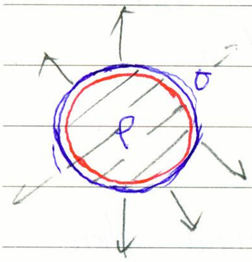
If in B.C., E_n is the normal component of $\vec{E}$ out of volume V

$$E_n = \frac{\sigma}{\epsilon_0}, \quad W = -\frac{1}{2}\int V\sigma\, d\vec{s} + \frac{\epsilon_0}{2}\int \vec{E}\cdot\vec{E}\, dv' + \frac{1}{2}\int \sigma V\, ds$$

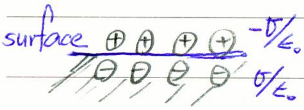


Hence the work

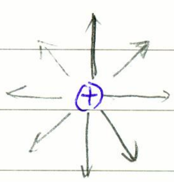
$$W \equiv \frac{\epsilon_0}{2}\int \vec{E}\cdot\vec{E}\, dv' = \frac{\epsilon_0}{2}\int E^2 dv' = \int u\, dv'$$

u: energy density $= \frac{1}{2}\epsilon_0 E^2$

$$W = mc^2 \Rightarrow W/mc^2 = \text{work}/\text{charge}$$



We can not apply $W = \frac{\epsilon_0}{2}\int E^2 dv'$ to a point charge for it shows that energy of a point charge is

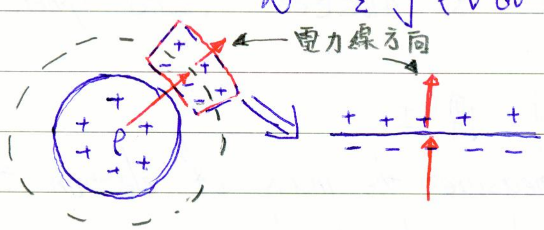


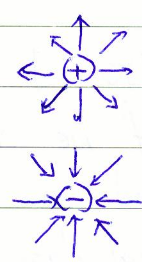
$$W = \frac{\epsilon_0}{2}\int_0^r \int_0^{\frac{\pi}{2}} \int_0^a \left(\frac{q}{4\pi\epsilon_0 r^2}\right)^2 r^2 dr \sin\theta\, d\theta\, d\phi$$

$$= \frac{q^2}{8\pi\epsilon_0}\int_0^\infty \frac{1}{r^2} dr = \infty$$

10/9 Maxwell's comments

$$W = \frac{1}{2}\int \rho V dv' + \frac{1}{2}\int \sigma V ds = \frac{1}{2}\int \rho V dv'$$





It integral over the charge distribution.
Is the energy stored in the charge?
or it stored in the field?

Feynman: The energy is located in space,
where the electric field is.

Einstein model

Find the energy of a uniformly charged sphere of radius R.

$$dq = \rho dv' = \rho \left[d\left(\tfrac{4}{3}\pi r^3\right) \right] = 4\pi \rho r^2 dr$$

Then the dW is

$$dW = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\frac{4}{3}\pi r^3 \rho}{r} \, dv$$

$$W = \int dW = \frac{4\pi \rho^2}{15\varepsilon_0} R^5$$

Electron Mass then can be calculated as

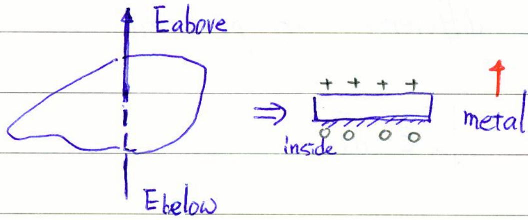
$$m_e = \frac{W}{c^2} = \frac{3}{5} \cdot \frac{q^2}{4\pi\varepsilon_0 R c^2} \quad \text{(solid sphere)}$$

* 1980 丁肇中博士

If we can measure the mass of electron 9.11×10^{-31} kg.
And we didn't know the distribution, we can calculate the radius of electron

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m c^2} \doteqdot 2.82 \times 10^{-15} \text{ m} \quad (\text{Exp. } 1 \times 10^{-15} \text{ m})$$

§ 2.5.3 Surface charge & the force on a conductor.



$\Rightarrow \sigma \hat{n}$

$$\vec{E}_a - \vec{E}_b = \frac{\sigma}{\varepsilon_0} \hat{n}$$

Introduce the BC.

* Because the force inside a conductor is zero.
If in B.C. condition, requires that the E-field outside $\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n}$, and the potential.

$$\boxed{\frac{\partial V}{\partial n} = \frac{-\sigma}{\varepsilon_0}}$$

* There is a problem, for the E-field is discontinous at a surface charge. The force per unit area (ΔA) should be the average of two

$$\boxed{f = \frac{F}{A} = \frac{qE}{A} = \sigma E}$$

$$\vec{f} = \sigma \vec{E}_{total} = \sigma \left[\frac{1}{2} (\vec{E}_{above} + E_{below}) \right]$$

$\vec{E} = \sigma/\varepsilon_0 \, \hat{n}$, $\sigma = \varepsilon_0 \vec{E} \cdot \hat{n}$

$$= \frac{1}{2} \sigma \vec{E}_{above} = \frac{\sigma^2}{2\varepsilon_0} \hat{n}$$

then $f = \frac{1}{2} \varepsilon_0 \vec{E}^2$

Is called the outward electrostatic pressure

$$P = \frac{1}{2} \varepsilon_0 E^2$$

2.35
2.47
Mid Exam 11/12 AM 10~12. Chapter 1.2 2.37

§ 2.5.4 Capacitors

* We can find the difference of potential

$$V = V_+ - V_- = -\int_-^+ \vec{E} \cdot d\vec{l}$$

* Parallel-plate

if we apply the Gauss' law and the ΔV

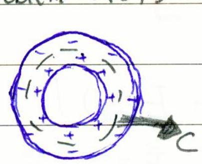
$$\oint \vec{E} \cdot d\vec{A} = E \cdot A = q/\varepsilon_0 \quad . \quad Ed = \Delta V$$

We can change the potential difference which is described below

$$C = \frac{q}{\Delta V} = \frac{\varepsilon_0 A}{d} \quad \text{(form factor)}$$

Example : cylindrical tube : length L, charge density λ

(Problem 2.39)



① $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \Rightarrow E \cdot 2\pi r L = q/\varepsilon_0$

$$E = \frac{q}{2\pi\varepsilon_0 r L} \rightarrow \text{find } \vec{E}\text{-field}$$

② The difference of potential

$$\Delta V = -\int \vec{E} \cdot d\vec{r} = -\int_a^b \frac{q}{2\pi\varepsilon_0 L r} dr = -\frac{q}{2\pi\varepsilon L} \ln\frac{b}{a}$$

So the ratio of $q/\Delta V$ is the capacitor $C = \left|\frac{q}{\Delta V}\right|$

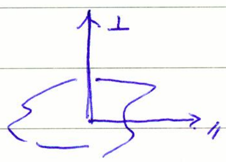
$$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \quad \text{if } b > a$$

$$C = -\frac{2\pi\varepsilon_0 L}{\ln(b/a)} \quad \text{if } b < a$$

Example 2.4 + Example 2.5 + Problem 2.30

An infinite plane carries a uniform surface charge density σ

(1) Find the E-field



From B.C.

$E_A^{\parallel} = E_B^{\parallel}$. $\vec{E}_A^{\perp} - \vec{E}_B^{\perp} = \frac{\sigma}{\varepsilon_0}\hat{n}$

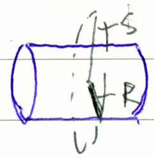
$\vec{E} = -\vec{\nabla}V \Rightarrow (\vec{E}^{\perp} \neq 0) \Rightarrow E^{\perp} = -\frac{\partial}{\partial r}V\hat{n}$

$\Rightarrow -\frac{\partial V}{\partial n}\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}$

Then the $\oint \vec{E}\cdot d\vec{a} = Q/\varepsilon_0$

$E = \frac{\sigma}{2\varepsilon_0} \times 2 = \frac{\sigma}{\varepsilon_0}$

(2) For a Hollow cylindrical sphere

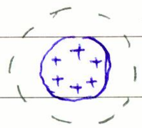


$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = E\cdot(2\pi s)\,l$

$Q_{enc} = (2\pi R\, l)\cdot\sigma$

Then $\vec{E} = \frac{R\sigma}{s\varepsilon_0}\hat{n}$

13) Ex 2.11 Find the capacitance of two concentric metal shells



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{r}$$

$$\Delta V = -\int_b^a \vec{E} \cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0} \int_b^a \frac{1}{r^2} dr$$

$$= \frac{Q}{4\pi\varepsilon_0} \cdot \left(\frac{1}{a} - \frac{1}{b}\right)$$

The ratio of $Q/\Delta V = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right)$

* Electro magnetism $\leftrightarrow$ Newton

Unit	q charge	$\leftrightarrow$	x	displacement
	I current	$\leftrightarrow$	v	volecity
	$\frac{1}{C}$ capactor	$\leftrightarrow$	k	coupling constant
	V voltage	$\leftrightarrow$	$\vec{F}$	force

Chapter 3. Special Techniques.

§3.1 Laplace's Equation

(From the continous equation of $\vec{E}$-field and potential)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho'}{|\vec{r}-\vec{r}'|} dv'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho'}{|\vec{r}-\vec{r}'|} dv'$$

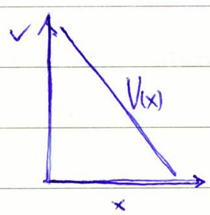
→ We can get the relation with $\boxed{\nabla^2 V = -\rho/\varepsilon_0}$ differential equation.

(Ps. Green function ⇒ 解決空間上点電荷分佈問題)

Where, There may be plenty charge elsewhere, if we confining in chargelse. the Possion eq. will reduce to Laplace's eq.

* For <u>One-dimensional</u> Laplace's eq.

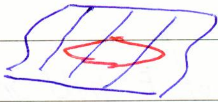
We propose the V depends on only one varible x



$$\frac{d^2V}{dx^2} = 0 \rightarrow \text{The general solution is } V(x) = mx + b$$

(I) is a kind of average instruction.
(II) no local minimun or maximun.

* For two-dimensional



$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) V = 0$. It's a partial eq of (X.Y)

Average definition : $V_{AVE} = \frac{1}{2\pi R} \oint_{circle} V \, dl$

A. It's the average of those around the point

B. V has no local minimum and maximum occurs at Boundaries.

* Laplace condition :

Find out the electrostatic boundary condition and value.

* In a region where $\rho = 0$ for $\nabla^2 V = -\rho/\varepsilon_0$.

Then ∇^2 is a scalar operator, different forms in different coordinate system.

A. ∇^2 : for Rectangular coordinate (X. Y. Z)

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

B. ∇^2 : for spherical polar coordinate ($r . \theta . \phi$)

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

C. ∇^2 : for cylindrical polar coordinate ($r . \theta . z$)

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$