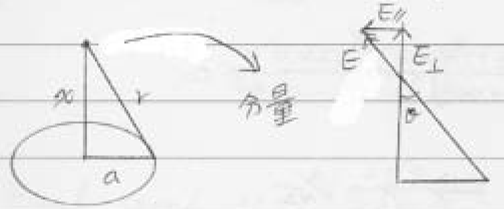
Ex : Prove the maximum of ElecEric field locates at $x = a/\sqrt{2}$



① $\Sigma E_{\parallel} = 0$

② $\Sigma E_{\perp} = E_{total}$

③ $E_{\perp} = E\cos\theta$

① $dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}$, $\boxed{\lambda ds = dq}$ $\Rightarrow$ $\frac{Q}{2\pi a} = \lambda$

② $dE\cos\theta = dE_{\perp} = \frac{\lambda ds \cdot \cos\theta}{4\pi\epsilon_0 \cdot r^2}$

③ $\cos\theta = \frac{x}{r}$, $r = \sqrt{a^2+x^2}$

④ $\int dE_{\perp} = \int \frac{\lambda ds}{4\pi\epsilon_0}\frac{x}{(a^2+x^2)^{3/2}} = \frac{2\pi a\lambda \cdot x}{4\pi\epsilon_0 (a^2+x^2)^{3/2}}$

⑧ if $x \ll a$, near the ring center.

$$E = \frac{qx}{4\pi\epsilon_0 a^3}\left[\frac{1}{(1+\underbrace{\frac{x^2}{a^2}}_{\to 0})^{3/2}}\right] \simeq \frac{qx}{4\pi\epsilon_0 a^3}$$

⑨ if the test change is $(-q')$ moving along the x-axis.

$$F = (-q')E = -q'\frac{qx}{4\pi\epsilon_0 a^3} = -Ax$$

⑩ Looking for the equation of motion

$$F = m\ddot{x} = m\frac{d^2x}{dt^2} = -Ax \quad , \quad m\ddot{x} + Ax = 0$$

$$\ddot{x} + \frac{A}{m}x = 0 \quad , \quad \ddot{x} + \omega^2 x = 0 .$$

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⑪ if $E = \dfrac{qx}{4\pi\epsilon_0 (a^2+x^2)^{3/2}}$ $\qquad \dfrac{d(\pm)\cdot \tau - \pm \cdot d(\tau)}{(\tau)^2}$

Maximum $\dfrac{dE}{dx} = \dfrac{q}{4\pi\epsilon_0} \dfrac{(a^2+x^2)^{3/2} - x(\frac{3}{2})(a^2+x^2)^{1/2}(2x)}{(a^2+x^2)^3} = 0$

Apply for a ring.

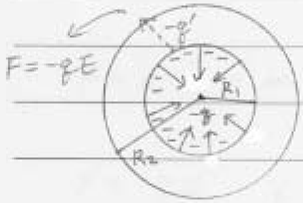
at $a^2 = 2x^2 \Rightarrow x = a/\sqrt{2}$

if the ring is half q_1, half q_2 $\Rightarrow$ total $q = q_1 + q_2$

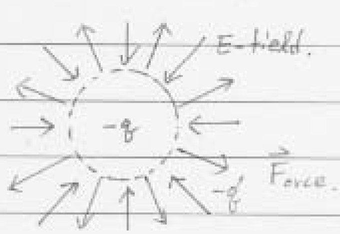
$q/2\pi a = \lambda$, $\therefore$ $\boxed{\lambda = \dfrac{q_1+q_2}{2\pi a}}$

the same reslt.

* Application for two sphere shells.



if one electric escapes from inner to outter shell what is the final speed? of electron at outter shell.



① 电力线朝内 $E = \dfrac{-q}{4\pi\epsilon_0 r^2}$

② $\vec{F} = -q'E = \dfrac{q'q}{4\pi\epsilon_0 r^2}$

③ $\int_{R_1}^{R_2} \vec{F}\cdot dr = \frac{1}{2} m v^2$

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$$\int_{R_1}^{R_2} F \cdot dr = \frac{q'q}{4\pi\epsilon_0} \int \frac{1}{r^2}\, dr$$

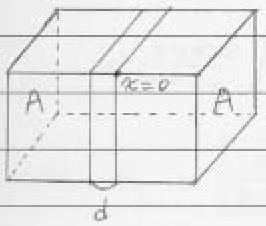
$$= \frac{q'q}{4\pi\epsilon_0} \left(-\frac{1}{r}\right)\Big|_{R_1}^{R_2} = \frac{q'q}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{1}{2} m V^2$$

Final speed $V = \left[\frac{2q'q}{4\pi\epsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{1/2}$

if $\left.\begin{array}{l} R_1 = 0.145\,m \\ R_2 = 0.207\,m \end{array}\right]$ $-q = -6 \times 10^{-8}\,C = -q'$

$\frac{1}{2} m V^2 = 1.78 \times 10^{-16}$ Joule , $\boxed{V = 2 \times 10^{7}\,m/s}$.

*



A nonconducting plane of thickness d, volume charge density ρ.

What is the E-field at x.

$\phi = \int E \cdot dA = E \cdot A = Q'/\epsilon_0 = \frac{\rho A x}{\epsilon_0}$

$E = \frac{\rho x}{\epsilon_0}$

one electron inside the plane.

$F = -e\vec{E} = -e\frac{\rho x}{\epsilon_0}$, $m\ddot{x} = -e\frac{\rho x}{\epsilon_0}$

$\ddot{x} + \frac{e\rho}{m\epsilon_0} x = 0$, $\omega = \sqrt{\frac{e\rho}{m\epsilon_0}}$.

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Review for last week.

* if we calculate out the E-field is proportional to x-axis, then $\boxed{\vec{E} \propto \vec{x}}$.

* Then we can find out the equation of motion.

$$F = m\ddot{x} = -e\vec{E}$$

* We can calculate the frequency of charge $(-e)$ in the electric-field.

$$\boxed{\ddot{x} + \frac{e}{m}\vec{E} = 0} \quad , \quad \omega = 2\pi f \ , \ f = \frac{\omega}{2\pi}$$

Summary:

1. In nonconducting plate

$$\vec{E} = \frac{\rho x}{\epsilon_0} \quad , \quad \omega = \sqrt{\frac{e\rho}{m\epsilon_0}}$$

2. In a ring, as $x \ll a$

$$E = \frac{q x}{4\pi\epsilon_0 a^3} \quad , \quad \omega = \sqrt{\frac{e\, q}{4\pi\epsilon_0 m a^3}}$$

3. In a disc.

~ potential

$\Downarrow$

Decay

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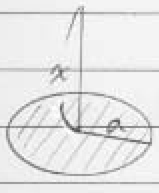
* In a Disc of charge.



$$E_z = \frac{q x}{4\pi\epsilon_0 (a^2+x^2)^{3/2}}$$ at a ring of radius $\underline{a}$.

Disc charge density.

$$\sigma = \frac{Q}{\pi a^2}, \quad \begin{cases} \Rightarrow Q = \pi a^2 \sigma \\ \Rightarrow dq = \sigma\, 2\pi a\, da \\ \qquad \equiv \sigma\, 2\pi r\, dr \end{cases}$$

From a ring charge to a disc, then $\underline{a} \longrightarrow \underline{r}$.

$$E_z = \frac{r}{4\pi\epsilon_0}\int_0^a \frac{\sigma\cdot 2\pi r\,dr}{(r^2+x^2)^{3/2}}$$

$$= \frac{2\pi\sigma x}{4\pi\epsilon_0}\int_0^{a^2+x^2} \frac{\frac{1}{2}\, d(r^2+x^2)}{(r^2+x^2)^{3/2}} = \frac{2\pi\sigma x}{4\pi\epsilon_0}\left[\frac{1}{x} - \frac{1}{\sqrt{a^2+x^2}}\right]$$

$$\Rightarrow \quad E_z = \boxed{\frac{\sigma}{2\epsilon_0}\Big[1} - \frac{x}{\sqrt{x^2+a^2}}\Big]$$

$\downarrow$

$$E_0 - \frac{\sigma}{2\epsilon_0}\frac{x}{\sqrt{x^2+a^2}}$$

Mathematical Method : $\int \frac{dk}{k^{3/2}} = (-2)\, k^{-1/2}$

Then, if $a \ll x$

$$E_z = E_0 - \frac{\sigma}{2x\epsilon_0}\,\hat{z}, \quad E_0 = \frac{\sigma}{2\epsilon_0}$$

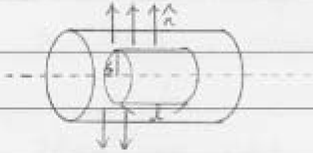
$E_z - E_0 = -\frac{1}{x}$ ~ potential

$\Downarrow$

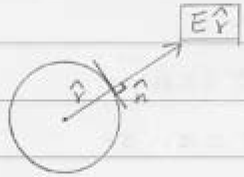
Decay.

Page 12. A long cylinder carries a charge density

$\sim \boxed{\rho = ks}$, k : constant.



1. if we circle a surface, then we apply Gauss's law.



$$\oint \vec{E} \cdot da = |\vec{E}| \cdot \pi s^2 \cdot l = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

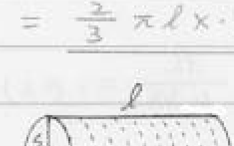
2. Our enclosed charge $Q_{\text{enclosed}} = ?$

$$\boxed{\begin{array}{l} Q_{enc} = \int \rho' \, \underline{dv'} = \int \rho \cdot \underbrace{s' ds' d\phi' dz'}_{= dv'} \\ \text{In cylinder coordinate: } \phi : 0 \sim 2\pi \\ \qquad\qquad\qquad\qquad\quad z : 0 \sim l \end{array}}$$

$$Q = \int_0^s \underline{k} \, s'^2 ds' \cdot \int_0^{2\pi} d\phi' \int_0^l dz'$$

$$= 2\pi l k \int_0^s s'^2 ds' = \underline{\frac{2}{3} \pi l k \cdot s^3}$$

$$\int E \cdot da = \frac{Q}{\epsilon_0} \Rightarrow$$



$$E \cdot \pi s l = \frac{1}{\epsilon_0} \cdot \frac{2}{3} \pi l k s^3$$

圓柱面積

$\underline{2\pi s} \cdot l$ = area

$$\vec{E} = \frac{1}{3\epsilon_0} k s^2 \hat{s}$$

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Problem 2.14, Find the electric field inside a sphere which carries charge density, $\rho = kr$.

1. $\oint \vec{E} \cdot d\vec{a} = E \cdot \underbrace{4\pi r^2}_{\text{球面積}}$

2. $\boxed{Q_{\substack{enclosed \\ sphere}} = \int \rho' \, \underbrace{r'^2 dr' \int \sin\theta' d\theta' \int d\phi'}_{\int \rho' dv'}}$

$= 4\pi \int \rho' r^2 dr'$

$4\pi \cdot \int_0^r \rho' r'^2 dr' = \int_0^r kr' \cdot r'^2 dr'$

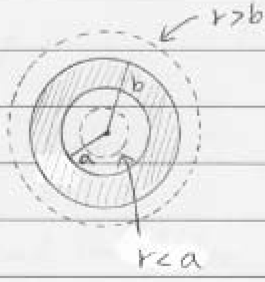
$= 4\pi k \cdot \frac{r^4}{4} = \pi k r^4$

$E \cdot 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \pi k r^4 / \epsilon_0$, $\vec{E} = \frac{1}{4\epsilon_0} k r^2 \hat{r}$

Ex: A hollow spherical shell carries charge density,

Problem 2.15 $\rho = \frac{k}{r^2}$, in the regin of $a \le r \le b$.

$r < a$ & $r > b$, $\rho = 0$.



* $\vec{E}$ - field at

$r < a \longrightarrow \oint \vec{E} \cdot da = \frac{Q_{enc}}{\epsilon_0} = \frac{0}{\epsilon_0}$, $\vec{E} = 0$

$r > b \longrightarrow \oint \vec{E} \cdot da = \frac{Q_{total}}{\epsilon_0}$

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$a<r<b$, $Q_{total} = \int \rho \, r'^2 dr' \sin\theta' d\theta' d\phi'$

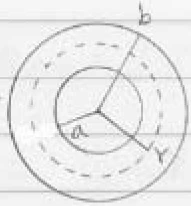
$$= 4\pi \int_a^b \frac{k}{r'^2} \cdot r'^2 dr' = \underline{4\pi k(b-a)}$$

Then, if $r>b$, $E \cdot 4\pi r^2 = \frac{4\pi k(b-a)}{\epsilon_0}$

$$E = \frac{k(b-a)}{r^2 \epsilon_0} \hat{r}$$

if $a<r<b$, $Q_{enc} = \int_a^r \rho r'^2 dr' \sin\theta' d\theta' d\phi'$

$$= 4\pi k(r-a)$$



$$E \cdot 4\pi r^2 = \frac{4\pi k(r-a)}{\epsilon_0}$$

$$\vec{E} = \frac{k(r-a)}{r^2 \epsilon_0} \hat{r}$$

2.2.4. The Curl of $\vec{E}$.

$\vec{\nabla}_r \times \vec{E} = \vec{\nabla}_r \times \left(\frac{\hat{r}}{r^2}\right) = 0$, Prove it.

↳ classical Electrodynamics.

In Physical concept.

* $\vec{F} = m\ddot{x}$, $\vec{F} = q\vec{E}$, Lorentz Force.
(Newton)

* $\vec{F} \cdot d\vec{l} \equiv \Delta W$, $\boxed{\text{if In closed path.}}$

作用功在重力 $mg \cdot h = \Delta W$

$-mg \cdot h = -\Delta W$

closed path $= 0$

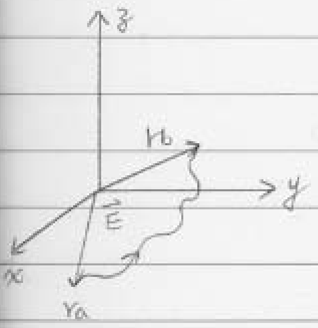
* Lorentz Force :

$\oint q \cdot \vec{E} \cdot d\vec{l} \equiv$ Force along a closed path.

$= 0$ (不作功)

$\oint \vec{E} \cdot d\vec{l} = ?$

If a charge $+q$ moving.



a electric-field (point charge)

$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$, if in spherical coordinate.

$d\vec{l} \equiv dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

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$$\text{if } \vec{E}\cdot d\vec{l} \begin{cases} dr\,\hat{r} \longrightarrow \oint E_r\,dr = \int \frac{q}{4\pi\epsilon_0 r^2}\,dr \\ d\theta\,\hat{\theta} = 0 \\ d\phi\,\hat{\phi} = 0 \end{cases}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{r_a}^{r_b}$$

Then, $\oint_{r_a}^{r_b} \equiv r_a = r_b$ at point origin.

$$\oint \vec{E}\cdot d\vec{r} = \frac{q}{4\pi\epsilon_0}\underbrace{\left[\frac{1}{r_a} - \frac{1}{r_b}\right]}_{=0} = 0$$

Stoke's Theorem:

$$\nabla\times\vec{E} = 0,\ \oint \vec{E}\cdot d\vec{l} = 0.$$

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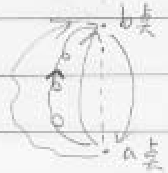
$\boxed{\nabla\times\vec{E} = 0}$



From a to b point, independent of path.

$$\equiv -\int_a^b \vec{E}\cdot d\vec{l} = -\int_0^b \vec{E}\cdot d\vec{l} - \left(-\int_0^a \vec{E}\cdot d\vec{l}\right)$$

$\oint \vec{E}\cdot d\vec{l} = 0$

Example: Calculate $\nabla\times\vec{E}$ directly.

A. $\vec{E}(r-r') = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|^2}\,dv'$

$$\vec{\nabla}_r \times \vec{E}(r-r') \equiv \frac{1}{4\pi\epsilon_0}\int \rho(r')\left(\vec{\nabla}_z \times \frac{1}{|\vec{r}-\vec{r}'|^2}\right)dv' = 0$$

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$\underline{\text{B.}}$ Prove a vector $\boxed{\vec{E}'(r) = r^n \hat{r}}$ is similar to Electric field.

In spherical coordinate

$$\nabla r \times \vec{E}'(r) = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \cdot r^n \hat{r} = \frac{1}{r^2}\underline{\frac{\partial}{\partial r} r^{n+2}}\,\hat{r}$$

$$\nabla \times \vec{E}'(r) = \frac{1}{r^2}\cdot (n+2)\, r^{n+2-1}\,\hat{r}$$

$$= \underline{(n+2)}\, r^{n-1}\,\hat{r}$$

if $\vec{E}' \sim E(r-r')$, $(n+2) = 0 \xrightarrow{\text{then}} \underline{n=-2}$ tt' a

$\boxed{\text{special case}}$

$$\vec{E}(r) = r^n \hat{r} \;,\quad \vec{E}_{(r)} = r^{-2}\,\hat{r}.$$

2.3.1 Introduction to Potential.

a, b, a=b

* Because of curl of $\vec{E}$, $\nabla \times \vec{E} = 0$
  then the integral form.

$$\oint \vec{E} \cdot d\vec{l} = 0 \quad , \ a=b \ , \text{ closed path.}$$

We define a function integrate the line,

the function $\equiv$ work
potential.

$$-\int_0^r \vec{E} \cdot dl \equiv V(r)$$

O $\equiv$ reference point to r

$\Downarrow$

define at $r = \infty$ ~~infinity~~.
at r = origin.

$$V(a) = -\int_0^a \vec{E} \cdot d\vec{l}$$
$$V(b) = -\int_0^b \vec{E} \cdot d\vec{l}$$

We have calculate the differences. (ΔV)

Define as $V(b) - V(a)$, then the ~~infinity~~ O can be neglected.

$$V(b) - V(a) = \left[-\int_0^b \vec{E} \cdot d\vec{l}\right] - \left[-\int_0^a \vec{E} \cdot d\vec{l}\right]$$

$$= \;{''}\; - \left[+\int_a^0 \vec{E} \cdot d\vec{l}\right]$$

$$= -\int_a^b \vec{E} \cdot d\vec{l}$$

$$\Downarrow$$

$$= \int_a^b \boxed{\nabla V} \cdot d\vec{l} \Rightarrow \vec{E} = -\nabla V.$$

Form the fundamental theorem of <u>Gradient</u> is
[the difference of electric potential]

$$V(b) - V(a) = \int_a^b \underline{\nabla V} \cdot d\vec{l} = -\int \vec{E} \cdot d\vec{l}. \quad \vec{E} = -\nabla V$$

push to divergency of electric field,

$$\nabla \cdot \vec{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V = \rho/\epsilon_0$$

chapter 3.
(Laplace $\rho = 0$
poisson $\rho \neq 0$)

P.S. Chapter 1.6 Helmholtz theorem
+ Chapter 2.3 Electric potential.

(1) $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \Sigma Q_i$. 有限的 (相疊加)

(2) $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int \rho \, dv'$. 連續的 (continuous)

(3) 無窮遠的向量場.

NO. 10/17 小考 2.2.3 ~ 2.2.4

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2.3.2 comments on potential.

(1) The name : 位能, $\boxed{\text{电位能}}$, 勢 (功)

potential, potential energy.

A. $\vec{F} = q\vec{E}$, observed charge point q on electric field $\vec{E}$.

B. $\int_a^b \vec{E} \cdot dl$, E-field along the path $(a \to b)$

(2) Advantage of the potential Formulation

A. if we define $V(b) - V(a) = -\int \vec{E} \cdot d\vec{l}$ → Closed path.
& satisfy $V(a) - V(b) = \int \vec{E} \cdot d\vec{l}$

B. $\boxed{\text{Curl of } \vec{E} = 0}$, $\nabla \times \vec{E} = 0$, $\oint \vec{E} \cdot d\vec{l} = 0$, $\vec{E} = -\nabla V$

C. $\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x}$, $\frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z}$, $\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x}$

⟹ Independent of path.

(3) The reference of O, we define the infinity point of O's potential is ZERO.

Example :

$$V(z) = -\int_{\infty}^{z} \frac{\sigma}{2\epsilon_0} dz = \frac{-\sigma z}{2\epsilon_0}.$$

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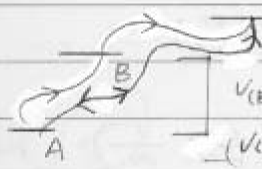
(4) Potential obey's the superposition principle classical Mechanism.

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \cdots\cdots = q(\vec{E}_1 + \vec{E}_2 + \cdots\cdots)$$

<u>special case</u>

if follows the potential, satifies.

$$V = V_1 + V_2 + V_3 \cdots\cdots$$



$V(C) - V(B)$ ....(1)

$-(V(C) - V(B))$ ....(2)

$V(B) - V(A)$ ....(2)

$-(V(B) - V(A))$ ....(4)

Total V ①+②+③+④ = 0.

From A to C

Total potential eq(1) + eq(2) $= V(C) - V(A)$

(5) Unit of potential：单位电荷的能量.(eV) ≡ 电压.

$$F = q\vec{E} \Rightarrow F\cdot\vec{l} = q\vec{E}\cdot dl \equiv \text{Newton - meter}$$

F ↓ Newton, q ↓ columb

$$\text{potential} \equiv \frac{F\cdot l}{q} = \frac{\text{Newton - meter}}{\text{Colomb}} = \frac{\text{Joule}}{\text{columb}}$$

Problem <u>2.20</u>, 1) $\vec{E} = k[\,xy\,\boxed{\hat{x}} + 2yz\,\boxed{\hat{y}} + 3xz\,\boxed{\hat{z}}\,]$

2) $\vec{E} = k[\,y^2\,\hat{x} + (2xy + z^2)\,\hat{y} + 2yz\,\hat{z}\,]$

One of these is an impossible electrostatic field?

$$\nabla\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = ?$$

$$\nabla\times\vec{E}_1 = k[\hat{x}(0-2y) + \hat{y}(0-3z) + \hat{z}(0-x)] \neq 0$$

$$\nabla\times\vec{E}_2 = 0.$$

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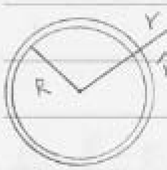
Example 2-6, Find the potential inside & outside a spherial shell or radius R. carries a uniform surface charge.



if $r>R$, $\vec{E}=\frac{q}{4\pi\epsilon_0 r^2}\hat{r}$; if $r<R$, $\vec{E}=0$.

$$V=-\int_{\infty}^{r}\vec{E}\cdot dr \ (\text{if } r>R) = \frac{-q}{4\pi\epsilon_0}\left(\int\frac{dr}{r^2}\right)$$

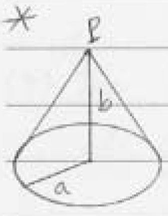
$$=\frac{q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{\infty}\right),\ r>R.$$

if $r<R$, 在球內,它的積分是從無窮遠處帶到 r 處.

$$V=-\int_{\infty}^{r}\vec{E}\cdot d\vec{l} = -\int(\vec{E}_{r>R},\vec{E}_{r<R})\,d\vec{l}$$

$$=-\int_{\infty}^{R}\vec{E}_{r>R}\cdot d\vec{l}+\left(-\int_{R}^{r}\vec{E}_{r<R}\cdot dl\right)$$

$$=-\int_{\infty}^{R}\frac{q}{4\pi\epsilon_0 r^2}\,dr+\left(-\int_{R}^{r}0\cdot dr\right)=\frac{q}{4\pi\epsilon_0 R}$$

* 

charge density λ of ring. What's the electric potential at point P.

$$V=\frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{a^2+b^2}}\right),\quad q=2\pi a\cdot\lambda$$

the $V=\dfrac{a\lambda}{2\epsilon_0\sqrt{a^2+b^2}}$

① $\boxed{\text{if } a\ll b}$, $\boxed{V=\dfrac{\lambda a}{2\epsilon_0 b}}$ $=\dfrac{\lambda a}{2\epsilon_0 b\sqrt{\frac{a^2}{b^2}+1}}$, $\frac{a^2}{b^2}=\frac{1}{\infty}\approx 0$

⇒ ring changes to point change.

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电磁学期中考 11/14 (三) 10:00 AM

範圍 ~ present

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② $\vec{E} = -\nabla V = \dfrac{qb}{4\pi\epsilon_0 (a^2+b^2)^{3/2}}$.

Problem 2.21, (A) Find the potential inside/outside a uniformly charged solid sphere, whose radius is R, & total charge q. (B) Prove $\vec{E} = -\nabla V$

P.S. use infinity as reference point $V(\infty) = 0$.

In solid sphere, outside $\Rightarrow \vec{E} = \dfrac{q}{4\pi\epsilon_0 r^2}$

從無窮遠處開始積分. at $r=R \Rightarrow \vec{E} = \dfrac{q}{4\pi\epsilon_0 R^2}$

inside $\Rightarrow \vec{E} = \dfrac{q'}{4\pi\epsilon_0 r^2}$

$$q' = q \cdot \frac{V'}{V} = q\,\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{q r^3}{R^3}$$

So, Inside E-field

$$\vec{E} = \frac{q\, r^3/R^3}{4\pi\epsilon_0 r^2} = \frac{q r}{4\pi\epsilon_0 R^3}$$

A. if the potential locates at point r, the r>R.

$$V(r) = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r} = -\int \frac{q}{4\pi\epsilon_0 r^2}\, dr = \boxed{\frac{q}{4\pi\epsilon_0 r}}$$

B. $r<R$, $V = -\int_{\infty}^{R} \vec{E}_{out}\cdot dr + \left(-\int_{R}^{r} \vec{E}_{in}\cdot dr\right)$

$$-\int_{\infty}^{R} \vec{E}\cdot dr = \frac{q}{4\pi\epsilon_0 R}$$

$$-\int_{R}^{r} \frac{q r}{4\pi\epsilon_0 R^3}\cdot dr = \frac{\frac{1}{2} q}{4\pi\epsilon_0 R} - \frac{\frac{1}{2} q r^2}{4\pi\epsilon_0 R^3}$$

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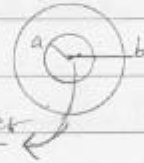
Total $V = \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left[3 - \frac{r^2}{R^2}\right]$ ※

at $r=0$, $V(0) = \frac{3q}{4\pi\epsilon_0 R}$

check $\vec{E} = -\nabla V = \frac{qr}{4\pi\epsilon_0 R^3}\hat{r}$, at $r<R$

* Problem combine P2.15 →



Find the potential at center 球心

$V(0) = -\int_\infty^0 \vec{E}\cdot d\vec{r}$

$= -\int_\infty^b \vec{E}_1\cdot d\vec{r} + \left(-\int_b^a \vec{E}_2\cdot d\vec{r}\right) + \left(-\int_a^0 \vec{E}_3\cdot d\vec{r}\right)$

$\vec{E}_3 = 0$, $E_1 = \frac{k}{\epsilon_0}\left(\frac{b-a}{r^2}\right)$, $E_2 = \frac{k}{\epsilon_0}\left(\frac{r-a}{r^2}\right)$.

$V(0) = \frac{k}{\epsilon_0}\left(\frac{b-a}{b}\right) - \frac{k}{\epsilon_0}\left[\ln\left(\frac{b}{a}\right) + a\left(\frac{1}{a}-\frac{1}{b}\right)\right]$

$= \boxed{\frac{k}{\epsilon_0}\ln\left(\frac{b}{a}\right)}$

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2.3.3 ?

(1) $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

(2) $\nabla\times\vec{E} = 0$ (closed path), $\vec{E} = -\nabla V$, V is scalar.

Then combine divergency and curl theorem.

$$\vec{\nabla}\cdot(-\nabla V) = \rho/\epsilon_0 \Rightarrow \nabla^2 V = \rho/\epsilon_0$$ ，拉普方程式.

in (x, Y, z) - aixs $\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right) V(x, y, z) = \rho/\epsilon_0 \Rightarrow$ $\boxed{\text{chapter } 3}$

2.3.3, if $\rho = 0 \longrightarrow$ Laplacian eq $\nabla^2 V = 0$,

if $\rho \neq 0 \longrightarrow$ Poisson eq $-\nabla^2 V = \rho/\epsilon_0$

(1) $\nabla\times\vec{E} = 0 \longrightarrow -\int\vec{E}\cdot d\vec{l} = V$ (Scalar)

conclude $\vec{E} = -\nabla V$

$\Downarrow$

(2) $\nabla\cdot\vec{E} = \rho/\epsilon_0 \Rightarrow \nabla^2 V = \rho/\epsilon_0$

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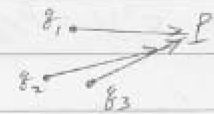
2.3.4 The potential of Localized charge distribution

$\equiv$ superposition $(\vec{F}, \vec{E}, V)$ + continuos (ρ, Q)

Case 1, point charge

Case 2, Multi-point charges



$\Sigma \equiv$ Sum over charges

Case 3, charge distribution.

Integral of charge density $\rho(r)$

1. Point charge (Multi-charges) $\Rightarrow \boxed{\oint \vec{E} \cdot d\vec{l} = \frac{1}{\epsilon_0} \Sigma Q_i}$ + $\rightarrow \boxed{V(r) = \frac{-1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}}$

Case 1 + Case 2

$\Rightarrow$ 2.3.2 comm 5^{th}

2. Charge distribution $\Rightarrow \boxed{\oint \vec{E} \cdot d\vec{l} = \frac{1}{\epsilon_0} \int \rho(r') dv'}$ +

Case 3

$$\rightarrow V(r) = \frac{-1}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{q}{|r-r'|} dr'$$

$$\boxed{V(r) = \frac{-1}{4\pi\epsilon_0} \int \frac{\rho(r') dv'}{r}}$$

2.7 $\Rightarrow$ 研究所範圍

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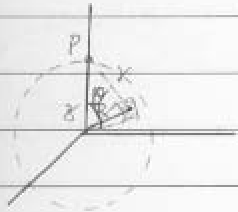
Ex 2.7 Find the potential of uniformly charged spherical shell of radius R.



Case 3 : charge distribution

<u>A.</u> Total charge $\equiv$ charge density $\times$ A (surface)

$Q = \sigma A$, $dq' = \sigma dA'$

<u>B.</u> $\vec{r} = \vec{z} - \vec{R}$, $r^2 = z^2 + R^2 - 2\,\vec{z}\cdot\vec{R}$

$= z^2 + R^2 - 2zR\cos\theta$

<u>C.</u> $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq'}{r}$ $\left(\sum_n \frac{q_n}{r_n}\right)$

$= \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, dA'}{(z^2+R^2-2zR\cos\theta)^{1/2}}$

<u>D.</u> if $\sqrt{(R-z)^2} \Rightarrow$ $z-R$ for out side.

$\Updownarrow$ $R-z$ for inside.

$|R-z|$

<u>E.</u> $dA' = r^2 d\Omega$, $dA' = R^2 \sin\theta\, d\theta\, d\phi$

$(\int d\phi = 2\pi)$

$d\Omega$ $\frac{dA'}{R^2} = \frac{dA''}{R''^2} = d\Omega$

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$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \cdot 2\pi R^2 \sin\theta \, d\theta}{(z^2+R^2-2zR\cos\theta)^{1/2}}$$

Math : method

$$\int \frac{\sin\theta}{(z^2+R^2-\underbrace{2zR\cos\theta}_{x})^{1/2}}$$

let $zR\cos\theta = x$, $d(zR\cos\theta) = dx$

$$zR\sin\theta = dx \quad \Rightarrow \quad \int \frac{\frac{dx}{Rz}}{\sqrt{z^2+R^2-2x}}$$

$$V(r) = \frac{2\pi\sigma R^2}{4\pi\epsilon_0} \int \frac{\sin\theta \, d\theta}{\sqrt{z^2+R^2-2zR\cos\theta}} \qquad \left(\int \frac{dx}{\sqrt{y-2x}} \right)$$

$$= \frac{2\pi\sigma R^2}{4\pi\epsilon_0} \left(\frac{1}{Rz}\right) \sqrt{z^2+R^2-2zR\cos\theta} \, \Big|_0^\pi$$

$$\Downarrow$$

$$\underbrace{\sqrt{(z-R)^2}}_{\theta=\pi} , \text{ or } \underbrace{\boxed{\sqrt{(R-z)^2}}}_{\theta=0} \quad \Longleftrightarrow \quad \boxed{\begin{matrix} \cos(\pi) = -1 \\ \cos(0) = 1 \end{matrix}}$$

Point D

$$V_{out} = \frac{R\sigma}{2\epsilon_0 z} \left[\underbrace{(R+z) - (z-R)_{out}}_{2R} \right]$$

$$V_{in} = \frac{R\sigma}{2\epsilon_0 z} \left[\underbrace{(R+z) - (R-z)_{in}}_{2z} \right]$$

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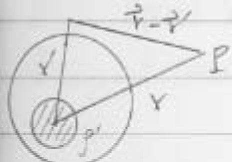
① Expansion of potential, $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|r-r'|}\,dv'$



$\boxed{\text{if } r >> r'}$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|\vec r|}\left(\frac{1}{1-\frac{r'}{r}}\right)dv'$$

→ Taylor Expansion as $\frac{r'}{r}$ is small enough.

$$= \frac{1}{4\pi\epsilon_0}\int \rho(r')\left[\frac{1}{r} - \vec r'\cdot \underbrace{\nabla\left(\frac{1}{r}\right)}_{-\frac{1}{r^2}\cdot\frac{\vec r}{r}} + \frac{1}{2!}\Sigma + \dots\right]$$

1^{st} term, $\frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{r}\,dv$

2^{nd} term, $\frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')\,\vec r'\cdot\vec r}{r^3}\,dv' = \frac{1}{4\pi\epsilon_0}\left(\frac{\vec p\cdot\vec r}{r^3}\right)$

② Superposition ⟷ Continuous $\bigcirc + \begin{matrix}\ominus\\ \oplus\end{matrix} + \begin{matrix}\ominus & \oplus\\ \oplus & \ominus\end{matrix} + \cdots\cdots$

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* Charged distribution $\rightarrow$ 積分 $\int \frac{\rho' dv'}{r'} \Rightarrow$ if $r \gg r'$

* Summary : Concluse that the charge distribution will convert to two ways of superposition & continuous.

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{|\vec{r}-\vec{r}'|} dv'$$

$$\frac{1}{|r-r'|} \Rightarrow |r-r'|^{-1} = \frac{1}{r}\left[1 + \frac{\vec{r}'\cdot\vec{r}}{r^2} + \cdots\cdots\right]$$

$$\Rightarrow |r-r'|^{-1} = \frac{1}{r} + \vec{r}'\cdot\left(-\nabla\frac{1}{r}\right) + \cdots\cdots$$

$$|r-r'|^{-1} = \frac{1}{r} + \vec{r}'\cdot\left(\frac{1}{r^2}\hat{r}\right) + \cdots\cdots$$

$$|r-r'|^{-1} \doteq \frac{1}{r} + \vec{r}'\cdot\left(\frac{1}{r^2}\hat{r}\right) + \cdots\cdots$$

$$= \frac{1}{r} + \vec{r}'\cdot\left(\frac{\vec{r}}{r^3}\right) + \cdots\cdots$$

$$= \frac{1}{r}\left[1 + \frac{\vec{r}'\cdot\vec{r}}{r^2} + \cdots\cdots\right]$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\left[\int \frac{\rho(r')dv'}{r} + \frac{\vec{P}\cdot\vec{r}}{r^3}\right] + \cdots\cdots$$

mono $\rightarrow$ point, dipole.

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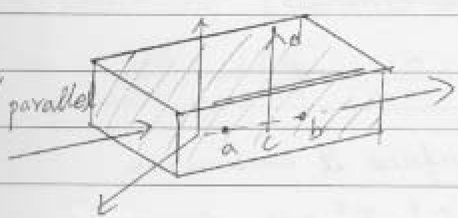
2.3.5 Materials : two types

A. Conductor (導体)
B. Dielectric (介电) (电介) 大陆 → Conductivity 来決定.

C. Charge density on the surface of conductor ; $\sigma A \equiv Q$

D. The distribution of of charge.



From a to b

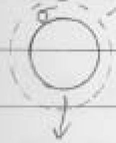
E-field in continuous $\Longrightarrow E_a^{\parallel} = E_b^{\parallel}$

E. From c to d, the E-field is discontinuous.

① $E^{\perp}_{c,\text{above}} \neq E^{\perp}_{d,\text{below}}$

② $E_a^{\parallel} = E_b^{\parallel}$

$* \oint \vec{E} \cdot d\vec{a} = \frac{\sigma A}{\epsilon_0} = \frac{Q_{enclosed}}{\epsilon_0}$ (Divergency of $\vec{E}$)



Gauss's Law

$|\vec{E}| \cdot A = \frac{\sigma A}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0}$

Gauss pill box

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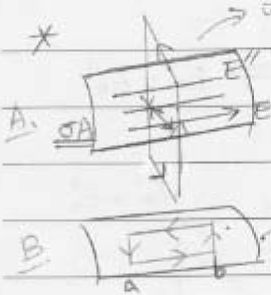
If a conductor : inside the box contribute nothing to the flux as the limit $d \to 0$, we can get

$$E_a^{\perp} - E_b^{\perp} = \frac{\sigma}{\epsilon_0} \quad \left(\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}\right) \quad eq\,2$$

where, $E^{\perp}$ is perpendicular to the surface, then we can conclude that the normal component of $\vec{E}$ is discontinuous.

*



$E^{\parallel}$ - field on the surface & continuous,

$$E_a^{\parallel} = E_b^{\parallel} \implies E_a^{\parallel} - E_b^{\parallel} = 0.$$
eq 1

$\nabla \times \vec{E} = 0$ on the surface $\implies \oint \vec{E} \cdot d\vec{l} = 0$

Summary : We can sum of all components. The B.C. on $\vec{E}$ can be combined into a single formula.

above : $\vec{E}_a = E_a^{\perp} + E_a^{\parallel}$

below : $\vec{E}_b = E_b^{\perp} + E_b^{\parallel}$

We can write within eq 1 + eq 2.

$Eq_1 \implies E_a^{\parallel} - E_b^{\parallel} = 0\,\hat{t}$ (切面) $\quad \nabla \cdot \vec{E}$

$Eq_2 \implies E_a^{\perp} - E_b^{\perp} = \frac{\sigma}{\epsilon_0}\,\hat{n}$ (法線) $\quad \nabla \times \vec{E}$

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we define the normal vector is $\hat{n}$

sum : $\vec{E}_a - \vec{E}_b = \frac{\sigma}{\epsilon_0}\hat{n}$

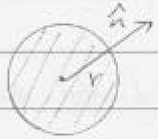
From curl of $\vec{E}$. $\Rightarrow \vec{E} = -\nabla V$

$$\vec{E}_a - \vec{E}_b = \left(-\underbrace{\nabla_n}_{\frac{\partial}{\partial n}} V_a\right) - \left(-\nabla_n V_b\right) = \frac{\sigma}{\epsilon_0}\hat{n}$$

* Combined $\nabla \cdot \vec{E}$ & $\nabla \times \vec{E}$

$$\boxed{\frac{\partial V_b}{\partial n} - \frac{\partial V_a}{\partial n} = -\frac{\sigma}{\epsilon_0}\hat{n}}$$

$\frac{\partial}{\partial r}$



$\hat{n} = \hat{r}$

2.4 Work & Energy

$\vec{F} = q\vec{E}$, $\Rightarrow \vec{E} = \frac{\vec{F}}{q}$

* The electric potential is the potential energy per unit charge.

* The potential difference between 1 & 2 in an electric field is equal to the work per unit charge.

$$\Delta V = V(\vec{r}_2) - V(\vec{r}_1) = \frac{W}{q}$$

or $W = \int \vec{F} \cdot d\vec{l} = \int q\vec{E} \cdot d\vec{l} = qV\Big|_a^b = q[V_b - V_a]$

* prove superposition of Work.

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P.27 2.3.2 ~ 2.3.4

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* if q_1 be placed r_1, no work.

* bring q_2 from infinity to r_2 & place it at a distance r_{12} from q_1.

Work $W_{12} \equiv \underset{\text{unit charge}}{\underset{\Downarrow}{q_2}} \cdot \boxed{\frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}}$

As q_3 is added to the system at $\vec{r}_3$

$$W_{123} = q_3 \left[V_1(\vec{r}_{13}) + V_2(\vec{r}_{23}) \right] = q_3 \left[\frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right) \right]$$

$$W = \frac{1}{4\pi\epsilon_0} \sum_i q_i \sum_{j>i} \frac{q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0} \frac{1}{2} \sum_i \sum_j \frac{q_i q_j}{r_{ij}}$$

$(\frac{1}{2})$ The factor appears in the expression because each pair is accounted twice.

* Work. $\boxed{W = \frac{1}{2} \sum q_i V(r_i)}$

Superposition of all q_i.

* If the changes are not localized at point, but are distributed with volume density ρ & surface density σ.

$$\boxed{W = \frac{\epsilon_0}{2} \int \vec{E} \cdot \vec{E} \, dV}$$

P.93 → 不太对

2.4.3 The Energy of a continuous charged distribution.

$\boxed{W = \frac{1}{2}\Sigma qV}$

* Is the energy stored in the charge?
 or
 is stored in the field?

* Feynman : The energy is located in space, where the electric field is.

* charge are distributed with volume density ρ & surface density σ.

* superposition ⟷ continuous.

* Then we can write down the work.

$$W = \frac{1}{2}\int \rho V dv' + \frac{1}{2}\int \sigma V ds$$

From Gauss's law :

$$* \quad \frac{1}{2}\int \rho V dv' = \frac{1}{2}\int \epsilon_0 (\vec{\nabla}\cdot\vec{E}) V dv'$$

$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$ 静电学 1st law

P.S. $\nabla\cdot(V\vec{E}) = V(\nabla\cdot\vec{E}) + \vec{E}\cdot(\nabla V)$

we can get $\nabla\cdot(V\vec{E}) - \vec{E}\cdot(\nabla V) = \underline{(\nabla\cdot\vec{E})V}$

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$$\frac{1}{2}\int \rho V\, dv' = \frac{1}{2}\int \epsilon_0 \left[\underline{\nabla\cdot(V\vec{E})} - \vec{E}\cdot\nabla V \right] dv'$$

$$\int \nabla\cdot(V\vec{E})\, dv' = \int V\vec{E}\cdot d\vec{s} \quad \text{——} \quad (\mathrm{I})$$

$$\int -\vec{E}\cdot\nabla V\, dv' = \int \vec{E}\cdot(-\nabla V)\, dv'$$

$$= \int \vec{E}\cdot(\vec{E})\, dv' \quad \text{——} \quad (\mathrm{II})$$

$$\frac{\epsilon_0}{2}\int V \underline{\vec{E}\cdot d\vec{s}} = -\int \frac{\epsilon_0}{2} V\cdot \frac{\sigma}{\epsilon_0}\, d\vec{s} = \underline{\frac{-1}{2}\int \sigma V ds}$$

↳ B.C. condition. $E_a^\perp - E_b^\perp = \frac{\sigma}{\epsilon_0}$ 从 ① total W

$b - a$ (?)

$$W = \frac{1}{2}\epsilon_0 \int \vec{E}\cdot\vec{E}\, dv' + \frac{1}{2}\int \sigma V ds' - \frac{1}{2}\int \sigma V ds'$$

$$= \frac{1}{2}\epsilon_0 \int \vec{E}\cdot\vec{E}\, dv'$$

Prove Energy is stored in field!

From B.C. condition.

E_n is normal component of $\vec{E}$ at of volume v

$$\boxed{E_n = \frac{-\sigma}{\epsilon_0}}$$

Hence the total work

$$W = \frac{\epsilon_0}{2}\int \vec{E}\cdot\vec{E}\, dv' = \frac{\epsilon_0}{2}\int E^2 dv'$$

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* When an electric field is present, there is located in space an energy whose density is Energy density.

$\Rightarrow u_E \equiv \frac{1}{2}\epsilon_0 E^2$

* We can not apply $W = \frac{1}{2}\epsilon_0 \int \vec{E}^2 dv'$ to a point charge?

For it shows that the energy of a point charge $\boxed{2.4.4}$ is infinite. (Integrate the all space).

$$W = \frac{\epsilon_0}{2}\int_0^r \int_0^{2\pi} \int_0^{\pi} \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)^2 r^2 \sin\theta \, d\theta \, dr \, d\phi = \frac{q^2}{8\pi\epsilon_0}\int_0^{\infty}\frac{1}{r^2}dr$$

$\Rightarrow \int_0^{\infty}\frac{1}{r^2}dr = \boxed{\infty}$ (Can not apply for point charge).

(Ex 2.8) W_{total} for $\boxed{\text{spherical shell}}$
sphere

$$W = \frac{1}{32\pi^2\epsilon_0} q^2 4\pi \int_R^{\infty}\frac{1}{r^2}dr = \frac{1}{8\pi^2\epsilon_0}\frac{q^2}{R}$$

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Ex: Electro Magnetic Mass (电磁質量)

$\boxed{\text{Chapter 2.4.4}}$: For point charge we can obtain the total work is infinity.

$$W \propto \int_0^\infty \frac{1}{r^2} dr = \infty = \left(-\frac{1}{r}\right)_0^\infty = 0 - \left(\frac{1}{0}\right) = \infty$$

$$\Rightarrow \frac{q^2}{4\pi\epsilon_0 r}\Big|_{r=0} = \infty$$

$\boxed{\text{Einstein}}$ if we know the mass of electron,

$m_e = 9.11 \times 10^{-31}$ kg

$W \neq \infty$

$$m_e c^2 = \frac{q^2}{4\pi\epsilon_0 r} \Rightarrow r_e = \frac{q^2}{4\pi\epsilon_0 m_e c^2}, \quad r_e \neq 0$$

$$r_e = 2.82 \times 10^{-15} m$$

丁肇中 (1980), $r_e = 1 \times 10^{-15} m$

$\boxed{\text{Superposition}}$: $W_{total} = \frac{\epsilon_0}{2} \int (\vec{E}_1 + \vec{E}_2)^2 dv'$

$$= \frac{\epsilon_0}{2} \int E_1^2 + E_2^2 + 2 \cdot \vec{E}_1 \cdot \vec{E}_2 \, dv'$$

$$= W_1 + W_2 + \epsilon_0 \int \vec{E} \cdot \vec{E}_2 \, dv'$$

Last week, $W = \frac{\epsilon_0}{2} \int E^2 dv' = \int u \, dv'$

$$u = \frac{\epsilon_0}{2} E^2$$

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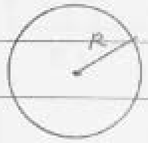
Ex: Find the energy of a uniformly charged sphere of radius $\underline{R}$.



1. $dq = \rho\, dV = \rho\, d(\frac{4}{3}\pi r^3) = 4\pi r^2 dr \cdot \rho$.

2. $dW = \frac{1}{4\pi\epsilon_0} \frac{\frac{4}{3}\pi r^3 \rho}{r} dq$.

$dW = \frac{4\pi\rho^2 r^4}{3\epsilon_0} dr$, $\int dW = \frac{4\pi\rho^2}{15\epsilon_0} R^5$ (for sphere)

3. if we know, $m_e = \frac{W}{c^2}$. Find out m_e?

$m_e = \frac{W}{c} = \frac{3}{5} \frac{q^2}{4\pi\epsilon_0 c^2 r}$

For spherical shell, $m_e = \frac{1}{2} \frac{q^2}{4\pi\epsilon_0 r\, c^2}$.

Chapter $\underline{2.5}$ Conductor

I. Basic properties: $E = 0$ inside a conductor.

II. $\rho = 0$. volume charge density $= 0$

$\sigma \neq 0$, surface 〃 $\neq 0$

III. Net charge resides on the surface.

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IV. A conductor is an equi potential (For B.C. condition)

$\vec{E}_A^{\parallel} = \vec{E}_B^{\parallel}$ 

$\Downarrow$

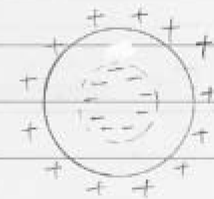
$V_{(a)} - V_{(b)} = 0$

V. $E_A^{\perp} - E_B^{\perp} \neq 0$. (perpendicular to the surface).

Note :

Chapeer 2.3.3 , poisson + Laplace Eq

$$\vec{E} = -\nabla V,\ \nabla \cdot \vec{E} = \rho/\varepsilon_0 \Rightarrow \nabla^2 V = -\rho/\varepsilon_0$$

+ Chapter 2.5.2 + 2.5.3. Induced charge. 

Force on a charged conducting surface.

* 

1. at $\underline{x}$ distance from the surface of conductor.

2. the charge density $\underline{\rho}$ & E-field $\underline{E}$

3. This is a poisson eq.

$$\nabla_x^2 V = \frac{d^2 V}{dx^2} = \frac{d}{dx}\left(\frac{-d}{dx} V\right) = -\rho/\varepsilon_0$$

$$\boxed{+\frac{d}{dx}(\vec{E}_x) = +\rho/\varepsilon_0}$$ [eq. of motion]

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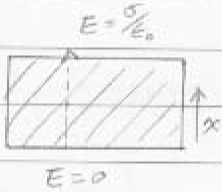
Ex: 1. Consider the charge at different distance.



2. The amount of charge between the plane $\underline{x \ \& \ x + dx}$.

$\Downarrow \rho dx$

The Force on this charge $\underline{\rho dx}$ is

$\vec{F} = q\vec{E}$, then $\vec{F} = \vec{E}(\rho dx)$

$\frac{d}{dx} E = \rho/\epsilon_0$

Then, the force $\vec{F} = \vec{E}(\epsilon_0 \frac{d\vec{E}}{dx}) dx$ (2.5.3 ? 錯 ?)

$$\vec{F} = \int_0^t \epsilon_0 E \frac{dE}{dx} \underline{\underline{dx}} = \epsilon_0 \int_{E=0}^{E=\sigma/\epsilon_0} E \, dE$$

$$= \epsilon_0 \cdot \frac{1}{2} E^2 \Big|_0^{\sigma/\epsilon_0} = \boxed{\frac{\sigma^2}{2\epsilon_0}} \hat{n}$$

$E = \frac{\sigma}{\epsilon_0}$, $E^2 = \frac{\sigma^2}{\epsilon_0^2}$

Electrostatic pressure

$\underline{P} = \frac{1}{2} \epsilon_0 E^2$ (tension)

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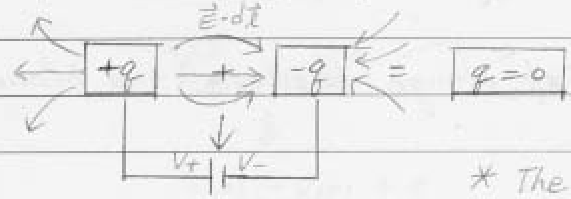
Chapter 2.5.4 Capacitors within a conductor

(电容)

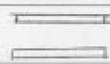


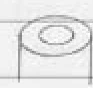
(deviation)

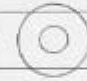
* The difference of potential is proportional to eleceric field. The ratio is defined as capacity $\underline{\underline{C}}$

$V_a - V_b = -\int \vec{E}\cdot d\vec{l}$

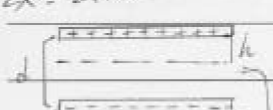
$\Delta V = \int_-^+ \vec{E}\cdot d\vec{l}$

Ex: Parallel - plate 

cylindrical tube 

two Hollow spherical shell 

Ex: 2.10



From Gaussian Law

The Gauss surface.

$\oint \vec{E}\cdot d\vec{A} = E\cdot A = q/\epsilon_0 \quad , \quad E = \frac{q}{A\epsilon_0}$

$V = -\int_-^+ \vec{E}\cdot d\vec{l} = E\cdot d = \frac{qd}{A\epsilon_0}$

$C \Rightarrow \frac{q}{V} = \frac{\epsilon_0 A}{d}$ (Capacity)

Work, $dw = v\,dq \quad , \quad w = \frac{1}{2}\sum_i V_i q_i$

$dw = \frac{q}{c}dq \quad , \quad w = \int_0^Q \frac{q}{c}dq = \frac{1}{2}\frac{Q^2}{c} = \frac{1}{2}cV^2$

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Capacitance $C \equiv \frac{Q}{V}$, How to calculate ?

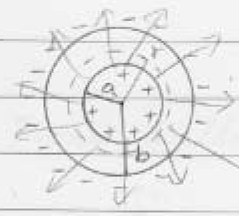
1. Compute $\vec{E}$: field (Gauss's Law)

2. Compute the potential V.

$$V = -\int \vec{E} \cdot d\vec{l}$$

3. $C = \frac{Q}{V}$, get the C.

If in case of cylindrical tube (Hollow) of radius inner a & outter b.



length L

1. Gauss Law ⇒ E - field.

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ Gauss surface

$\nabla \cdot \vec{E} = \rho/\epsilon_0$

$$\oint \vec{E} \cdot d\vec{A} = E \cdot (2\pi r L) = \frac{Q}{\epsilon_0} \quad , \quad \vec{E} = \frac{Q}{2\pi r L \epsilon_0} \hat{r} \ (\hat{n})$$

2. $V = -\int_a^b \vec{E} \cdot d\vec{l}$

$V = -\int_b^a \vec{E} \cdot d\vec{l}$ → $\boxed{\text{positive value}}$

$$= -\int_a^b \frac{Q}{2\pi\epsilon_0 L} \frac{1}{r} dr$$

$$= \frac{-Q}{2\pi\epsilon_0 L} \left[\ln b - \ln a \right]$$

(内為高电位)

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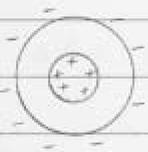
3. $C = \frac{Q}{V} = \left|\frac{Q}{V}\right| = \frac{Q}{\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$

Ex 2.11, Find the capacitance of two concentric metial shells with radius a & b.



1. Find the E - field

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{r}$$

2. Find the potential (Difference)

$$V = \Delta V = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_b^a \vec{E}\cdot d\vec{l}$$

$$= \frac{-Q}{4\pi\epsilon_0}\int \frac{1}{r^2}\,dr$$

$$= \frac{-Q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Big|_{小}^{大} = \frac{Q}{4\pi\epsilon_0 r}\Big|_{小}^{大}$$

$\Rightarrow \left(\frac{1}{大} - \frac{1}{小}\right) < 0, \quad \left(\frac{1}{小} - \frac{1}{大}\right) > 0$

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) > 0$$

then we can get $C = \left|\frac{Q}{V}\right| = \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}$

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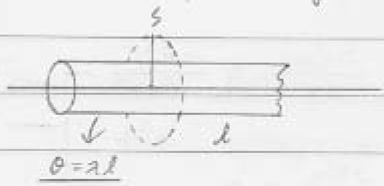
Ex: an finity long wire, charge density λ.



1. $\vec{E}$ - field, length l

$$\int \vec{E}\cdot d\vec{a} = E\cdot(2\pi s)l = \frac{Q}{\epsilon_0}$$

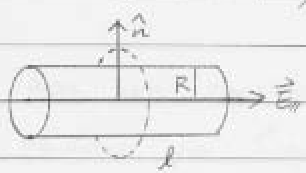
$$\vec{E} = \frac{Q}{2\pi l\epsilon_0}\frac{1}{s}\hat{s}$$

$$V = -\int \vec{E}\cdot d\vec{s} = -\int_a^s \frac{Q}{2\pi\epsilon_0 l}\frac{1}{s}ds$$

$$= \frac{-Q}{2\pi\epsilon_0 l}\cdot \ln\frac{s}{a} \quad 判别 \begin{matrix} >0 \\ <0 \end{matrix}$$

$$C = \frac{Q}{V} = \left|\frac{Q}{V}\right| = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{s}{a}\right)}$$

* From $\boxed{\text{Hollow}}$ cylindrical sphere. → conductor → metal



$$\underset{\text{Above}}{\vec{E}}\cdot 2\pi s l = \frac{Q_{enclosed}}{\epsilon_0} = \frac{2\pi R\sigma}{\epsilon_0}$$

$$\vec{E}_A^{\perp} - \vec{E}_B^{\perp} = \boxed{\frac{\sigma}{\epsilon_0}}, \quad \underset{\text{Above}}{\vec{E}} = \frac{R\sigma}{\epsilon_0 s}\hat{n}\ (\hat{s})$$

if s on the surface at $s = R$.

$$\vec{E}_A = \frac{\sigma}{\epsilon_0}\hat{n}, \quad \vec{E}_B = 0$$

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P.107 : 2.46 ; problem 2.47   2.42

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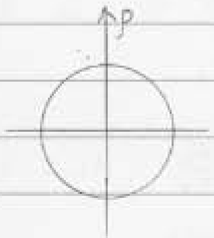
Ex 2.7 , Spherical shell / metal / conductor



potential $V_{out} = \frac{R^2\sigma}{\epsilon_0 r}$, $\frac{\partial V_{out}}{\partial r} = \frac{-R^2\sigma}{\epsilon_0 r^2}$

$V_{in} = \frac{R\sigma}{\epsilon_0}$, $\frac{\partial V_{in}}{\partial r} = 0$

$\longrightarrow \left(\frac{-\partial V_{out}}{\partial n}\right) - \left(\frac{-\partial V_{in}}{\partial n}\right) = \frac{R^2\sigma}{\epsilon_0 r^2}\hat{n}$

at $r=R$ on the surface

$\frac{R^2\sigma}{\epsilon_0 r^2} = \frac{\sigma}{\epsilon_0}\hat{n} = E_A^{\perp} - E_B^{\perp}$

prove $\frac{-\partial V_{out}}{\partial n} = \vec{E}_{above}$

~~B.C~~

$-\nabla V = \vec{E}$ $\frac{-\partial V_{in}}{\partial n} = \vec{E}_{Below}$

$\longrightarrow E_a^{\perp} - E_b^{\perp} = \frac{\sigma}{\epsilon_0}\hat{n}$.

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