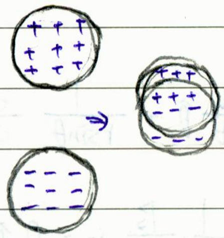
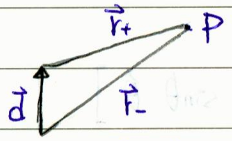
No.
Date.

Example 4.3.



A positive sphere and negative sphere without polarization, two are superimosed all the positive charges more sightly upward negative slightly downward.



Then the $E_+ \cdot 4\pi r_+^2 = \frac{4}{3}\pi r_+^3 \cdot \frac{\rho}{\varepsilon_0}$

$E_- \cdot 4\pi r_-^2 = \frac{4}{3}\pi r_-^3 \cdot \frac{\rho}{\varepsilon_0}$

① $\vec{d} + \vec{r}_+ = \vec{r}_-$

② $\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0}$

$$\vec{E}_{total} = \vec{E}_+ + \vec{E}_- = \frac{\rho}{3\varepsilon_0}\vec{r}_+ + \frac{-\rho}{3\varepsilon_0}\vec{r}_-$$

So the final $\vec{E}_{total} = \frac{\rho}{3\varepsilon_0}(\vec{r}_+ - \vec{r}_-)$

$= \frac{\rho}{3\varepsilon_0}(-\vec{d})$

$$\vec{E} = -\frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{\vec{d}}{3\varepsilon_0} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{qd}{R^3} = -\frac{\vec{P}}{3\varepsilon_0}$$

(2) Then the $\vec{p} = V \cdot \vec{P}$

$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p} \cdot \hat{r}}{r^2}$ if $r > R$.

§ 4.3 The electric displacement $\vec{D}$ (Maxwell)

In sec. 4.2, we found that bound charges.

1 Volume density of bound charge $\rho_b = -\vec{\nabla}\cdot\vec{P}$
2. Surface 〃 〃 〃 $\sigma_b = \vec{P}\cdot\hat{n}$
3. dipole moment = volume · polarization $\vec{p} = \vec{P}\cdot V$
4. $k \equiv$ Dielectric constant

ch. 2.3.

ρ_f metallic. Grounded

Free charge contains of $\boxed{\text{electrons on a conductor}}$ / $\boxed{\text{Ions embedded in the Dielectric}}$

$\rho_b . \sigma_b$ uniform polarized non-conductor.

ch 4.

Within the medium, then, the total charge density can be rewritten by $\rho = \rho_f + \rho_b$

* Review the Gauss' law $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$.

$$\varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho = \rho_f + \rho_b$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \Rightarrow \varepsilon_0(\nabla\cdot\vec{E}) = -\nabla\cdot\vec{P} + \rho_f$$

Then the $\vec{E}$ is total Electric field not a part of polarization.

$$\varepsilon_0(\nabla\cdot\vec{E}) + \nabla\cdot\vec{P} = \rho_f$$

$$\Rightarrow \nabla\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_f$$

$$\nabla\cdot\vec{D} = \rho_f$$

The expression $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

No.
Date.

Rewritten the Gauss' Law as "Electric displacement"
$\boxed{\vec{\nabla}\cdot\vec{D} = \rho_f}$ & $\boxed{\oint \vec{D}\cdot d\vec{s} = Q_f}$. Q_f is total charge enclosed in the volume.

$$\int \vec{D}\cdot\hat{n}\, da = \int \rho_f\, d\tau'$$

* Maxwell introduced $\vec{D}$ in isotropic materials.

$$\varepsilon_0 \boxed{\text{Isotropic } \vec{E}} + \boxed{\begin{matrix}\text{Isotropic} \\ \text{Anisotropic}\end{matrix} \ \vec{P}} = \vec{D}$$

$\vec{D}$ contrains one for scale. tensor. k 常數 k 張量

In isotropic materials (then $\vec{P}$ is isotropic)
$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\vec{E} + \varepsilon_0\chi\vec{E} = \varepsilon_0(1+\chi)\vec{E}$

If we write $\epsilon = \varepsilon_0(1+\chi_e)$, then $\vec{D} = \epsilon\vec{E}$
ϵ permitivity of the dielectric materials.

* If in an anisotropic dielectric medium, then
ε is a function of $\vec{E}$ $\epsilon = \epsilon(\vec{E})$,
where ε is a tensor of ε_{ij}.

* We introduce a dimensionless quantity k to characterize the electric behavior of materials.

$$\left.\begin{matrix}\vec{D} = \epsilon\vec{E} \\ \vec{D} = \varepsilon_0(1+\chi_e)\vec{E}\end{matrix}\right] \rightarrow k = 1+\chi \qquad \begin{aligned}\vec{D} &= \varepsilon_0 k\vec{E} \\ &= k\boxed{\varepsilon_0\vec{E}}\end{aligned}$$

conductor

No.
Date.

Let consider the electric field in an isotropic dielectric surrounding a spherical charge Q in which the charge density is a function of distance from the center only

$$\oint \vec{D}\cdot d\vec{s} = \oint k\varepsilon_0 \vec{E}\cdot d\vec{s} = \Sigma q_i$$

$$k \oint \varepsilon_0 \vec{E}\cdot d\vec{s} = \Sigma q_i$$ (Back to Gauss' Law)

$$\oint \vec{E}\cdot d\vec{s} = \Sigma q_i / k\varepsilon_0$$

The differs from Gauss' Law for charges in empty space only in appearance of the factor k.

$$\vec{E} = \frac{1}{k}\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)$$

if $k > 1$, $\vec{E}$ larger.
$k > 1$ $\vec{E}$ smaller

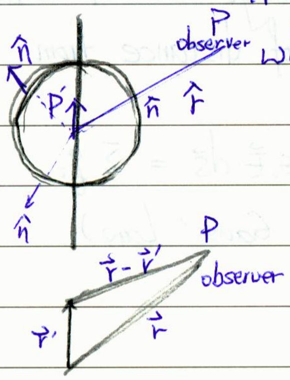
$\vec{P}$ can be rewritten by

$$\vec{P} = \varepsilon_0 (k-1)\vec{E}$$
$$= \varepsilon_0 (k-1)\frac{1}{k}\frac{Q}{4\pi\varepsilon_0 r^2}$$
$$= \frac{k-1}{k}\varepsilon_0 \frac{Q}{4\pi\varepsilon_0 r^2}$$
$$= \boxed{\frac{k-1}{k}\frac{Q}{4\pi r^2}}$$

No.
Date.

考卷討論 Problem 4.10.

A sphere of radius R carries a polarization $P(r) = k\vec{r}$ where k is a constant.



* $\rho_b = -\nabla \cdot P$ & $\sigma_b = \vec{P} \cdot \hat{n}$

① $\sigma_b = \vec{P} \cdot \hat{n}$, $\hat{n}$ is a normal direction

$= k\vec{r} \cdot \hat{n} = kr$ at the surface

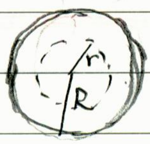
$= kR$ at $r = R$

$\rho_b = -\nabla \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r} r^2(kr)$ ($\theta . \phi$ independent)

$= -\frac{1}{r^2}\frac{\partial}{\partial r} kr^3 = -3k$

② Find the E-field at $r<R$ & $r>R$

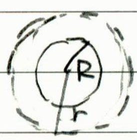
$r<R$



$$\oint \vec{E} \cdot d\vec{a} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon} = \frac{1}{\varepsilon_0}\frac{4}{3}\pi r^3 \rho_b$$

$$\vec{E} = \frac{1}{3\varepsilon_0} \rho \vec{r} = \boxed{-\frac{k}{\varepsilon_0}\vec{r}}$$

$r>R$



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0} = \frac{0}{\varepsilon_0}$$

$$\vec{E} = 0 \quad \text{at } r>R$$

③ Calculate the total $Q = 0$

Total $Q_b = \sigma_b \cdot A + \rho_b \cdot V$

$= kR \cdot 4\pi R^2 + (-3k) \cdot \frac{4}{3}\pi R^3$

$= 0$

另一種做法: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = 0$

$= \varepsilon_0 \left(-\frac{k}{\varepsilon_0}\vec{r}\right) + k\vec{r} = 0$

Problem 4.15

A thick spherical shell inner radius a, outter radius b is made of dielectric materials with a polarization

$$\boxed{P(r) = \frac{k}{r}\hat{r}}$$

(a) Calculate σ_b & ρ_b

$$\rho_b = -\nabla\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2 P = -\frac{k}{r^2}$$

$$\sigma_b = \vec{P}\cdot\hat{n} = \frac{k}{r}$$

$\sigma_b = -\frac{k}{a}$ at $r=a$

$\sigma_b = \frac{k}{b}$ at $r=b$.

1. Using Gauss Law

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}\hat{r}$$

at $r<a$, $Q=0$

$r>b$, $Q=0$

2. $a\leq r<b$

$$Q = \oint \sigma_b\, ds + \int \rho_b\, dv$$

$$= \boxed{\left(-\frac{k}{a}\right)\cdot 4\pi a^2} + \boxed{\int_a^r -\frac{k}{r^2}\cdot 4\pi r^2\, dr}$$

Total Q at $a\leq r<b$

$$= \left(-\frac{k}{a}\right)4\pi a^2 + \int_a^r -\frac{k}{r^2}\cdot 4\pi r^2 dr$$

$$= -4\pi k a + -4\pi k(r-a)$$

$$= -4\pi k r$$

then the E-field

$$E\cdot 4\pi r^2 = -4\pi k r/\epsilon_0$$

$$\vec{E} = -\frac{k}{\epsilon_0 r}\hat{r}$$

3. Calculate the electric displacement

$$\oint D\cdot da = Q_f = 0, \quad D=0 \text{ everywhere}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$

$$= \epsilon_0\left(-\frac{k}{\epsilon_0 r}\hat{r}\right) + \vec{P} = -\frac{k}{r}\hat{r} + \frac{k}{r}\hat{r} = 0$$