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* A comparsion Between Electratic & Magnetostatics.
It should be compared between two static fields.

1. Both $\vec{B}$ and $\vec{E}$ are defined in terms of forces.
$\vec{F}_E = q\vec{E}$, $\vec{F}_B = q(\vec{v}\times\vec{B})$
$\vec{E}$ = polar vector $\vec{v}$ = axial vector
2. Where $\vec{E}$ is defined via $\vec{F} = q\vec{E}$ on a stationary Charge and is consequently a polar vector.
3. $\vec{B}$ is defined via $\vec{F} = q(\vec{v}\times\vec{B})$. The cross-product of two polar vectors. is a axial vector.

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix} = V_x B_y \hat{z} \iff \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -V_x & 0 & 0 \\ 0 & -B_y & 0 \end{vmatrix} = V_x B_y \hat{z}$$

* Now $\vec{B}$ is defined via Lorentz Force
$\vec{F} = q(\vec{v}\times\vec{B})$ on a moving charge
$\vec{F} = (q\vec{v})\times\vec{B}$

4. Gauss' law appear different for these two static fields.
$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$
$$\int \vec{\nabla}\cdot\vec{E}\, d^3r = \int \rho/\varepsilon_0 \, d^3r = Q/\varepsilon_0$$
5. The B-field is divergence-less at all points & its solenoid. It is lines form closed loop.
6. $\vec{E}$ is curl-free ($\vec{\nabla}\times\vec{E} = 0$) solenoid $\vec{E}$-field is easy to make.

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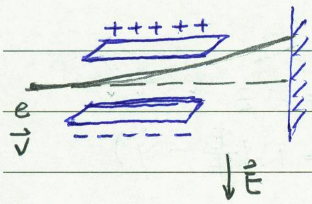
* J. J. Thomson 1897.

The measurement of e/m



Using Lorentz force $\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B}$

(1) If the sum of electron's force is zero

then $q\vec{E} = q(\vec{v}\times\vec{B}) \sim |E/B| = v$

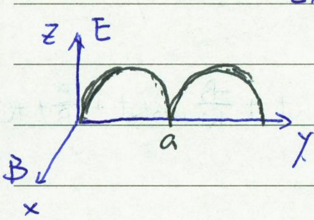
(2) The axial in x and y is

$x = vt$, $y = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}t^2 = \frac{1}{2}\cdot\frac{eE}{m}t^2$

$\boxed{y = \frac{eE}{2mv^2}x^2}$. 代入 $\boxed{\frac{e}{m} = \frac{2yE}{B^2x^2}} \simeq 1.7\times10^{11}\ C/kg$.

Millikan $e \sim 1.602\times10^{-19}C = 9.1\times10^{-31}kg$.

* Example 5.2 Cycloid motion p. 205



If $\vec{B}$ is points in x - direction] ⇒
" $\vec{E}$ " " " z "

⇒ then A particle at rest
is released from the origin.

① The $V_p(0, \dot{y}, \dot{z})$. $V_x \times B_x = 0 \Rightarrow V_p(\dot{y}\hat{y}, \dot{z}\hat{z})$

② We can calculate magnetic force

$$\vec{F}_B = q\vec{v}\times\vec{B} = q\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \dot{y} & \dot{z} \\ B_x & 0 & 0 \end{vmatrix} = qB_x(\dot{z}\hat{y} - \dot{y}\hat{z})$$

③ We can calculate the E - force

$$\vec{F}_E = q\vec{E} = qE_z\hat{z}$$

④ The total force

$$\vec{F} = m\vec{a} = m(\ddot{y}\hat{y} + \ddot{z}\hat{z}) = (qB\dot{z})\hat{y} + (qE - qB\dot{y})\hat{z}$$

So the equation of F = ma can compare each other

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$$\boxed{\begin{aligned}\hat{y}:&\quad m\ddot{y} = qB\dot{z}\\ \hat{z}:&\quad m\ddot{z} = qE - qB\dot{y}\end{aligned}}$$

$\ddot{y} = \frac{qB}{m}\dot{z}$

Define $\frac{qB}{m} = \omega$ then $\ddot{y} = \omega\dot{z}$
$\ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$

① If $\ddot{y} = \omega\dot{z} \Rightarrow \dddot{y} = \omega\ddot{z}$
$\ddot{z} = \omega\left(\frac{E}{z} - \dot{y}\right) \Rightarrow \dddot{y} = \omega^2\left(\frac{E}{B} - \dot{y}\right)$

② Let $\dot{y} = s$ then the equation can be rewritten as $\boxed{\ddot{s} = \omega^2\left(\frac{E}{B} - s\right)}$

get the solution of s
$s = A\cos\omega t + B\sin\omega t + \frac{E}{B} = \dot{y}$

$\therefore y(t) = \int \dot{y}(t)\,dt = \frac{A}{\omega}\sin\omega t + \frac{-B}{\omega}\cos\omega t + \frac{E}{B}t + C$

Insert to $\ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$
$= \omega\left(\frac{E}{B} - A\cos\omega t - B\sin\omega t - \frac{E}{B}\right)$
$= -\omega\left(A\cos\omega t + B\sin\omega t\right)$

So the $\dot{z} = -A\sin\omega t + B\cos\omega t + C'$
$z = \frac{A}{\omega}\cos\omega t + \frac{B}{\omega}\sin\omega t + C''$

check the initial condition
$t = 0 \Rightarrow \dot{y}(0) = 0 \quad \dot{z}(0) = 0$
$y(t) = \frac{E}{\omega B}(\omega t - \sin\omega t) = R(\omega t - \sin\omega t)$
$z(t) = \frac{E}{\omega B}(1 - \cos\omega t) = R(1 - \cos\omega t)$
$R \equiv$ radius.

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§. 5.1.3 Current vs charge.

A. Electric current $\longleftrightarrow$ charge at rest

$\downarrow$ I $\qquad\qquad$ $\downarrow$ Q

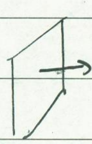
Definition : Electric current is the flow of electric charge.

B. The magnitude of an electric charge at a point is defined as the time deviation of electric charge.

$I(t) = \dot{Q}(t)$ Formly this is written as

$I(t) = dQ/dt \iff Q(t) = \int_{t_0}^{t} I(t)\, dt + Q.$

C. Current density



$\vec{J} = \frac{\vec{I}}{A}$

* It is a measure of the density of electrical current. It is defined as a vector whose magnitude is I/A.
* The total charges pass through the area A in a time, then the $\Delta Q = q(\vec{v}\Delta t)\vec{A}\cdot n$

n = charge particle per unit volume.

if $A \parallel \vec{v}$, $I = \frac{\Delta Q}{\Delta t} = nqA\vec{v}$

$$J = \frac{I}{A} = nq\vec{v} \iff \vec{J} = \rho\vec{v}$$

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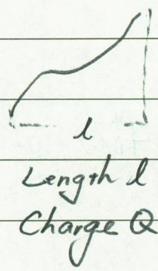
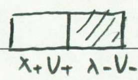
* Current density $J = \rho V$ ≡ 電子流的密度

§ 5.1.3 Electric Current



1. Charge density $\lambda = Q/l$
2. Current $I = Q/t$
3. $I/\lambda = \frac{Q}{t} / \frac{Q}{l} = \vec{v} \Rightarrow \vec{I} = \lambda \vec{V}$

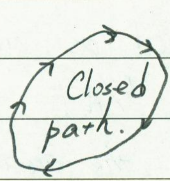
For Example of Ampere's experiment
then non-uniform



$I = \lambda V = \lambda_+ V_+ + \lambda_- V_-$

$F_{mag} = \begin{cases} q\,\vec{v} \times \vec{B} \\ \int \vec{v} \times \vec{B}\, dq \end{cases}$ → Columb ⊕→ ←⊖

$$\Rightarrow \int (\vec{v} \times \vec{B})\, dq = \int (\vec{v} \times \vec{B})\, \lambda\, dl$$
$$= \int (\lambda \vec{v}) \times \vec{B}\, dl = \int (\vec{I} \times \vec{B})\, d\vec{l} \qquad \boxed{\vec{I} \parallel d\vec{l}}$$

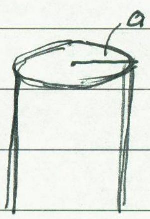


* For example of closed path
if current is independent on position
$\vec{F}_{mag} = I \int \vec{B} \times d\vec{l}$

* For $\vec{B}$ independs on the position.
if $\vec{B}$ is uniform
$\vec{F}_{mag} = I \left[\oint d\vec{l} \right] \times \vec{B}$ for closed path.
$= 0$

Example 5.4

(a) A current I is uniformly distributed over a wire of circuit cross section which radius a,
截面



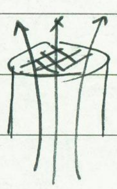
Then we define the current density

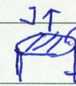
$$J = I/A \Rightarrow I = \int \vec{J}\, da \qquad J = I/\pi a^2$$

(b) Find the total current if $\vec{J} = ks$

$$I = \int ks\, da = \int_0^a ks \cdot 2\pi s\, ds = 2\pi k \int s^2 ds = 2\pi k \tfrac{1}{3} a^3$$

P.S. If the current crossing a surface S can be written as



$$I = \int \vec{J}\, da =$$ 

$$\rho\, dv = \int \vec{\nabla} \cdot \vec{J}\, dv$$

flux ⇒ current $= -\frac{d}{dt} \int \rho\, dv = -\frac{Q}{t} = \frac{\Delta Q}{\Delta t}$

流出. flux out

$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

$\Rightarrow \int \nabla \cdot E\, dv = \frac{\int \rho\, dv}{\varepsilon_0}$

The final form of continuity equation is described as

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}.$$