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$$\text{If} \quad \oint \vec{E}\cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$Q_{enclosed} = \int \rho \, dv' = \int ks' \, s' ds' d\phi \, dz'$$

$$d\phi \rightsquigarrow 0 \to 2\pi$$
$$dz \rightsquigarrow 0 \to l$$

$$= 2\pi k l \int_0^s s'^2 ds'$$

$$= \frac{2}{3}\pi k l s^3$$

$$|\vec{E}| \cdot 2\pi s l = \frac{2}{3\epsilon_0}\pi k l s^3$$

$$\Rightarrow \vec{E} = \frac{k}{3\epsilon_0} s^2 \hat{s}$$

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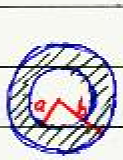
* Problem 2.15

A hollow spherical shell carries charge density $\boxed{\rho = \frac{k}{r^2}}$ in the region $a \le r \le b$.



a. $r<a$ * If $r<a$, using Gauss' Law = flux
Enclosed charge $Q_{enclosed} = 0$
then E-field $= 0$

b. $r > b$ * if $r > b$. enclosed charge is

$$Q = \int_a^b \rho' dv'$$

$$= \int_a^b \frac{k}{r'^2} r'^2 dr' \underbrace{(d\theta' \sin\theta' d\phi')}_{4\pi}$$

$$= 4\pi k \int_a^b dr' = 4\pi k (b-a)$$

Gauss' method ⇒ calculate flux

$$\vec{E} \cdot \vec{A} = \int \vec{E} \cdot d\vec{A} = \vec{E}\, 4\pi r^2 \hat{r}$$

$\vec{A}$ ⇒ along $\hat{r}$-direction.

$$\frac{Q}{\epsilon_0} = \int \vec{E} \cdot d\vec{A} \Rightarrow \vec{E}\, 4\pi r^2 = \frac{1}{\epsilon_0} 4\pi k (b-a) \hat{r}$$

$$\vec{E} = \frac{1}{\epsilon_0 r^2} k (b-a) \hat{r}$$

c. $a < r < b$ * if $a < r < b$ then enclosed charge

$$Q = \int_a^r \rho' dv' = 4\pi k (r-a)$$

$$\vec{E} = \frac{1}{\epsilon_0 r^2} k (r-a) \hat{r}$$

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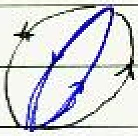
§ 2.3 Electric Potential

2.3.1 Introduction to Potential



Because $\oint \vec{E} \cdot d\vec{l} = 0$

The line integral of $\vec{E}$ from $\left.\begin{matrix} a \text{ to } b \\ b \text{ to } a \end{matrix}\right]$ is
is the same path.

(1) So the line integral is independent of path.

$\boxed{\text{Scalar}}$ $V(r) = -\int_{O}^{r} \vec{E} \cdot d\vec{r}$

(2) Definition of reference point to r $\left(\begin{matrix} 觀察者 & Obs \\ 原點 & Ref \end{matrix}\right)$
It's called electric potential.
Such as $\boxed{V(a) - V(b) \equiv 計算梯度}$

$$V(b) - V(a) = \left[-\int_{o}^{b} \vec{E} \cdot d\vec{r} \right] - \left[-\int_{o}^{a} \vec{E} \cdot d\vec{r} \right]$$

$$= -\left[\int_{o}^{b} \vec{E} \cdot d\vec{r} + \int_{a}^{o} \vec{E} \cdot d\vec{r} \right]$$

$$= -\int_{a}^{b} \vec{E} \cdot d\vec{r} = \boxed{\int_{a}^{b} \vec{\nabla} V \cdot d\vec{r}} = V(b) - V(a)$$

* The defintation of the Gradient of Potential (Scalar)

$$-\int \vec{E} \cdot d\vec{r} = \boxed{\int \vec{\nabla} V \cdot d\vec{r}} \qquad \nabla_r \equiv \frac{\partial}{\partial r}$$

* Then $\vec{E}$ field is $-\vec{\nabla} V$, it says that Gradient of a Scalar potential.

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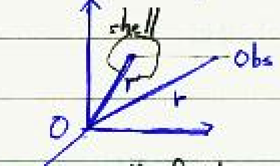
Chapter 1.6 Helmholtz theorem
Chapter 2.3 Electric field

We can define the vector field (divergency & curl)
$\boxed{\vec{E}, \vec{B}, \vec{F}}$ must be the sum of two parts. of
$= -\vec{\nabla}V - \vec{\nabla}\times\vec{A}$

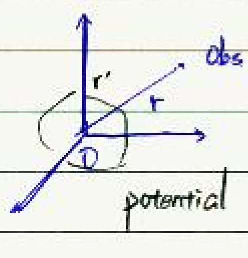
- one scalar potential ➡ The Gradient of potential
  ➡ $\boxed{\vec{E} = -\vec{\nabla}V}$
- one vector's curl ➡ The curl of vector potential
  ➡ $\boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$

Example 2.6
Find the potential inside and outside a spherical shell of radius R, where carries a uniform surface cha



$\vec{E}$ - field

$\Downarrow$



potential

$* \ r > a \qquad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q\hat{r}}{r^2}$

$$V(r) = -\int \vec{E}\cdot d\vec{l} = -\int \frac{q\hat{r}}{4\pi\epsilon_0 r^2}\cdot d\vec{r}$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}$$

$* \ r < a.$

$$V(r) = \frac{-1}{4\pi\epsilon_0}\left[\int_{\infty}^{R}\frac{q}{r'^2}dr' - \int_{R}^{r}(0)\,dr'\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

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2.3.2 Comments on Potential

(I) The name : potential or potential energy $V . U . \Phi$

$$-\int \vec{E} \cdot d\vec{l} = \int \vec{\nabla} V \cdot d\vec{l}$$

(II) Advantage of potential formulation

$\vec{E} = -\vec{\nabla} V$

$\vec{\nabla} \times \vec{E} = 0 \Rightarrow$ potential $\Rightarrow \frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \Rightarrow$ No vector potential

(III) The reference point Q

(IV) Potential obeys the superposition principle.

If $\vec{F} = \Sigma \vec{F}_i$ (Hemholtz)
the Integrating from the common reference point r it follows that the potential also satisfies $V = \Sigma V_i$

(V) Unit of potential

A joule per columb is called a volt.
$U = -\vec{F} \cdot \vec{r}$ = Newton · meter
$\vec{F} = q\vec{E} \Rightarrow \vec{E} = \vec{F}/q$ = Newton / columb
Then $\int \vec{E} \cdot d\vec{l}$ = Newton · meter / columb $= -\int \vec{\nabla} V \cdot d\vec{l}$
Voltage = Joule / columb.

§ 2.3.3 Poisson's Equation and Laplace's Equation.

§ 2.1 → 2.3 Introduced two things of curl and divergency of E-field.

$\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\varepsilon_0}$. $\oint \vec{E}\cdot d\vec{l} = 0$ if non-closed path.

§ 2.3.2 Introduced the potential $-\int \vec{E}\cdot d\vec{l}$

Curl

$\Rightarrow \vec{E} = -\nabla V = \frac{\rho}{\varepsilon_0}$

The Gradient of potential, then combine divergency theorem

We get $\vec{\nabla}\cdot(-\vec{\nabla}V) = \frac{\rho}{\varepsilon_0}$, we said the Divergency of $\vec{E}$ is the Laplacian (作用) of V is known as Possion eq.

$\boxed{\frac{\partial}{\partial x}\ ,\ \nabla} \Rightarrow \frac{\partial^2}{\partial x^2} \Rightarrow \frac{\partial}{\partial x}\boxed{\left(\frac{\partial}{\partial x}\right) = ?}$

page 376. $\nabla^2(\ ) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(\ )$ (9.41)

if $\rho = 0$ is known as Laplacian equation.

$\nabla^2 V = 0$ Laplacian equation.

$\nabla^2 V = -\rho/\varepsilon_0$ Possion equation.

$\vec{\nabla}\times\vec{E} = 0$ Curl 獨立性

$\Downarrow$ 單數性

$-\int \vec{E}\cdot d\vec{l} = V$ 相加性.

$\Downarrow$

$\vec{\nabla}\cdot(-\vec{\nabla}V) = \frac{\rho}{\varepsilon_0}$ Divergency

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§ 2.3.4 The potential of a Localized Charge distribution.

Target : Isolated ↔ continous

Prove : superposition rule.

* For a single charge can be written as

$$\int \vec{E}\cdot d\vec{l} = \frac{1}{\varepsilon_0}\Sigma Q_i \Rightarrow V(r) = \frac{-1}{4\pi\varepsilon_0}\Sigma \frac{q_i}{r_i}$$

* For continous charge can be written as

$$\int \vec{E}\cdot d\vec{l} = \frac{1}{\varepsilon_0}\int \rho' dv' \Rightarrow V(r) = \frac{-1}{4\pi\varepsilon_0}\int_{\infty}^{r} \frac{q}{r^2} dr'$$

So the volume charge is

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r)}{r} dv'} \quad (2.30)$$

Prove the potential Expansion ⇒ chapter 4
can be obtained the sum of single charges.

if $r >> r'$, $V(r) = \frac{\vec{p}\cdot\vec{r}}{r^3}$,

where $\vec{p} = \int \rho(r')\vec{r}' dr'$

E - Polarization.

§ 2.3.5 Summary : Electrostatic Boundary Conditions

18 century , Gray

⇒ a. conductor ➡ surface  ← charge. 只存在表面

b. Dielectric ➡ volume  ← charge 分布体质内