

§ 6.4.2 Ferromagnetism 鐵磁

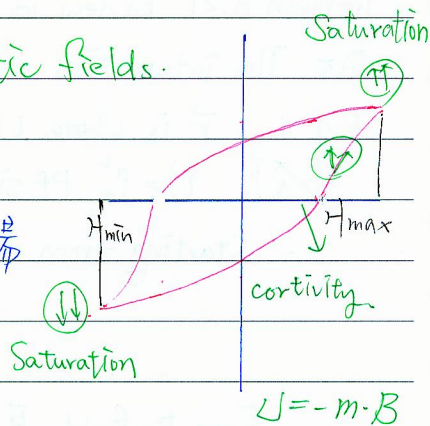
The magnetic domains under applied magnetic fields.

$$B = \mu_0 (H + M)$$

* Induced magnetisation M 磁矩

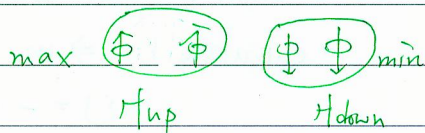
& Saturated applied fields H 外加磁場

$$U = -m \cdot B_z \leftrightarrow \text{calculate the stability}$$



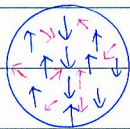
Problem 6.21

$$U = -m \cdot B$$



* Paramagnetism 統計力學 (順磁)

for statistical analysis, it may be shown that the probability of finding any one atom having energy



due to orientation in the field is proportional to the

Domain of

maxwell distribution at $k_B T$

paramagnetic material in nature.

① $U = -\vec{m} \cdot \vec{B}$ potential energy

② probability $\sim e^{-\frac{U}{k_B T}}$

③ Where θ is the angle between \vec{m} and \vec{B} , then $U = -mB \cos \theta$

\therefore The probability is $e^{-\frac{mB \cos \theta}{k_B T}}$

From this, curie can calculate the average value of $\cos \theta$.

$$\langle \cos \theta \rangle = \frac{\int_0^\pi \cos \theta e^{-\frac{mB \cos \theta}{k_B T}} \sin \theta d\theta}{\int_0^\pi e^{-\frac{mB \cos \theta}{k_B T}} \sin \theta d\theta} \cdot m, \text{ where } \frac{mB}{k_B T} \ll 1.$$

The magnetization of a paramagnetic with N atoms per unit volume

$$\vec{M} = 4N \frac{m^2 B}{3k_B T} \Leftrightarrow \vec{M} = \chi_m \vec{H} \sim \chi_m = \frac{4\mu_0 N m^2}{3k_B T} \text{ Curie Law}$$

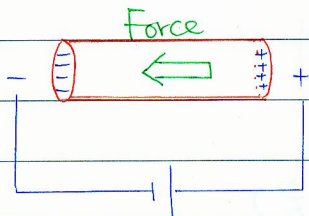
$\chi \sim \frac{1}{T}$ paramagnetism (↑↓)

susceptibility

$\chi \sim \frac{1}{T \pm \theta}$ Curie Weiss Law

$\chi \sim \frac{1}{T - \theta}$ Antiferromagnetism (↑↓)

$\chi \sim \frac{1}{T - \theta}$ ferromagnetism (↑↑)



When electric current in material is proportional to the voltage.

$I \sim V$ called as Ohm's Law

$f \equiv$ Force per unit charge $IR = V$ "macroscopic View"

* Into microscopic view

The current density defined as electric current per unit area $J = \frac{I}{A}$, can be expressed in terms of the electron density.

$$J \sim I \sim V \sim f \equiv \sigma f$$

static, steady, dynamics.

* Microscopic view

* The number of atoms per unit volume $n \equiv$

$$n \equiv \frac{N_A \left(\frac{\text{atoms}}{\text{mole}} \right) \cdot \rho' \left(\frac{\text{kg}}{\text{m}^3} \right)}{A \left(\frac{\text{kg}}{\text{mole}} \right)}, \quad \rho' \equiv \text{密度} = D$$

* Ohm's law, $I = \frac{V}{R} = \frac{\text{Voltage}}{\text{resistance}}$, $R = \frac{\rho L}{A}$,

$\rho \equiv$ resistivity 電阻率

$$\text{Then the current density is } J = \frac{I}{A} = \frac{V}{R \cdot A} = \frac{V}{\frac{\rho L}{A} \cdot A} = \frac{E}{\rho}$$

$$\Leftrightarrow J A = I = \sigma E A$$

* The current density is proportional to the E & $\frac{1}{\rho}$.

Then define a conductivity $\sigma \equiv \frac{1}{\rho}$ 電導率

$\rho \rightarrow 0, \sigma \rightarrow \infty \Rightarrow$ Superconductivity.

5/27 小考: 6.2 ~ Ch6 End.

§ 7.1.1 Ohm's Law

* If force on charge will fast/slow velocity.

So the current density J is $\sim \sigma E$.

* From the Lorentz force $F = q(\vec{v} \times \vec{B} + \vec{E})$

Force per unit charge $f = \frac{F}{q} = (\vec{v} \times \vec{B} + \vec{E})$

* Then the current density $J = \sigma \vec{E} = \sigma f$. (7.1) R+c

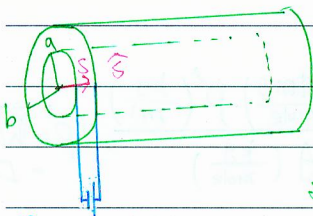
* If we consider the electrodynamic of electron/magnetic force,

then $J = \sigma(\vec{v} \times \vec{B} + \vec{E})$. (7.2) R+c + L

steady \Downarrow static State
 $= \frac{\Delta Q}{\Delta T} = I$ $I = \frac{Q}{T}$

Ex 7.2

Two long cylinders (radius a & b) are separated by material of conductivity σ . If they are maintained at a potential difference V , what current flows from one to the other in a length L .



force voltage

* The Electric field between a and b cylinder

is $E = \frac{\lambda}{2\pi\epsilon_0 s} \hat{s}$ [See P. 2.35]

where λ = charge per unit length on the linear cylinder

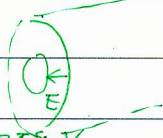
$I = \int J da$

The total current $I = \int J da = \int \sigma E da$, $da = 2\pi s L$
 $= \int_0^L \frac{\lambda}{2\pi\epsilon_0 s} 2\pi s ds = \frac{\sigma \lambda}{\epsilon_0} L$

The potential between the cylinder is

$V = -\int \vec{E} \cdot d\vec{s} = -\int \frac{\lambda}{2\pi\epsilon_0 s} ds$

$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a} \Rightarrow \lambda = \frac{2\pi\epsilon_0 V}{\ln \frac{b}{a}}$



* If the current is $I = \frac{\sigma \lambda}{\epsilon_0} L$ & $\lambda = \frac{2\pi\epsilon_0 V}{\ln \frac{b}{a}}$

* So the current $I = \frac{2\pi\sigma b}{\ln \frac{b}{a}} V$, Then the resistance is $\frac{\ln \frac{b}{a}}{2\pi\sigma b} = R$

If a free charge toward the end of the conductor and establish an electric field \vec{E} given by $\vec{E} = \vec{v} \times \vec{B}$.

The end of the conductor is $V_{ba} = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$

Then the difference of voltage is dependent on $\vec{v} \times \vec{B}$

The V_{ab} can be changed as: $V_{ab} = \frac{1}{q} \int q (\vec{v} \times \vec{B}) \cdot d\vec{l}$ average method

$$\mathcal{E} = \frac{1}{q} \int q (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\langle B \rangle = \frac{1}{V} \int B \cdot d^3r$$

if v, B, dl are constants, then $\mathcal{E} = vBL$, called emf.

§ 7.1 (IV)

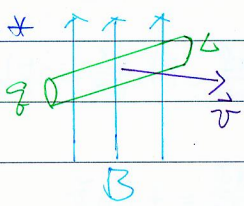
§ 7.1.2.

V * Voltage \equiv Applied force F on charges / per charge $\Rightarrow f$

J * Current density $\Rightarrow J = \sigma f$, $\sigma =$ conductivity $= \frac{1}{\rho}$

emf * Electromotive Force $\mathcal{E} = V = \int \vec{E} \cdot d\vec{l} \rightarrow$ Electricstatics 靜電

* $F = q\vec{E} + q\vec{v} \times \vec{B} \rightarrow$ introduce a steady state \rightarrow Electrodynamics 電動



Now we consider a conductor of length L moving v velocity \vec{v} perpendicular to a uniform field $\vec{F} = q\vec{v} \times \vec{B}$

The free charge establish an electric field. $q\vec{E}$. Static \rightarrow Steady State then $qE \rightarrow q\vec{v} \times \vec{B}$ the potential difference between the ends of the conductor is $V = \int \vec{v} \times \vec{B} \cdot d\vec{l}$ [Electrodynamics 電動]

We obtain $\mathcal{E} = \frac{1}{q} \left[\int q v \times B \cdot dl \right]$ (平均法) \Rightarrow Average Voltage / per charge

§ 7.1.3 Motional emf

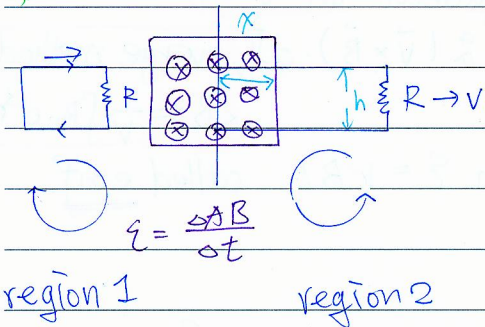
$$\Rightarrow \mathcal{E} = \vec{v} B L \text{ 剛好!}$$

We consider the processes Static \rightarrow Steady \rightarrow dynamics (motional)

$$\mathcal{E} = \frac{q}{L} BL$$

$$= \frac{\Delta x}{L} BL \Rightarrow \frac{\Delta A}{\Delta t} B ?$$

*



If the B is point into the page, connecting a resistance R , moving with velocity \vec{v}

Let us define the flux of area through B -field.

1. Then we know $\mathcal{E} = v \times B h = \int f_{\text{mag}} \cdot dx$

2. Due to the changing of area, we introduce a ideal of magnetic flux $\Phi = BA$.

* Considering the moving \vec{v} .

1. In region 2 the loop is moving out.

Then the flux is decreasing 通量减少 $\Rightarrow \frac{d\Phi}{dt} = Bh \frac{dx}{dt} = -Bh\vec{v}$ flux decrease

2. In region 1. the loop is moving in.

Then the flux is increasing. 通量增加 $\Rightarrow \frac{d\Phi}{dt} = +Bh\vec{v}$

3. So the emf generate in loop is minus the rate of change of flux through the loop in region 2. $\mathcal{E} = -\frac{d\Phi}{dt}$.

Conclusion $\mathcal{E} = -\frac{d\Phi}{dt}$

Problem 7.12 A long solenoid of radius a is driven by an alternating current. So that the field inside is $B(t) = B_0 \cos(\omega t) \hat{z}$. A circuit $\Phi = BA \rightarrow$ changing area \rightarrow changing B -field \rightarrow dynamic. loop of wire of radius $\frac{a}{2}$

1. $\Phi = \frac{\pi a^2}{4} B$

So the $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \left[\frac{\pi a^2}{4} B_0 \cos \omega t \right]$

$= -\frac{\pi a^2}{4} B_0 (-\omega) \sin \omega t =$

$= \frac{\pi \omega a^2}{4} B_0 \sin \omega t,$

then $I = \frac{\mathcal{E}}{R} = \frac{\pi \omega a^2}{4R} B_0 \sin \omega t$

2. loop area $\frac{\pi a^2}{4}$