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Chapter 4 Electric field in matter

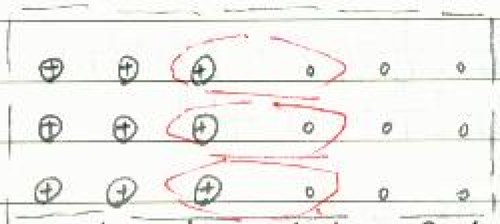
$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right] \Leftrightarrow \vec{p} = \underline{\underline{\alpha}}\vec{E}$$

α_1

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}$$

α_2

17/11. Electrostatics of Dielectric Media (介質)



under the applied E-field

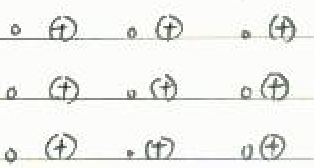
* To calculate the displacement by an external E-field.

* To calculate the force.

* To calculate the potential / Torque.

➡ Into the continous media of dielectric.

The final result as.



As a result of polarization (極化) each atom becomes a tiny dipole. whose strength depends on $\boxed{\vec{E}}$

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* The vector electro cloud is displaced by a distance $\vec{\delta}$
Then the dipole moment $\vec{p} = q\vec{\delta} \Rightarrow \vec{p} = \alpha\vec{E}$.
if the dipole moment of polarized atom has
$\boxed{\text{Z - atomic number.}}$ we can define a dipole moment
per unit volume

$$\vec{P} = N \cdot q \cdot \vec{\delta}$$

* $\vec{E}$ is not too large, the $\vec{p} = \alpha\vec{E}$.
α is called the atomic polarizability.

* We often write the polarization $\vec{P}$ for isotropic materials in terms of a scalar quantity denoted as χ

$$\vec{P} = N \cdot \alpha \cdot \vec{E} = \varepsilon_0 \chi \vec{E}$$

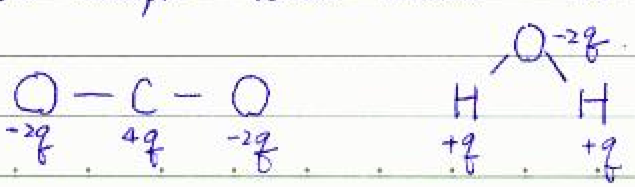
χ is susceptibility (極化率)

* if applied field $\vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z$, the linear relation between $\vec{E}$ & $\vec{P}$ can be replaced by

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

for anisotropic dielectric media.

* For example : Carbon dioxide CO_2



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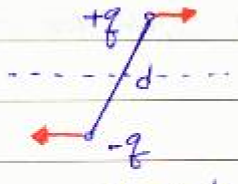
Case 1. Uniform external field :
How to calculate the force.

* Calculate the total force on the dipole as

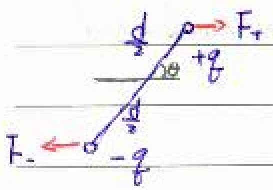


$\vec{p} = q\vec{d}$

$$\boxed{\begin{aligned}\vec{F} &= q\vec{E} = (q\vec{E}_+) + (-q\vec{E}_-)\\ &= q(\vec{E}_+ - \vec{E}_-) = q\cdot\Delta\vec{E}\end{aligned}}$$

Ex 4.1.3 the force on the positive $\vec{F}_+ = q\vec{E}$

negative $\vec{F}_- = -q\vec{E}$



The torque can be represented as

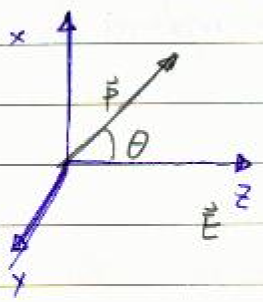
$$\boxed{\vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- = N}$$

$$\Rightarrow \frac{\vec{d}}{2} \times q\vec{E} + \frac{-\vec{d}}{2} \times (-q\vec{E}) = q(\vec{d}\times\vec{E}) = \vec{p}\times\vec{E}$$

For example the vectors of $\vec{A}$ & $\vec{B}$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\vec{A}\times\vec{B} = AB\sin\theta$$

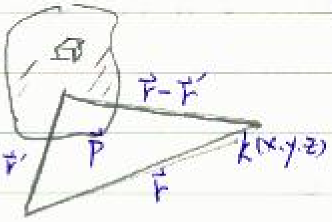


$$N = \vec{p}\times\vec{E} = -PE\sin\theta\,\hat{y}$$

4.4.2.1 Bound charges $\leftrightarrow$ Potential

We can calculate the potential for such a polarized dielectric system, we may now replace the dielectric itself by the volume polarization.

$$\vec{p} = \vec{P} \cdot v' = \vec{P} \cdot d\vec{r}^{3\prime} = \vec{P}\, d\tau'$$



We know that the potential at any point due to a single dipole $\vec{p}$ is

$$V = \frac{1}{4\pi\varepsilon_0} \left(\frac{\vec{p} \cdot \vec{r}}{r^3} \right) \quad \text{(chapter 3)}$$

If the left case the dipole placed at point $k(x, y, z)$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{P} \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

Note : $\dfrac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \dfrac{\hat{r}}{r^2} = -\nabla_r \dfrac{1}{|\vec{r} - \vec{r}'|} = \nabla_{r'} \dfrac{1}{|\vec{r} - \vec{r}'|}$

Rewritten $V(r) = \dfrac{1}{4\pi\varepsilon_0} \vec{p} \cdot \vec{\nabla}' \left(\dfrac{1}{|\vec{r} - \vec{r}'|} \right)$ and to apply the system of Integral.

$$\boxed{\vec{p} = \int \vec{P} \cdot d\vec{r}^{3\prime}}$$

dipole moment
$\rightarrow$ polarization

$$V(r) = \frac{1}{4\pi\varepsilon_0} \int \vec{P} \cdot \nabla' \frac{1}{|\vec{r} - \vec{r}'|}\, d\vec{r}^{3\prime}$$

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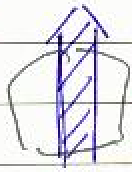
Math for $\nabla' \cdot \left[\frac{\vec{P}(r')}{|\vec{r}-\vec{r}'|} \right] = \vec{P}(r') \cdot \nabla' \frac{1}{|\vec{r}-\vec{r}'|} + \frac{\nabla' \cdot \vec{P}(r')}{|\vec{r}-\vec{r}'|}$

then the potential at any point is.

$$V(r) = \frac{1}{4\pi\varepsilon_0} \int \nabla' \cdot \left(\frac{\vec{P}(r')}{|\vec{r}-\vec{r}'|} \right) d^3r' - \int \frac{\nabla' \cdot \vec{P}(r')}{|\vec{r}-\vec{r}'|} d^3r'$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\vec{P}(r')}{|\vec{r}-\vec{r}'|} \cdot d\vec{s}' - \int \frac{\nabla' \cdot \vec{P}(r')}{|\vec{r}-\vec{r}'|} d^3r'$$

For ① $\vec{P}(r') \cdot d\vec{s}'$



$d\vec{s} = \hat{n}\, da'$

The total potential is.

$$V(r) = \frac{1}{4\pi\varepsilon_0} \left[\int \frac{P(r') \cdot \hat{n}'}{|\vec{r}-\vec{r}'|} da + \int \frac{-\nabla' \cdot \vec{P}(r')}{|\vec{r}-\vec{r}'|} d^3r' \right]$$

$\vec{P}(r') \cdot d\vec{s}'$
$= \vec{P}(r') \cdot \hat{n}\, da'$

Compare the result with

$$\left. \begin{aligned} V(r) &= \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{|\vec{r}-\vec{r}'|} d^3r' \\ V(r) &= \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(r')}{|\vec{r}-\vec{r}'|} da' \end{aligned} \right] = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Then the $\rho_b = -\nabla \cdot \vec{P}$ volume density
$\sigma_b = \vec{P} \cdot \hat{n}$ surface density

for bound charge

They are perfectly genuie accumulation of charge

$$V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma_b}{|\vec{r}-\vec{r}'|} da' + \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_b}{|\vec{r}-\vec{r}'|} d^3r'$$

$$Q_p = \int \rho_b\, d^3r' + \int \sigma_b\, da' = \int (-\nabla' \cdot P')\, d^3r' + \int P \cdot \hat{n}\, da' = \underline{0}.$$

prove.

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Polarization charge density

Polarized

$\sigma_b = \vec{P} \cdot \hat{n}$ Polarized surface charge density

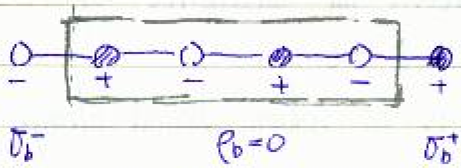
$\rho_b = -\nabla \cdot \vec{P}$ 〃 volume 〃 〃

Case I : If the polarization is constant through the medium, the charge per unit volume in its interior is zero because of the derivation in $\nabla \cdot \vec{P}$ vanishes.

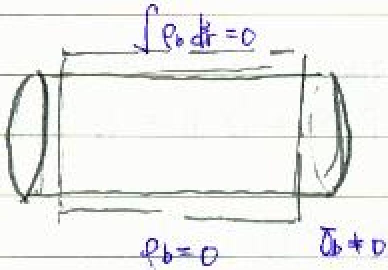
Case II : The total polarization charge is always zero follow simply from the divergence theorem.

$$Q_z = \int \sigma_b \cdot ds + \int \rho_b \cdot d\tau$$
$$= \int \vec{P} \cdot \hat{n}\, da + \int -\nabla \cdot \vec{P}\, dv = 0$$

§ 4.2.2 Uniform Polarization.



under uniform polarization.



* Net charge at the End (surface) ≡ bound charge.

* To difference with free charges.

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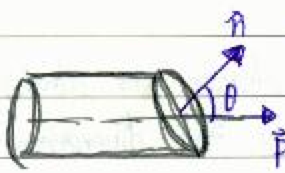
* Then we can calculate the bound charges at the End (or surface)

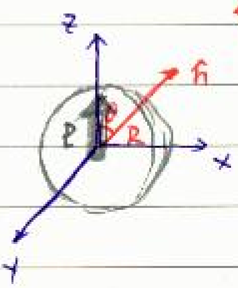
Dipole moment $\underline{\vec{P}_{+-}(Ad)} = q \cdot d$

$\vec{p} = qd$

then $q = \vec{P} \cdot A \Rightarrow \vec{P} = q/A$



for an oblique cut
the amount of charge is still the same
but $A = A_{end} \cdot \cos\theta$ $\therefore$ $\sigma_b = q/A_{end} = q/A \cdot \cos\theta$



☆ Example 4.2
Find the electric field produced by a uniformly polarized sphere of radius R.

* We known the divergence of $\vec{P}$ is zero
then $\nabla \cdot \vec{P} = 0$
There is no volume bound charge $\rho_b = 0$

* The surface charge density
$\sigma_b = \vec{P} \cdot \hat{n} = \vec{P_z} \cdot \hat{n} = P_z \cdot \cos\theta$

* From Ex. 3.9 $\sigma(\theta) = k \cos\theta$

$V(r,\theta) = \frac{\overset{= P_z}{k}}{3\varepsilon_0} r \cdot \cos\theta \ (r \le R)$, $V(r,\theta) = \frac{\overset{= P_z}{k} R^2}{3\varepsilon_0} \frac{1}{r^2} \cos\theta \ (r \ge R)$

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Then we can calculate Electric field.

$\vec{E} = -\nabla V$

If $r \le R$, $\vec{E} = -\left[\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}\right]$

$= -\left[\frac{P_z}{3\epsilon_0}\cos\theta\,\hat{r} - \frac{1}{r}\cdot\frac{P_z}{3\epsilon_0} r\sin\theta\,\hat{\theta}\right]$

$= -\frac{P_z}{3\epsilon_0}\left[\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right]$

$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$

$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

$\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta} = (\cos^2\theta + \sin^2\theta)\,\hat{z}$

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- P4.12

So the E - field $= -\frac{P_z}{3\epsilon_0}\hat{z}$ (4.14)

If $r \ge R$. The potential V is

$V = \frac{P}{3\epsilon_0}\frac{R^3}{r^2}\cos\theta = \frac{P}{3\epsilon_0}\frac{4\pi R^3}{3}\cdot\frac{\cos\theta}{r^2}\cdot\frac{3}{4\pi}$

$= \frac{P\cdot V\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{P}\cdot\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{P}\cdot\hat{n}}{4\pi\epsilon_0 r^2}$

$\vec{E} = -\nabla V = \frac{P}{4\pi\epsilon_0}(2\cdot\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$