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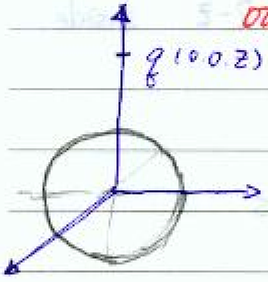
§3.1.4 Laplaces Equtation in Three Dimensions.

In general case of a charge q locate at z outside of a sphere.



$$V_{average} = \frac{1}{4\pi R^2}\int V\,da$$

（有限條件.面積較好應用）
（唯一解.邊界條件.有限面積）

Then the r can be represented as

$$r^2 = z^2 + R^2 - 2zR\cos\theta$$

$$V_{average} = \frac{1}{4\pi R^2}\int \frac{q}{4\pi\varepsilon_0}\cdot\frac{R^2\sin\theta\,d\theta\,d\phi}{\sqrt{z^2+R^2-2zR\cos\theta}}$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{2zR}\sqrt{z^2+R^2-2zR\cos\theta}\,\Big|_0^{\pi}$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{2zR}\left[(z+R)-(z-R)\right] = \frac{q}{4\pi\varepsilon_0 z}$$

Problem 3.1
Find the average potential over a spherical surface of riduas R due to a point charge q located inside $(z<R)$

If $z<R$. $V_{average} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2zR}\right)\left[(z+R)-(R-z)\right]$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R} \sim \frac{1}{R}$$

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if $\sqrt{z^2+R^2-2zR\cos\theta}$ if $\theta=0$ $\sqrt{(z-R)^2} = z-R$

π $\sqrt{(z+R)^2} = z+R$

$\sqrt{(z-R)^2} > 0$ if $z>R$ then $z-R$ outside

$z<R$ then $R-z$ inside.

Problem 3.3

Laplace's eq. in spherical coordinates

" " " cylindrical "

for V dependent only nor

if $\nabla^2 V = 0$, $\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} V = 0$ spherical coord.

then $\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} V = 0$

$r^2 \frac{\partial}{\partial r} V = C'$, So that $\boxed{V = \frac{C'}{r^2}}$

So the solution for spherical Laplace's eq. is

$\frac{\partial}{\partial r} V = \frac{C'}{r^2}$, the

$\boxed{V = -\frac{C}{r} + k}$. For C, k are constants.

if in cylindrical coordinate

$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} V = 0$

$\frac{\partial}{\partial r} r \frac{\partial}{\partial r} V = 0$

$r\frac{\partial}{\partial r} V = C'$

then the solution of cylindrical Laplace's eq. is

$\frac{\partial}{\partial r} V = \frac{C'}{r}$

$\Rightarrow \boxed{V = C' \ln r + k}$

* Solution of Laplace's eq.

The solution to Laplacian eq. have two basic properties given below:

A. Principle of superposition

If $V_1 . V_2 . V_3 \cdots V_n$ are all solution of Laplacian equation, then $V = C_1 V_1 + C_2 V_2 + \cdots$

$$V = \sum_n C_n V_n$$

V_n is known

C_n is variable.

B. Uniqueness Theorem

If two solutions of Laplacian eq. that satify the same $\boxed{\text{boundary conditions}}$ → $\frac{\partial \Phi}{\partial n}$

Prove : Two solutions of Φ_1 and Φ_2 in V with the same B.C.

If : $\phi = \Phi_1 - \Phi_2$ then $\nabla^2 \Phi_1 = \nabla^2 \Phi_2 = 0$

Because $\nabla^2 \Phi = 0$ ⇒ So the $\nabla^2(\Phi_1 - \Phi_2) = 0$

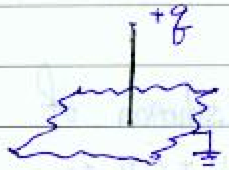
Furthermore, either ϕ or $\frac{\partial \phi}{\partial n}$ vanishes on the boundaries. So the B.C. are.

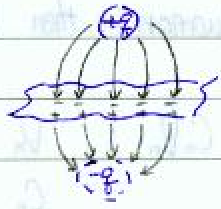
① $\phi = \Phi_1 - \Phi_2$, $\phi = \Phi_1 - \Phi_2 = c$

② $\frac{\partial \phi}{\partial n} = \frac{\partial \Phi_1}{\partial n} - \frac{\partial \Phi_2}{\partial n} = 0$

§ 3.2 Image method

Question : a point charge q is located a distance d above an $\boxed{\text{infinite}}$ ground conducting plane.





$\sigma = ?$ then $\int \sigma \, da = -q$.