Diving by $\frac{dI}{dt}$, $\mathcal{E} = -M_{21}\frac{dI_1}{dt}$

we could get the definition of M $M_{21} \equiv \frac{\mu_0}{4\pi}\oint_\lambda \oint_s \frac{d\vec{s}\cdot d\vec{\lambda}}{r}$

where $d\vec{s}$ & $d\vec{\lambda}$ are two element of length (§7.2.3)

& r is the distance between them. eq 7.22

This is known as Neumann's Formula. $M_{12} = \frac{\mu_0}{4\pi}\oint_s \oint_\lambda \frac{d\vec{\lambda}\cdot d\vec{s}}{r}$

6/3 (三) $\Rightarrow M_{12} = M_{21}$

* Final Exam. 6/17 (三) 10:00 AM.

$\Phi = MI.$

flux mutual inductance

§ 7.2.4 Energy in magnetic field ⇒ Energy + Density of Energy

To obtain the energy of an electrostatic field, we calculate the work done by $\mathcal{E}$ in moving charge increment dq.

Then the increment of dq can be described as $dq = Idt$.

* $dW = -\mathcal{E}dt$, through the circuit is $\Rightarrow dW = -\mathcal{E}I = LI\frac{dI}{dt}$.

then we can integrated the time function of work.

$\boxed{W = \frac{1}{2}LI^2(t)}$

1. If $dW = Id\Phi$ is equal to the charge in magnetic energy.
2. If there are n turns circuit, then the work done against the induced emf is given by $\boxed{dW = \sum_{i=1}^{n} I_i d\Phi_i}$

* Magnetic Energy in terms of field vectors.

For simplicity, we assume that each circuit consist of a single loop, then the flux Φ

$$\Phi_i = \int \vec{B}\cdot\hat{n}\,da = \int \nabla\times\vec{A}\cdot\hat{n}\,da = \int_{closed\ loop} \vec{A}\cdot d\vec{l}$$

⇒ input the magnetic energy.

GEE-JUMP

For n turns, the function of $dW = \Sigma I_i \Phi_i$ can be replaced as $W = \frac{1}{2} I \Phi$. $\Rightarrow$ Magnetic energy

$\Rightarrow U = \frac{1}{2} \Sigma I_i \Phi_i$ $\quad U = \frac{1}{2}\int I_i \vec{A}_i \cdot d\vec{l}$

* The second step: we can change $I_i dl_i = (\vec{J}\cdot da)\cdot dl = \vec{J}\, dv_i$

& $\sum_i \left[\oint_{C_i \,\text{closed path}} \right] = \int_{\text{Volume}} \Rightarrow$ then $\boxed{U = \frac{1}{2}\int \vec{J}\cdot\vec{A}\, dv.}$

* Mathematical method of $\vec{B} = \nabla\times\vec{A}$, $\nabla\times\vec{B} = \mu_0\vec{J}$.

Re-calculated $\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$
$= \vec{B}\cdot\vec{B} - \vec{A}\cdot\mu_0\vec{J}$

$\Rightarrow \boxed{\vec{B}\cdot\vec{B} - \nabla\cdot(\vec{A}\times\vec{B}) = \vec{A}\cdot\mu_0\vec{J}}$ VIP

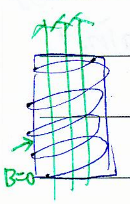
$U = \frac{1}{2}\int \vec{J}\cdot\vec{A}\, dv$, we obtain the expression of

$$U = \underbrace{\frac{1}{2\mu_0}\int \vec{B}\cdot\vec{B}\, dv}_{\text{OK!}} - \underbrace{\frac{1}{2\mu_0}\int \nabla\cdot(\vec{A}\times\vec{B})\, dv}_{? = 0}$$

$$= \frac{1}{2\mu_0}\int \vec{B}\cdot\vec{B}\, dv - \frac{1}{2\mu_0}\underline{\int \vec{A}\times\vec{B}\cdot\hat{n}\, da} \hookrightarrow \frac{1}{r}$$

Explan.

The integrations on the right are to be taken over the entire volume occupied by the current with the surface.



Because $\vec{B}$ falls off at fast as $\boxed{\frac{1}{r^2}}$

$\vec{A}$ " $\boxed{\frac{1}{r}}$

Surface $da \sim \boxed{r^2}$

Then the second term of surface integral vanishes.

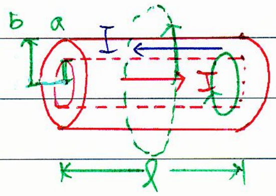
$U = \frac{1}{2\mu_0}\int \vec{B}\cdot\vec{B}\, dv$, when $\vec{B} = \mu_0\vec{H}$

$= \frac{1}{2}\int \vec{H}\cdot\vec{B}\, dv$, we may define the

* energy density in magnetic field by $u = \frac{1}{2}\vec{H}\cdot\vec{B}$

$u = \frac{B^2}{2\mu}$

§ 7.1.3 Example. A long co-axial cubles carries current I.



The outter current $I \leftarrow$

inner current $I \rightarrow$

Find the magnetic energy stored in a section of length l.

* According to Ampere's law. $B = \frac{\mu_0 I}{2\pi\rho_0}\hat{\phi}$

* The magnetic energy per unit volume

$$u = \frac{1}{2\mu_0}B^2 = \frac{\mu_0 I^2}{8\pi^2\rho^2}.$$

* The unit volume of cylinder $U = \int_{volume} dv$. $dv = l 2\pi\rho d\rho$.

So the magnetic energy is $U = \int u\,dv = \int_a^b \frac{\mu_0 I^2}{8\pi^2\rho^2} 2\pi\rho\, d\rho\, l$

$= \frac{\mu_0 I^2 l}{4\pi}\int_a^b \frac{1}{\rho}d\rho = \frac{\mu_0 I^2 l}{4\pi}\ln\frac{b}{a}$, represented with L. I.

$\boxed{U = \frac{1}{2}LI^2}$ $L = \frac{\mu_0 l}{2\pi}\ln\frac{b}{a}$

HW 06/10 return
P7.18 P7.24
P7.20 P7.30

§ 7.3 Maxwell Equation

7.3.1. Electrodynamics equation. Before Maxwell

A. Gauss's Law $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

: $\nabla\cdot\vec{B} = 0$

Faraday's Law $\nabla\times E = -\frac{\partial B}{\partial t}$

Ampere's Law $\nabla\times B = \mu_0 J$

There equations represent the state of EM theory over a century.

The old eqs. with the old rule that divergency of curl is always zero.

(1) $\nabla\cdot(\nabla\times\vec{E}) = \nabla\cdot\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla\cdot B) = 0.$

(2) $\nabla\cdot(\nabla\times B) = (\nabla\cdot\vec{J})\mu_0 = 0$?

Zero only at Static / steady state.; Non Zero at dynamics state.

Maxwell equation & Electromagnetic wave in vacuum.

1. Gauss's Law $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
2. Absence of magnetic monopole $\nabla \cdot \vec{B} = 0$.
3. Ampere's Law $\nabla \times \vec{B} = \mu_0 \vec{J}$
4. Faraday's Law $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$
5. The continuity eq. $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$

6/4 (IV) 6/17 (三) Final Exam of EM.

§ 7.3.1

(1) Ampere's Law : $\nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \nabla \cdot (\nabla \times B) = \mu_0 (\nabla \cdot \vec{J}) = 0$

at steady, vaild for dynamics ?

(2) Continuity equation : $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$. Charge increament $\neq 0$

§ 7.3.2.

How Maxwell fixed Ampere's Law for Electrodynamics ?

<1> Continuity equation & Ampere's Law, they were all vaild even in time varying situtation & realize Ampere's Law was in consist with the continuity equation.

<2> $\nabla \cdot (\nabla \times B) = 0$, & $\nabla \cdot J = 0$.

This indeed true of ρ does not change with time.
But it is not true when ρ is changing with time

velocity

$\boxed{\frac{\Delta \rho}{\Delta t} = \text{constant}}$

$\boxed{\frac{\Delta \rho + \rho}{\Delta t} \neq \text{constant}}$

Accel

$\boxed{\frac{\rho}{t} = \text{constant}}$

Speed

<3> Using Gauss's Law, we can rewrite the continuity eq.

as $\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial t}[\nabla\cdot\varepsilon_0\vec{E}]$

Replace $\nabla\cdot[\vec{J}+\frac{\partial}{\partial t}\varepsilon_0\vec{E}] = 0 \Rightarrow \nabla\cdot\vec{J}_{Maxwell} = 0.$

<4> So the divergency of curl B can be rewriten as

$\nabla\cdot(\nabla\times\vec{B}) = \nabla\cdot(\mu_0 J_{Maxwell}) = \nabla\cdot[\mu_0\vec{J}+\frac{\partial}{\partial t}\mu_0\varepsilon_0\vec{E}] = 0$

Maxwell saw that if Ampere's Law is modified by the addition of a new term the time derivative term.

$$\Rightarrow \boxed{\nabla\times\vec{B} = \mu_0\vec{J}+\frac{\partial}{\partial t}(\underset{\text{Mag.}}{\mu_0}\underset{\text{Ele.}}{\varepsilon_0}\vec{E})}$$

(1) Is vaild for Steady-state phenomena is also compatible with the equation of continuity of time-dependent fields. (dynamics)

(2) The term $\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ has the dimensions of current density

$\nabla\cdot(\varepsilon_0\vec{E}+\vec{P}) = \rho_f \Rightarrow \boxed{\nabla\cdot D = \rho_f}$

§ Have a look in Gauss's Law for Electric & Magnetic fields.

(1) $\nabla\cdot E = \frac{\rho_0}{\varepsilon_0}$ + (2) $\nabla\cdot B = 0$ $(\mu_0\rho_m = 0)$

Using the free space (e or $m = 0$)

$\nabla\cdot E = 0$ $\quad \nabla\times E = -\frac{\partial B}{\partial t}$

+

$\nabla\cdot B = 0$ $\quad \nabla\times B = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

replace E by B

replace B by $-\mu_0\varepsilon_0 E$

$\Rightarrow$ $\nabla\cdot B = 0$, $\nabla\times B = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$.

$\nabla\cdot E = 0$, $\nabla\times E = -\frac{\partial B}{\partial t}$

There are something missing from $\nabla\cdot B = 0$ & $\nabla\times E = -\frac{\partial B}{\partial t}$ if we had ρ_e, ρ_m & m. (In non-free space)

We have ① $\nabla\cdot E = \frac{\rho_e}{\varepsilon_0}$ ③ $\nabla\times E = -\mu_0 J_m - \frac{\partial B}{\partial t}$

② $\nabla\cdot B = \mu_0\rho_m$ ④ $\nabla\times B = \mu_0 J_e + \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

ρ_m : magnetic charge density.

J_m : magnetic current density.

"Maxwell's equation beg for the existance of magnetic charge"   7.35, 7.36