No.
Date.

Finally we have

$$\vec{F} = \frac{1}{2}\varepsilon_0 \chi \nabla^2 E = \frac{1}{2}\varepsilon_0 (k-1) \nabla E^2$$

$$k = \varepsilon/\varepsilon_0 = 1+\chi$$

$$\nabla E^2 = \nabla(\vec{E}\cdot\vec{E}) = \vec{E}\cdot(\nabla \vec{E}) + \vec{E}\cdot(\nabla \vec{E}) = 2\vec{E}\cdot\nabla\vec{E}$$

1/7 Ex. 4.5 + Prob. 4.26

A metal sphere of radius a carries a charge Q. It's suronded, out to radius b, by linear dielectric material of permitivity ε.

$$W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau$$

$\Uparrow$

Find the potential at the center, energy (work)

(1) For $r>a$. the Displacement ?

for dielectric material $D = \varepsilon E + P = \varepsilon E + 0$

$$\vec{D} = \varepsilon \frac{1}{4\pi\varepsilon}\cdot\frac{Q}{r^2}\hat{r} = \frac{Q}{4\pi r^2}\hat{r}$$

(2) Inside the material $r<a$.

$$E = D = P = 0$$

(3) For Gauss' laws $r>b$ $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$

$a<r<b$ $\vec{E} = \frac{Q}{4\pi\varepsilon r^2}$

Find the potential at the center is therefore

$$V = -\int_\infty^0 \vec{E}\cdot d\vec{l} = -\left(\int_\infty^b + \int_b^a + \int_a^0\right)\vec{E}\cdot d\vec{l}$$

No.
Date.

$$= -\frac{Q}{4\pi}\left[\int_{\infty}^{b}\frac{1}{\varepsilon_0 r^2}dr + \int_{b}^{a}\frac{1}{\varepsilon r^2}dr + \int_{a}^{0} 0\, dr\right]$$

$$= \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{\varepsilon a} - \frac{1}{\varepsilon b}\right)$$

If the work be done then $W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau$

$$W = \frac{1}{2}\left(\int_0^a + \int_a^b + \int_b^{\infty}\right)\vec{D}\cdot\vec{E}\, d^3r$$

$$= \frac{1}{2}\left(\frac{Q}{4\pi}\right)^2\left[\frac{1}{\varepsilon}\int_a^b \frac{1}{r^2}\cdot\frac{1}{r^2}\cdot 4\pi r^2 dr + \frac{1}{\varepsilon_0}\int_b^{\infty}\frac{1}{r^4}4\pi r^2 dr\right]$$

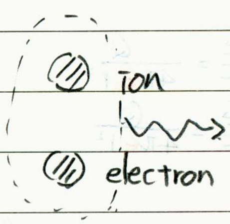
- So the final form of work is

$$W = \frac{Q^2}{8\pi}\left[\frac{1}{\varepsilon}\left(-\frac{1}{r}\right)\Big|_a^b + \frac{1}{\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_b^{\infty}\right]$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{(1+\chi_e)}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{b}\right]$$

§ Plasma oscillations

Plasma is a body of ions & electrons of sufficiently low density, they will oscillate about their mean position. $\vec{\nabla}\cdot\vec{D} = 0$



$$\Rightarrow \vec{P} = -\frac{N}{V}e\vec{l} = -ne\vec{l}$$

* So we can write down the equation of motion.

$\nabla\cdot\vec{D} = 0$ then $\varepsilon_0\vec{E} + \vec{P} = 0 \Rightarrow \vec{E} = -\vec{P}/\varepsilon_0$

No.
Date.

$$\text{So} \quad \boxed{-e\vec{E}_x} = m_e \ddot{x} = m_e \frac{d^2x}{dt^2}$$

$$-\frac{ne^2}{\varepsilon_0}\vec{x} = m_e \frac{d^2x}{dt^2}$$

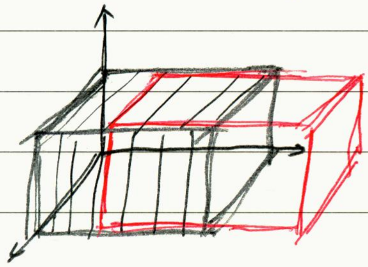
So the final eqution is

$$m_e \frac{d^2\vec{x}}{dt^2} + \frac{ne^2}{\varepsilon_0}\vec{x} = 0$$

the solution of $x(t) = x_0 \cdot \sin(\omega_p + \phi)$

$$\boxed{\omega_p = \left(\frac{ne^2}{\varepsilon_0 m_e}\right)^{1/2}} \sim \sqrt{n}$$

Example :
A slab of linear dielectric materials is partially inserded between the plates of a parallel-plate capacitor.



Force on dielectric materials

$$F = \frac{1}{2}\varepsilon_0 \chi \nabla E^2 = \frac{1}{2}\varepsilon_0 \chi \frac{\partial}{\partial x} E_x^2$$

Force per volume

$dz = 0 \rightarrow w$

$dx = 0 \rightarrow L$

$dy = 0 \rightarrow d$

$$F_v = \frac{1}{2}\varepsilon_0 \chi \left[\iiint_0^L \nabla E^2 \, dx\, dy\, dz\right]$$

No.
Date.

$$F_v = \frac{1}{2}\varepsilon_0 \chi \int_L^0 \frac{\partial}{\partial x} E^2 \, dx \cdot w\,d$$

$$= \frac{1}{2}\varepsilon_0 \chi \left(E_0^2 - E_i^2 \right) w d$$

where $E_i = V/d$, $E_0 = 0$

$$F_v = \frac{-1}{2}\varepsilon_0 \chi \left(\frac{v}{d} \right) w d$$