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法蘭克-赫茲

散度：牛頓要推廣流體力學，對速度場研究
提供一個向量場隨時間變化的知識
(1) 純量場 $\xrightarrow{time}$ 稱為向量場的散度
(2) 向量場 $\xrightarrow{time}$ 稱為向量場的旋度
實驗的定義：Point (點). charge (電荷)
electron (−) hole (+) plasma (+).(−)

$\Delta V \to 0$
散度：① 描述該點的發散程度 (点＝單位体積無窮小)
② 流入或流出該點的流速場，我們稱為通量
③ 封閉空間 (closed path) $\oint \cdot ds$
④ 流入與流出的淨量，稱為通量 (flux) $= \Phi$)
定義出散度：單位通量 $d\Phi = \vec{v} \cdot d\vec{s}$，$\Phi = \oint \vec{v} \cdot d\vec{s}$
極限：如果所圍体積 ΔV 趨近於無限小
其通量與 ΔV 之比，稱為該點的
"電場的散度"。 $\nabla \cdot \vec{E} = \frac{\Phi}{\Delta V}$
向量場

A. $\nabla \cdot \vec{E} > 0$ 電力線由點向外擴散 
B. $\nabla \cdot \vec{E} < 0$ 〃〃〃向該點凝聚 
C. $\nabla \cdot \vec{E} = 0$ 〃〃〃只是通過該點 Pass through.

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旋度：通常稱向量場的旋度（遇到障礙物時該點的旋轉程度）

1. 環繞該點的封閉曲線 $\oint dl$ — the same
2. 該點上的流速場是否存在"環流"
3. 環流的方向 = 法向量 (Normal vector) $\hat{n}\cdot d\vec{S} = dA\cdot\hat{n}$
4. 環線上切線分量的平均值
5. 向量場 × 單位長度 = $\vec{E}\cdot d\vec{l}$ ➡ 環流的密度
   $J = \oint \vec{E}\, dl \Rightarrow \vec{\nabla}\times\vec{E} = 0$ Stoke's theorem
6. 如果線面積為 $\Delta\vec{S}$，其方向為法向量
   (3&4) $\hat{e} = \hat{n}$, if $\Delta\vec{S}\to 0$, 旋度定義為
   $\frac{J}{\Delta S}$，沿著 $\hat{e}$ 的方向　　Superposition theorem
7. 只要是向量場皆有"場的疊加特性"

The LaPlacian : The divergence of the Gradient of a scalar function is called the Laplacian.

* Scalar function : potential U, V
* If we set the Gradient of scalar function $= -\vec{\nabla}U$ or $-\vec{\nabla}U$ as $\vec{E}$
* Then the Divergence of E-field $\Rightarrow -\vec{\nabla}\cdot(\vec{\nabla}U) = -\nabla^2 U = 0$ Laplace
  $= -\nabla^2 U = \frac{\rho}{\varepsilon_0}$ Possion

Frank - Hertz experiment

The Divergency of $\vec{E}$

1. Compute the flux $\Phi = \int \vec{E} \cdot d\vec{s}$
2. Define the ratio of flux with volume $\frac{\Phi}{\Delta V} = \vec{\nabla} \cdot \vec{E}$

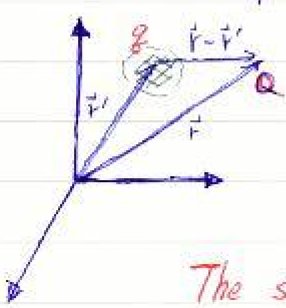
The curl of $\vec{E}$

1. Current density $J = \oint \vec{E}\, dl$
2. Define the ratio of J with ΔS as $\vec{\nabla} \times \vec{E} = \frac{J}{\Delta S}$

Chapter 2.1 Columb's Law

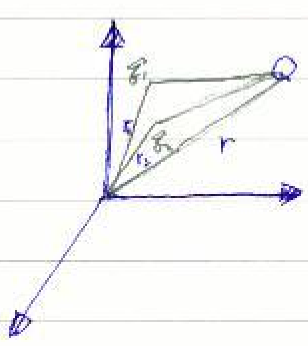
Using a test charge of Q in E-field
Measure the E-field of force lines.
Then define the force as

$$\vec{F} = \frac{q}{4\pi\epsilon_0 r^2} Q \hat{r} = \vec{E} Q \qquad \epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$



$$\vec{\imath} = \vec{r} - \vec{r}'$$

The superposition of $q(r)$



$$\vec{F} = \vec{E} Q = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-r')^2} Q$$

$$= \left[\frac{q_1}{4\pi\epsilon_0 (r-r_1)^2} + \frac{q_2}{4\pi\epsilon_0 (r-r_2)^2} + \frac{q_3}{4\pi\epsilon_0 (r-r')^2} \right] Q$$

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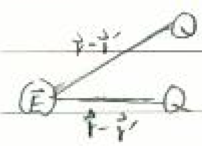
Charge element of $\vec{E}$ field can be described as three forms:

1. * The divergency $\vec{\nabla} \cdot \vec{E} = \frac{\Phi}{\Delta V} = \frac{\rho}{\varepsilon_0}$ [Columb + Divergency rule]
   * Define the charge density $\Rightarrow \rho = \frac{q}{V}$.
   $q = \int \rho(r') \, dv'$

2. * Differential function:
   $$d\vec{E}(\vec{r}-\vec{r}') = \frac{1}{4\pi\varepsilon_0} \frac{\rho(r') \cdot (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \quad ; \quad \hat{r}-\hat{r}' = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$

3. Intergral function of $\vec{E}$-field.



charge element

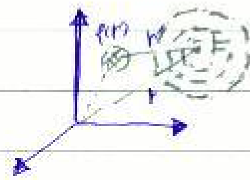
$$\vec{E}(\vec{r}-\vec{r}') = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \, dv' = \frac{1}{4\pi\varepsilon_0} \int \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \boxed{dq} \quad (2.1.5)$$

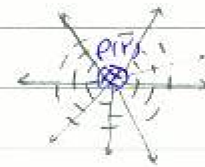
Force $\vec{F} = \vec{E} Q$

① $\rho(r') \, dv' = dq$ volume density
② $\sigma(r') \, da' = dq$ surface density
③ $\lambda(r') \, dl' = dq$ Line density
↳ Three Forms of charge element.

* charge elements : differential. Intergal Form. Divergency

* coordinate : $\rho(r')$





* $\vec{\nabla}_r \cdot \vec{E} = \frac{\rho(r)}{\varepsilon_0}$

➡ Divergency of $\vec{E}$ $\quad \vec{\nabla}_r \cdot \vec{E}(r-r')$

➡ Divergency of $\rho(r')$ $\quad \vec{\nabla}_{r'} \cdot \vec{E}(r-r') = -\vec{\nabla}_r \cdot \vec{E}$

* $\vec{\nabla}_r \cdot \vec{E}(r-r') = \frac{\rho(r)}{\varepsilon_0}$

↓

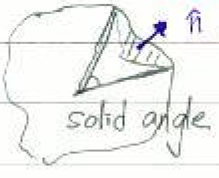
Divergency of $\vec{E}$ ↔ Gauss Law : 1. The existance of charge
2. proportional to charge
3. The existance of field line

↕ related to

The distribution of charge → Columbs Laws
$\vec{F} = Q\vec{E}$ ← 積分式

* Gauss' Law : Integral and Differential Forms

Case 1 : If we can prove $\rho > 0$ → point 1



$\hat{n}$. normal vector of ds (dA)

solid angle

Then the total flux

$$\oint \vec{E} \cdot d\vec{s} = \oint \vec{E} \cdot \hat{n}\, dA$$

$$\vec{E} \cdot \hat{n}\, dA = \frac{q}{4\pi\varepsilon_0} \frac{\hat{r} \cdot \hat{n}}{r^2} dA$$

Consider the projection of dA on a plane perpendicular to r & $d\Omega = dA/r^2$ $\quad \oint d\Omega = 4\pi$

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The integral Form

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\,\frac{\hat{r}\cdot\hat{n}}{r^2}\,dA = \frac{q}{4\pi\varepsilon_0}\,d\Omega$$

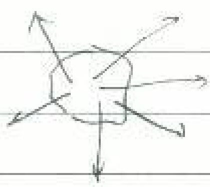
$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = \frac{q}{\varepsilon_0} = \Phi$$

$$\frac{\Phi}{V} = \frac{\rho}{\varepsilon_0}$$

⇒ Conclusion: The flux of $\vec{E}$ out of any closed surface containing a charge q is q/ε_0.

Then the divergency of $\vec{E}$ field can be described as the form of

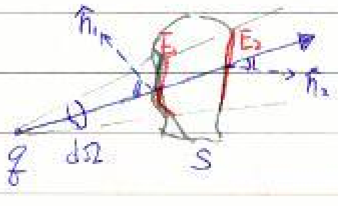
$$\vec{\nabla}\cdot\vec{E} = \frac{\Phi}{V} = \frac{\rho}{\varepsilon_0} \Rightarrow \text{Gauss' Law}$$



if $\rho > 0$. $\hat{r}\cdot\hat{n} = 1$ (out)

$\hat{r}\cdot\hat{n} = -1$ (in)

Case 2. $\rho = 0$, flux in & out through S



Using solid angle

$$d\Omega = dS_1\cos\theta_1/r_1^2 = dS_2\cos\theta_2/r_2^2$$
$$= \hat{n}_1\cdot\hat{r}_1\,dA_1/r_1^2 = \hat{n}_2\cdot\hat{r}_2\,dA_2/r_2^2$$

Because the $\pi-\theta_1$ between the direction of $\vec{E}$ and n_1 is obtuse (鈍角)

The flux through dS_1 is negative :

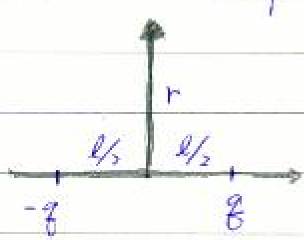
1. $\vec{E}_1 \cdot \hat{n}_1 dA_1 = E \hat{r}_1 \cdot \hat{n}_1 dA$,
   $\hat{r}_1 \cdot \hat{n} = \cos(\pi - \theta) = -\cos\theta$
2. $\vec{E}_2 \cdot \hat{n}_2 dA_2 = \frac{q}{4\pi r^2} (\hat{r}_2 \cdot \hat{n}_2) dA$

$r_1^2 d\Omega_1$ $\qquad$ $r_2^2 d\Omega_2$

$\therefore \vec{E}_1 \hat{n}_1 dA_1 + \vec{E}_2 \hat{n}_2 dA_2 = \frac{q}{4\pi} \frac{(-\cos\theta_1)}{r_1^2} dA_1 + \frac{q}{4\pi} \frac{\cos\theta_2}{r_2^2} dA_2$
$= \frac{q}{4\pi} d\Omega \left[-\cos\theta_1 + \cos\theta_2 \right]$

$\oint \vec{E} \cdot dS = 0 \qquad \therefore \rho = 0$

Problem 2.2. : Columb's Law
Find the E-field a distance r above the midpoint between two equal $+q, -q$.



We set the coordinate of
1. $+q \left(\frac{l}{2}\hat{x}, 0\hat{y}\right)$; $-q \left(\frac{-l}{2}\hat{x}, 0\hat{y}\right)$
2. Obs. $\vec{r} = z\hat{y}$
3. Using the superposition rule of E-field
   $\vec{E}_+ = +q \cdot (\vec{r} - \vec{r}_+) / 4\pi\varepsilon_0 |\vec{r} - \vec{r}_+|^3$
   $\vec{E}_- = -q \cdot (\vec{r} - \vec{r}_-) / 4\pi\varepsilon_0 |\vec{r} - \vec{r}_-|^3$

Then $\vec{r} - \vec{r}_+ = \left(-\frac{l}{2}\hat{x}, z\hat{y}\right)$
$\vec{r} - \vec{r}_- = \left(\frac{l}{2}\hat{x}, z\hat{y}\right)$
$|\vec{r} - \vec{r}_+| = |\vec{r} - \vec{r}_-| = \sqrt{z^2 + \frac{l^2}{4}}$

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Superposition rule :

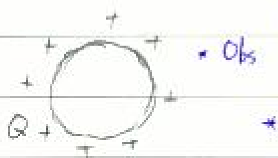
$$\vec{E} = \sum_i \vec{E}_i = \vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\varepsilon_0} \frac{1}{(z^2+\frac{l^2}{4})^{3/2}} \left[(z\hat{y} - \frac{l}{2}\hat{x}) - (z\hat{y} + \frac{l}{2}\hat{x}) \right]$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1}{(z^2+\frac{l^2}{4})^{3/2}} (-l\hat{x})$$

Then the $\vec{E}$-field is $\vec{E} = \frac{-q}{4\pi\varepsilon_0} \frac{l\hat{x}}{(z^2+l^2/4)^{3/2}}$

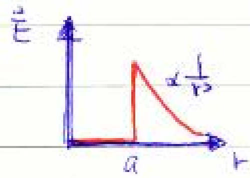
if $z \gg l$. then $\vec{E} = \frac{-1}{4\pi\varepsilon_0} \frac{ql}{r^3}\hat{x}$

Example. Calculate the $\vec{E}$-field of a sphere.



(continous media) + (symmetry)
isotropic field

* Case 1. $r > a$.

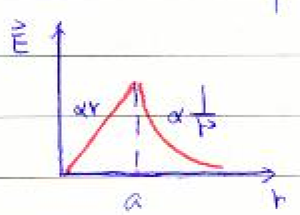


Using the flux rule

$$\oint \vec{E} \cdot d\vec{s} = \oint \vec{E} \cdot \hat{n}\, dA = \frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\hat{r}$$

if $r < a$. $\vec{E} = 0$

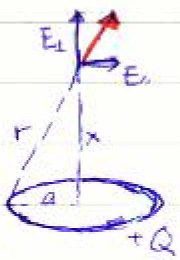
if a solid sphere $Q = \frac{4}{3}\pi a^3 \cdot \rho$



if $r < a$. $Q = \frac{4}{3}\pi r^3 \rho = \frac{r^3}{a^3} Q$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{rQ}{a^3}$$

Example : Write down the eqution of motion for a radius ring with a charge Q , $\lambda = \frac{Q}{2\pi a}$



$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{\lambda\, dl}{r^2} \quad ; \quad r^2 = x^2 + a^2$$

$$d\vec{E}_\parallel = 0 \; , \quad dE_\perp = \frac{\lambda}{4\pi\epsilon_0}\cdot\frac{dl\cdot\cos\theta}{r^2} \quad ; \quad \cos\theta = \frac{x}{r}$$

$$d\vec{E}_\perp = \frac{\lambda\, dl}{4\pi\epsilon_0}\cdot\frac{x}{r^3}$$

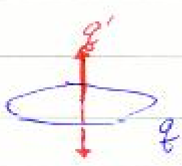
We know the integral of $\int dl = 2\pi a$
then

$$\int d\vec{E}_\perp = \frac{\lambda x}{4\pi\epsilon_0 r^3}\, 2\pi a \Rightarrow \vec{E} = \frac{qx}{4\pi\epsilon_0 (a^2+x^2)^{3/2}}\,\hat{z}$$

⇒ if $x << a$, ⇒ obs. ~ located at the center of ring

$$\vec{E} = \frac{qx}{4\pi\epsilon_0 a^3}$$

Eqution of motion for test charge $(-q')$



$$\vec{F} = -q'\vec{E} = -q'\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{qx}{a^3}\right)$$

From the Newton's Law

$$F = -q'E = m\ddot{x}$$

Eqution of motion : $m\ddot{x} + \frac{1}{4\pi\epsilon_0}\cdot\frac{qq'}{a^3}\,x = 0$

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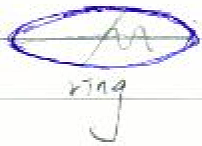
Case I

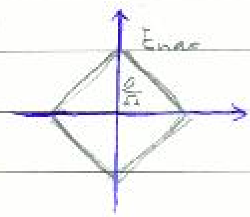


(1) $x \ll a$ $\ddot{x} + \omega^2 x = 0$ $\omega = \sqrt{\frac{qq}{4\pi\varepsilon_0 m a^3}}$

(2) Maximan E field

a. if $E = \frac{q}{4\pi\varepsilon_0} \frac{x}{(a^2+x^2)^{3/2}}$

b. Equation of motion parameter (x)
運動方程式軌跡



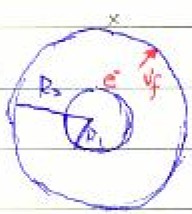
$$\frac{dE}{dx} = 0 = \frac{q}{4\pi\varepsilon_0} \frac{(a^2+x^2)^{3/2} - x \cdot \frac{3}{2}(a^2+x^2)^{1/2}(2x)}{(a^2+x^2)^{3}}$$

at $a^2 = 2x^2$ or $x = \pm a/\sqrt{2}$

Case II. Two sphere shell

inter radius $R_1 = 0.145$ m

outter radius $R_2 = 0.207$ m



if one electron esapes from inner to outter, what is the final speed of electron at outter shell.

Sphere shell E-field

$E_1 = -q_1/4\pi\varepsilon_0 R^2 = -Q/4\pi\varepsilon_0 R^2$

The force of E-field using one electron

$F = eQ/4\pi\varepsilon_0 R^2$ $(R \Rightarrow R_1 \rightarrow R_2)$

$\frac{1}{2} m v_f^2 = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \Rightarrow \frac{1}{2} m v_f^2 = \frac{|eQ|}{4\pi\varepsilon_0} \frac{1}{r} \Big|_{R_1}^{R_2}$

$v_f = 2 \times 10^7$ m/s

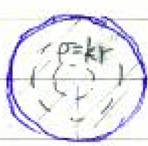
* Gauss' Law , $\vec{\nabla}_r \cdot \vec{E} = \frac{\rho(r)}{\varepsilon_0}$

a. if ρ is uniform → continous matter
ρ density is proportional to k
$\vec{\nabla} \cdot \vec{E} = kr'/\varepsilon_0$

b. if ρ is constant → charge
$\vec{\nabla}_r \cdot \vec{E} = \rho/\varepsilon_0$ $E \sim \frac{1}{r^2}$

c. if ρ is nonuniform → Integral function [Ch3.-4]
$\vec{\nabla}_r \cdot \vec{E} = \rho(r')/\varepsilon_0$

* Problem 2.14
Find the electric-field inside a sphere which carries a charge density proportional to the distance from the origin , $\rho = kr$



(1) $r > R$ → $E \sim \frac{1}{r^2}$
(2) $r < R$, Gauss' law
$\oint \vec{E} \cdot da = E \cdot 4\pi r^2 = Q_{enclosed}/\varepsilon_0$
$Q = \int \rho \, dv' = \iiint \rho' r'^2 dr' \sin\theta' d\theta' d\phi'$
$= 4\pi \int_0^r k r'^3 dr' = \pi k r^4$

Then the E-field $E \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = \pi k r^4$

⇒ The final E-field is $\vec{E} = \frac{k}{4\varepsilon_0} r^2 \hat{r}$

§ 2-2 The divergence of E

From the Gauss' law of Area-flux calculation
The E-field flux $\Phi \equiv \int_s \vec{E} \cdot d\vec{s}$ equal to $\frac{Q}{\varepsilon_0}$ $Q = \rho V$
So the description of $\vec{E}$ field can be written
as Columb's Law. $\vec{F} = Q\vec{E}$

$$\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \int \rho(r') \, d^3r'$$ 定題目常用

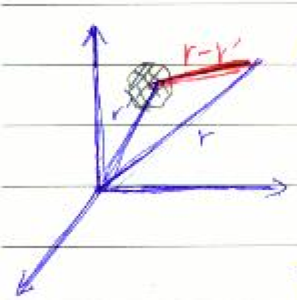
$\int \vec{E} \cdot \hat{n} \, da$

$$\int \vec{\nabla}_r \cdot \vec{E} \, dv = \frac{1}{\varepsilon_0} \int \rho(r') \, d^3r'$$ 推導常用

The Laplacian's equation (chapter 3)

* Thus we can compare the eqs. of left and right
So the resultan eq is

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ Gauss' 1st law



Write down the E-field directly

$$\vec{E}(r-r') = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{|r-r'|^2} \, dv' \quad (2.15)$$

The calculation of divergency of $\vec{E}$

$$\vec{\nabla}_r \cdot \vec{E}(r-r') = \frac{1}{4\pi\varepsilon_0} \int \vec{\nabla}_r \cdot \frac{\rho(r')}{|r-r'|^3} \, dv'$$

Then the relation

$$\vec{\nabla}_r \cdot \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \right) = 4\pi\, \delta^3(r-r') \quad \text{Dirac-Delta function}$$

if a point charge

Thus

$$\vec{\nabla}_r \cdot \vec{E}(r-r') = \frac{1}{4\pi\varepsilon_0} \cdot 4\pi \cdot \rho(r') = \frac{\rho(r)}{\varepsilon_0} \quad \text{when } r=r'$$

which Gauss' Law in different form of (2.14) as

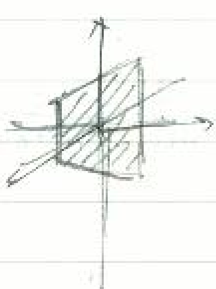
$$\int_v \vec{\nabla} \cdot \vec{E}\, dv = \int \frac{\rho}{\varepsilon_0}\, dv = \oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$$

§ 2.2.3 The Applications of Gauss' Law

The Gauss' Law is always true, but is not always useful, only in three types of candition for.

1. point charge of superposition rule
2. The symmetricmatters with spherical, cylindical and coordinates.
3. The integral of continous materical, using the integral function of charge density.

Example: A non-conductive plane of thickness d. with average volume of charge ρ what is the E-field of x $E(x)$



$\Phi = \int \vec{E} \cdot d\vec{A} = EA$ (using the flux rule), $\Phi = \frac{8}{\varepsilon_0} = \frac{\rho A}{\varepsilon_0}$

Thus the E-field can be represented as

$\vec{E}(x) = \rho x / \varepsilon_0$

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Case 2. The frequency of one-electron inside the plate.

➡ Write down the eqution of motion

$$\vec{F} = q\vec{E} = m\ddot{x}$$

$F = m\ddot{x}$, The eqution of motion as

$$\ddot{x} + \frac{\rho q}{m\epsilon_0} x = 0$$ compare with Hook's Law

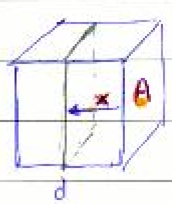
$$\ddot{x} + \frac{k}{m} x = 0 \ , \ \omega^2 = \frac{k}{m}$$

then the frequency is

$$\omega = \sqrt{\frac{\rho q}{m\epsilon_0}} = 2\pi f \ , \ \text{so the } f = \frac{1}{2\pi}\sqrt{\frac{\rho q}{m\epsilon_0}}$$

non-conductive：均勻分佈 ⇒ 算包含的体積

conductive：表面包含的面積 σA

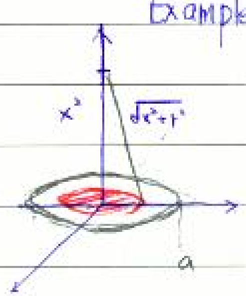


volume = $A x$

enclosed charge = $A \cdot x \cdot \rho$

$E(x)$ at x點上的大小

Example：Calculate the E-field at point p at disc of charge.



Well known：ring of charge $E_z = \frac{q}{4\pi\epsilon_0} \frac{z}{(a^2+z^2)^{3/2}}$

Surface density of charge σ . $\sigma \equiv \frac{Q}{\pi a^2}$

The unit of charge for surface $dq = 2\pi r \sigma dr$

$r = 0 \to \infty$

$$E_z = \int_0^a \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi r \sigma x}{(r^2+x^2)^{3/2}} dr$$

$$= \frac{2\pi x \sigma}{4\pi\varepsilon_0} \int_0^a \frac{r\,dr}{(r^2+x^2)^{3/2}}$$

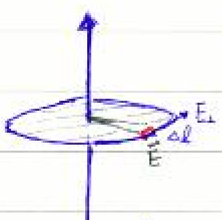
$$\int_0^a \frac{r\,dr}{(r^2+x^2)^{3/2}} = \int_{x^2}^{a^2+x^2} \frac{\frac{1}{2}d(r^2+x^2)}{(r^2+x^2)^{3/2}}$$

$$\Rightarrow \int_{x^2}^{a^2+x^2} \frac{\frac{1}{2}dy}{y^{3/2}} = \frac{1}{x} - \frac{1}{\sqrt{a^2+x^2}}$$

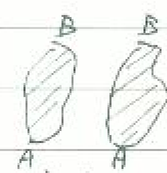
The E-field at point p is $E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]$

§ 2.2.4 The curl of $\vec{E}$-field
向量場

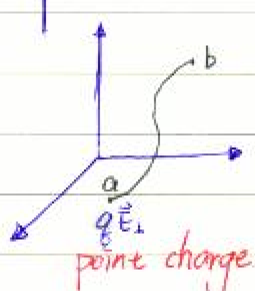
What is the curl?



$$\frac{\sum E_{\perp i} \cdot \Delta l_i}{\Delta S} = \text{The curl of } \vec{E}$$



$\vec{\nabla} \times \vec{E} = 0$



if an E-field along the path from point $a \to b$

if the point charge, the $\vec{E}$-field can be written as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

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* Then the line integral

$$q\left[\sum_i E_{1i}\cdot\Delta l_i\right] \Rightarrow \oint \vec{E}\cdot d\vec{l}$$

Sum ⇒ Integral] → the existance of closed path!

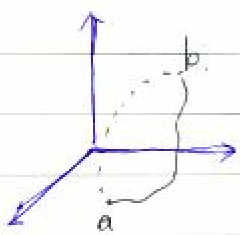
$\int_a^b \vec{E}\cdot d\vec{l}$. In spherical coordinate.

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

$$= \int_a^b \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{r}\cdot\hat{r}\,dr$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

* Special case : If the same areas and a closed path inside.



$$\oint \vec{E}\cdot d\vec{l} = \int_a^b \vec{E}\cdot d\vec{l} + \int_b^a \vec{E}\cdot d\vec{l} = 0$$

* Stoke's theorem ?

$$\sum_i \vec{E}_{1i}\cdot d\vec{l}_i = \sum \vec{\nabla}\times\vec{E}_i\cdot\Delta\vec{S}$$

$$\int \vec{E}\cdot d\vec{l} = \int \vec{\nabla}\times\vec{E}\cdot d\vec{S}$$

$$\oint \vec{A}\cdot d\vec{l} = \int \vec{\nabla}\times\vec{A}\cdot d\vec{S}$$

Problem 2.19 Calculate $\vec{\nabla} \times \vec{E}$ directly

From Eq. 2.8

if $\vec{E}$-field $\Rightarrow \vec{E}(\vec{r}-\vec{r}') = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|r-r'|^2}\, dv'$

then $\vec{\nabla}\times\vec{E}$. if in spherical coordinates

$$\vec{\nabla}_r \times (r^n \hat{r}) \Rightarrow \vec{\nabla}_r = \frac{1}{r^2}\frac{\partial}{\partial r} r^2$$

if $f(r)$ function can be presented as $r^n \hat{r}$
So the curl of f is

$E(r) \sim \frac{1}{r^2}$

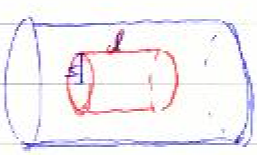
$$\boxed{\begin{aligned} \vec{\nabla}\times\vec{E} &= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{1}{r^2}\hat{r} \\ &= \frac{1}{r^2}\frac{\partial}{\partial r} 1\,\hat{r} \\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\hat{r} \\ \text{X} \quad &= 0 \end{aligned}}$$

$$\vec{\nabla}_r \times f(r) = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \cdot r^n \hat{r} = \frac{1}{r^2}(2+n)\, r^{n+1}\hat{r} = (2+n)\cdot r^{n-1}\hat{r}$$

Special case for if $n=-2$

$$\vec{\nabla}_r \times f(r) = (2+n)\, r^{n-1}\hat{r} = 0\cdot r^{-2-1}\hat{r} = 0\,\frac{1}{r^3}\hat{r} = 0$$

Example 2.3 A long cylindrical carries a charge density that is proportional to the distance from the axis $\rho = ks$. k = constant



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$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$Q_{enclosed} = \int \rho \, dv' = \int ks' \, s' ds' d\phi' dz'$$

$d\phi \rightsquigarrow 0 \to 2\pi$

$dz \rightsquigarrow 0 \to l$

$$= 2\pi k l \int_0^s s'^2 ds'$$

$$= \frac{2}{3} \pi k l s^3$$

$$|\vec{E}| \, 2\pi s l = \frac{2}{3\epsilon_0} \pi k l s^3$$

$$\Rightarrow \vec{E} = \frac{k}{3\epsilon_0} s^2 \hat{s}$$