In Electrostatic ⇒ Polarization $\mathbf{P}$

$P = \Sigma \frac{\Delta p}{\Delta V}$ then the $\boxed{\text{potential}}$ can be described as

$$V = \frac{1}{4\pi\varepsilon_0} \frac{P\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{1}{4\pi\varepsilon_0}\int \frac{P\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} d r'$$ P.167

$$= \underbrace{\boxed{\frac{1}{4\pi\varepsilon_0}\oint \frac{\vec{P}\cdot\hat{n}\, da'}{|\vec{r}-\vec{r}'|}}}_{\text{Surface}} - \underbrace{\boxed{\frac{1}{4\pi\varepsilon_0}\int \frac{\nabla'\cdot P}{|\vec{r}-\vec{r}'|} d^3r'}}_{\text{Volume}}$$

Define surface charge $\sigma_b = \vec{P}\cdot\hat{n}$

volume $\rho_0 = -\nabla\cdot\vec{P}$

] Bound Charge

Chapter 6

In Magneto static ⇒ Magnetization M.

$\boxed{M \equiv \Sigma \frac{\Delta m}{\Delta V}}$ ⇒ Using vector potential $\vec{A}$

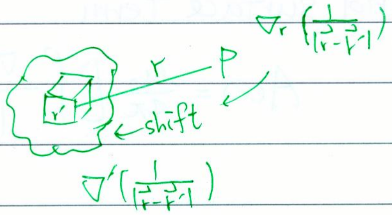
$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{M(r')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} d^3r'$$

$$= \boxed{\frac{\mu_0}{4\pi}\int \frac{\nabla'\times M}{|\vec{r}-\vec{r}'|} d^3r'} + \boxed{\frac{\mu_0}{4\pi}\int \frac{M(r')\times\hat{n}'}{|\vec{r}-\vec{r}'|} da'}$$ ⇒ Bound current

* How to separate the vector potential into two terms of surface and volume bound charges ?

* if $\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{(-1)(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}$

$$\nabla'_{r'}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^2}$$



* Then the vector potential can be rewritten as

$$A(r) = \frac{\mu_0}{4\pi}\int \vec{M}(r')\times\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) d^3r'$$

$$\vec{M}\times\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) \equiv \text{Vector}\times\nabla'(\underset{\text{scale}}{\text{function}})$$

Then using the exchange rule

$$\nabla' \times \left(\frac{\vec{M}(r')}{|\vec{r}-\vec{r}'|}\right) = \frac{1}{|\vec{r}-\vec{r}'|}(\nabla' \times M(r')) - \vec{M}(r') \times \left(\nabla' \frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$\Rightarrow \vec{A} = \frac{\mu_0}{4\pi}\int \frac{1}{|\vec{r}-\vec{r}'|}(\nabla' \times \vec{M}(r'))\, d^3r' - \boxed{\frac{\mu_0}{4\pi}\int \nabla' \times \left(\frac{\vec{M}(r')}{|\vec{r}-\vec{r}'|}\right) d^3r'}$$

Let the Bound current density for volume $J_b \equiv \nabla' \times \vec{M}(r')$

The second term can be rewritten by Problem P1.60

$* \int \nabla \cdot (\vec{v} \times c)\, d^3r = -\int c\, \vec{v} \times d\vec{S}'$

Then $-\frac{\mu_0}{4\pi}\int \nabla' \times \frac{\vec{M}(r')}{|\vec{r}-\vec{r}'|} d^3r' = -\left[\frac{-\mu_0}{4\pi}\int \frac{1}{|\vec{r}-\vec{r}'|}(\vec{M}(r') \times \hat{n}')\, da'\right]$

$$= +\frac{\mu_0}{4\pi}\frac{M \times n'}{|\vec{r}-\vec{r}'|}\, da'$$

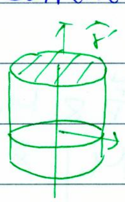
The final form of vector potential is

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\nabla' \times M(r')}{|\vec{r}-\vec{r}'|} d^3r' + \frac{\mu_0}{4\pi}\int \frac{\vec{M}(r') \times \hat{n}'}{|\vec{r}-\vec{r}'|} da'$$

The integer are taken over the volume and surface of the magnetization matter.

A. The vector potential produced by volume current density $J(r')$ is

$$\boxed{\begin{aligned} J(r') &\equiv \nabla' \times \vec{M}(r') \\ \vec{A}(r)_{\text{volume}} &= \frac{\mu_0}{4\pi}\int \frac{J(r')}{|\vec{r}-\vec{r}'|} d^3r' \end{aligned}}$$



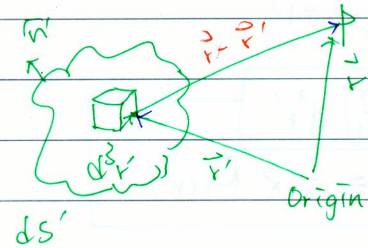
B. The vector potential produced by surface current density

$K(r') \equiv \vec{M}(r') \times \hat{n}'$

$\hat{n}' = \frac{\vec{r}'}{r}$

$A_{surface} \equiv \frac{\mu_0}{4\pi}\int \frac{K(r')\, da'}{|\vec{r}-\vec{r}'|}$

Replot the map of M

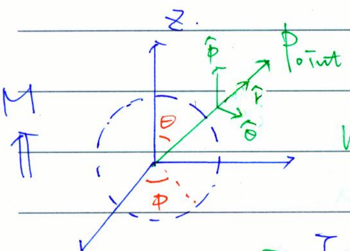


$\sim \left(\frac{1}{r-r'}\right)$

$$\left[\begin{array}{l} V=\frac{1}{4\pi\varepsilon_0}\oint\frac{\sigma_b}{|\vec{r}-\vec{r}'|}da'+\frac{1}{4\pi\varepsilon_0}\int\frac{\rho_b}{|\vec{r}-\vec{r}'|}d^3x' \text{ , 靜電} \\ A=\frac{\mu_0}{4\pi}\int\frac{K(r')}{|\vec{r}-\vec{r}'|}da'+\frac{\mu_0}{4\pi}\int\frac{J(r')}{|\vec{r}-\vec{r}'|}d^3r' \text{ , 靜磁} \end{array}\right.$$

$$\left[\begin{array}{l} \sigma_b \equiv \vec{P}\cdot\hat{n} \text{ , } \rho_b = -\nabla\cdot P \\ K(r') = M\times\hat{n} \text{ , } J=\nabla\times M \end{array}\right.$$

Example 6.1 Find the magnetic field of a uniformly magnetized sphere.



If we chose the z-axis along the direction $M \equiv M_0\hat{z}$

We have volume current density $J=\nabla\times M=0$

surface current density $K = M\times\hat{n}$.

① $J=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & \underset{\text{constant}}{\underline{M_0}}\end{vmatrix}=0$

② $K=M\times\hat{n}$ $\quad \hat{n}=\frac{\vec{r}}{r}$

$\vec{r} = r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}$

$\hat{n}=\frac{\vec{r}}{r}$ & $M_0\hat{z}\times\hat{n}\equiv K$.

$K=M_0\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 0 & M_0\\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\end{vmatrix} = \sin\theta\cos\phi M_0\hat{y} - M_0\sin\theta\sin\phi\,\hat{x}$

$K = M_0\sin\theta\,\hat{\phi}$ = $\boxed{\text{Ex 5.11 result.}}$

Ex 6.1

$K = M_0 \sin\theta \hat{\phi}$

If a rotating spherical shell of uniform surface

Compare the current density σ

$\vec{K} = \sigma v = \sigma \omega R \sin\theta \hat{\phi}$

Ex 5.11 $B = \nabla \times \vec{A} = \frac{2}{3} \mu_0 \sigma R \omega$

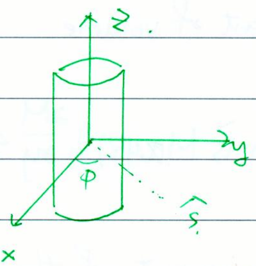
Then $M_0 = ?$ compare each other

$M_0 = \sigma \omega R.$

Ex 6.1's B-field is $B = \frac{2}{3} \mu_0 \sigma R \omega = \boxed{\frac{2}{3} \mu_0 M_0}$

Then the dipole moment $\vec{m} = \frac{4}{3} \pi R^3 M_0.$

Problem 6.8 A long circular cylinder of radius R carries a magnetization $M = ks^2 \hat{\phi}$ where k is constant. S is the distance from the axis and ϕ is the usual azimuthal unit vector.



$$J = \nabla \times M = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\sin\phi s^2 & \cos\phi s^2 & 0 \end{vmatrix}$$

$K = M \times \hat{n}$

$F = -\nabla U = \nabla(m\cdot B)$ for any case free current or dipole moment $\oint J$ $J_f + J_b$

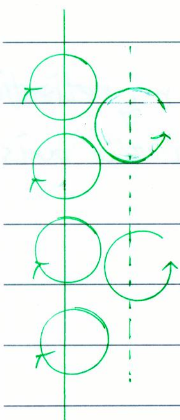
$F = \nabla(m\cdot B) \overset{?}{=} (m\cdot\nabla)B_{dipole}$ exist only for dipole moment case

$\Rightarrow$ $\boxed{\text{Bound current } J_b, K_b}$

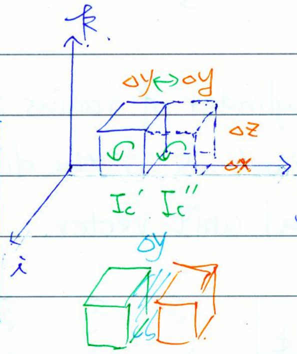
$F = \nabla(P\cdot E) = (P\cdot\nabla)E$ ok!

6.2.2 Physical Interpretation for nonuniform magnetization.

If the magnetization is non-uniform the cancellation is net complete, consider the abrupt charge of magnetization in Fig 6.12, 15, 17.



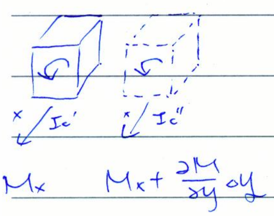
$$\boxed{\begin{array}{c} J\cdot A = I. \\ \Rightarrow \\ M = I\dfrac{\Delta m}{\Delta V} \end{array}}$$



To find the relationship between J_M and M, Let us consider two small volume element located next to each other in the direction of the y axis element of volume $\equiv \Delta x\Delta y\Delta z$.

* The magnetization is $M_{(x,y,z)} + \frac{\partial M}{\partial y}\Delta y$

Cacellation is net complete



* Then the x-component of magnetic moment of the first element $M_x\Delta x\Delta y\Delta z = m_x$, may be in terms of a circular current $I_c' \equiv \dfrac{m_x}{A} = \dfrac{M_x\Delta x\Delta y\Delta z}{\Delta y\Delta z} = M_x\Delta x$

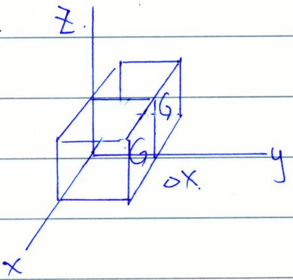
$$I_c'' = \frac{(M_x + \frac{\partial M_x}{\partial y}\Delta y)\cdot\Delta x\Delta y\Delta z}{\Delta y\Delta z} \quad \text{—①}$$

$$= M_x dx + \frac{\partial M_x}{\partial y}\Delta y\Delta x \quad \text{—②}$$

The net upward current in the middle region of the two volume elements is

$$\boxed{I_c' - I_c'' = -\frac{\partial M_x}{\partial y}\Delta x\Delta y} = I^x_{up}.$$

We next consider two adjacent volume element along x-axis & focus out attention on the compoment of the magnetization. $\hat{y}$.



$$I_{c\,up}^{y} = \frac{\partial M_y}{\partial x}\,\delta x\,\delta y$$

The net current, which comes about from non-uniform magnetization. $(J_M)_z = \frac{I}{\delta x \delta y} = (I_{up}^{x} + I_{up}^{y})/\delta x \delta y$

$$= \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}$$

effect current density for

$$J_M = \nabla \times \vec{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M_x & M_y & M_z \end{vmatrix}$$

For volume current density. Called "Bound current J_b".

6.3.1 Ampere's law in Magnetized materials.

a. Bond currents $J_b = \nabla \times \vec{M}$ for volume density.

b. Bond currents $K_b = \vec{M} \times \hat{n}$, for surface density.

We say the total current $J = J_b + J_f$, because the field attributable to the bond currents, plus the field due to everything else.

From Ampere's Law. $\nabla \times B = \mu_0 J$

$$\xRightarrow{\text{magnetized}} \frac{1}{\mu_0}(\nabla \times B) = J_b + J_f = J_f + \nabla \times M.$$

Then the equation is

$\nabla \times \left(\frac{B}{\mu_0} - M\right) = J_f$. The quantity in parentheses is designated by

$\underline{H = \frac{B}{\mu_0} - M}$. So the Ampere's law leads to $\nabla \times H = I_f$

$$\oint H \cdot dl = I_{f\,\boxed{\text{enclosed}}}$$

$$\boxed{\begin{matrix} M = 0 \rightarrow B \\ M \neq 0 \rightarrow H \end{matrix}} \Rightarrow \text{Magnetized object?}$$