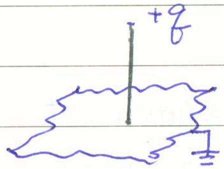
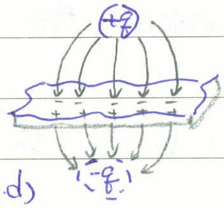
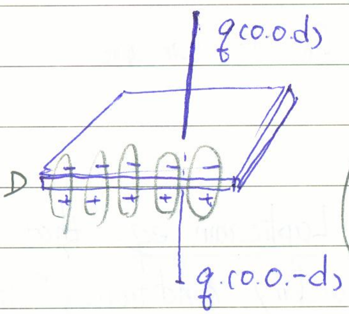
§ 3.2 Image method

Question : a point charge q is located a distance d above an $\boxed{\text{infinite}}$ ground conducting plane.





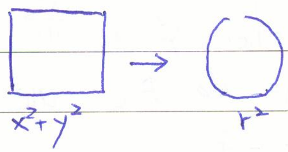
$\sigma = ?$ then $\int \sigma\, da = -q$.



(if D 夠大，則需考慮 ⊕
稱為 dipole.)

Case. In the region $z>0$. $V=0$ when $z=0$
$V \to 0$ for from the charge $x^2+y^2+z^2 \gg d$

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{-q}{\sqrt{x^2+y^2+(z+d)^2}}\right]$$



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* Case 1. What is the potential in the region above the plane V

2. The surface charge density σ

3. The charge $-q$

$$E_n = \sigma/\epsilon_0 \Rightarrow \boxed{\sigma = -\epsilon_0 \frac{\partial V}{\partial n}} \qquad E_n = -\nabla_n V$$

$$\frac{dV}{dz} = \frac{1}{4\pi\epsilon_0} \cdot \left[\frac{-q(z-d)}{[x^2+y^2+(z-d)^2]^{3/2}} + \frac{q(z+d)}{[x^2+y^2+(z+d)^2]^{3/2}} \right]$$

So $\sigma(x,y) = \dfrac{-qd}{2\pi(x^2+y^2+d^2)^{3/2}}$ is (x,y) dependent.

$$x^2+y^2=r^2, \quad da = dx\,dy \Rightarrow r\,dr\,d\phi = 2\pi r\,dr$$

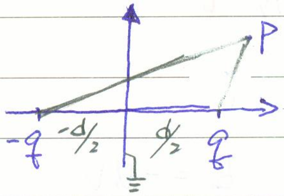
$$\sigma(r) = \frac{-qd}{2\pi(r^2+d^2)^{3/2}}$$

The total induced charge

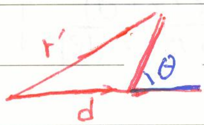
$$Q = \int \sigma\, da = \int_0^{2\pi}\int_0^{\infty} \frac{-qd}{2\pi(r^2+d^2)^{3/2}}\, r\,dr\,d\phi$$

$$= \frac{qd}{\sqrt{r^2+d^2}}\Bigg|_0^{\infty} = -q$$

* Point charge near an infinite ground conducting plane



Q : The potential at point P



$$\vec{r}' = \vec{d} + \vec{r} = \vec{d} + (\vec{r}\sin\theta + \vec{r}\cos\theta)$$

$$\Rightarrow d^2 + r^2 + 2d \cdot r\cos\theta$$

$$V_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{-q}{r'}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{-q}{\sqrt{r^2+d^2+2rd\cos\theta}}\right]$$

Case : Notice that $V=0$ where $r=r'$
P locates at z-axis.

Electric field E at P point are given by the component of $-\nabla V$

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r^2} - \frac{q(r+d\cos\theta)}{(r^2+d^2+2rd\cos\theta)^{3/2}}\right]$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q\,d\sin\theta}{(r^2+d^2+2rd\cos\theta)^{3/2}}\right]$$

If check the σ at $z = -\frac{d}{2}$

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$$\sigma = -\varepsilon_0 \frac{\partial V}{\partial z} = \varepsilon_0 \left(E_r \cos\theta - E_\theta \sin\theta\right)_{r=r'} = -\frac{qd}{4\pi r^3}$$

The total charge Q at $r=r'$, $z=-d/2$

$$Q = \int \sigma \cdot 2\pi r\,dr = -q.$$