(1) if the surface has a potential V_0 then will decay.
along the axis.

$$\boxed{X(x) = A\cdot e^{-kx} + Be^{+kx}}$$

(2) if the surface without potential $V=0$ between a BC $(-a.a)$. $(-b.b)$ then the solution

$$\boxed{Y(y) = C\cos ky + D\sin ky} \quad k = n\pi/a$$

$$Z(z) = C'\cos k'z + D'\sin k'z \quad k' = n\pi/b$$

(3) if has V_0 and B.C.

$$\boxed{X(x) = E\cosh kx + F\sinh kx}$$

* Potential of a uniform sphere of charge.

a. Outside the sphere

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r} \sim r^{-1}$$

b. Inside the sphere

$$\boxed{V = cr^2 + d}$$

then the Poisson eq. $\nabla^2 V = -\rho/\varepsilon_0$

在 $\hat{r}$.

$$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{\partial V}{\partial r} = -\frac{\rho}{\varepsilon_0} \Rightarrow \nabla_r^2 V = -\frac{\rho}{\varepsilon_0}$$

$$2c + \frac{2}{r}\cdot 2cr = -\rho/\varepsilon_0$$

$$\Rightarrow 6c = -\rho/\varepsilon_0 \Rightarrow c = -\frac{\rho}{6\varepsilon_0}$$

if at $r=R$

$$cR^2 + d = \frac{Q}{4\pi\varepsilon_0 R} \Rightarrow \frac{-\rho R^2}{6\varepsilon_0} + d = \frac{Q}{4\pi\varepsilon_0 R}$$

$$d = \frac{Q}{4\pi\varepsilon_0 R} + \frac{\rho R^2}{6\varepsilon_0}$$

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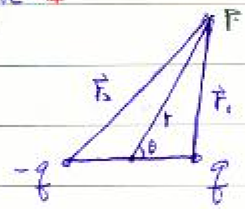
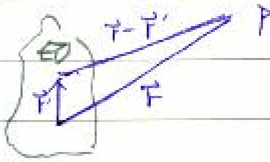
§ 3.4.1 Approximate potential at large distance

Example : I Example : II





$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_2} \qquad V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{|\vec{r}-\vec{r}'|}\,dv'$$

⋆ Example I and II also have the same solution of form.

For Example I :

$r_1 = r - \frac{l}{2}\cos\theta$ then the general form of potential.

$r_2 = r + \frac{l}{2}\cos\theta$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-\frac{l}{2}\cos\theta} + \frac{-1}{r+\frac{l}{2}\cos\theta}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{l\cos\theta}{r^2-\frac{l^2}{4}\cos^2\theta}\right]$$

Then the obtained potential of Example I is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,l\cdot\cos\theta}{r^2-\frac{l^2}{4}\cos^2\theta}$$

if l^2 is negilible compared with r^2,

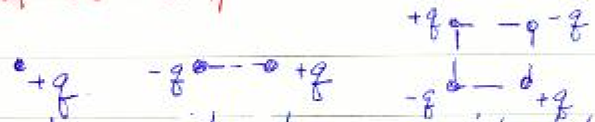
then

$$\boxed{V \simeq \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot l\cos\theta}{r^2}}$$

⇒ $q\cdot l$ is dipole moment p.

similar as. $m_e = \frac{m_1 m_2}{m_1+m_2}$

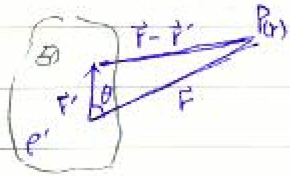
* Potential at multipoles.



mono-pole di-pole quadr-pole.

* Example 3.10

Systematic Expansion for the potential of an arbitiary localized charge distribution



If the form of

$$|\vec{r}-\vec{r}'|^2 = r^2 + r'^2 - 2rr'\cos\theta'$$

then $|\vec{r}-\vec{r}'|^2 = r^2\left[1 + \left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cdot\cos\theta'\right]$

$$= r^2\left[1+\epsilon\right]$$

then we get the form of

$|\vec{r}-\vec{r}'| = r\sqrt{1+\epsilon}$ Taylor Expansion.

$$\frac{1}{|\vec{r}-\vec{r}'|} = r^{-1}(1+\epsilon)^{-\frac{1}{2}}$$

$$= r^{-1}\left[1 - \frac{1}{2}\epsilon + \frac{3}{8}\epsilon^2 - \frac{5}{16}\epsilon^3 + \cdots\right]$$

$$= r^{-1}\left[1 - \frac{1}{2}\left(\frac{r'}{r}\right)\left(\frac{r'}{r} - 2\cos\theta'\right) + \frac{3}{8}\left(\frac{r'}{r}\right)^2\left(\frac{r'}{r} - 2\cos\theta'\right)^2 + \cdots\right]$$

* The Expression after Taylor expension of $\frac{1}{|\vec{r}-\vec{r}'|}$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\left[1 + \frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\left[\frac{3\cos^2\theta'-1}{2}\right] + \left(\frac{r'}{r}\right)^3\left[\frac{5\cos^3\theta - 3\cos\ldots}{2}\right.\right.$$

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where is the solution of Legendre polynomials.

$$\frac{1}{|r-r'|} = \frac{1}{r}\sum\left(\frac{r'}{r}\right)^n \cdot P_n(\cos\theta')$$

then $r' = l$

the second term is $\frac{r'}{r}\cos\theta' = \frac{l}{r}\cos\theta'$

dipole moment $l \cdot \cos\theta'$

quadra : $l^2 \cdot \left[\frac{3\cos^2\theta - 1}{2}\right]$

§ 3.4.2 Dipole moment terms

If the $V \sim \frac{1}{4\pi\varepsilon_0}\frac{q\, l\cos\theta}{r^2}$, then the Integral form can be expressed as.

$$q = \int \rho(r')\,dv' \;,\; l = r'\cos\theta'$$

Dipole-moment of potential can be rewritten as

$$V_D = V_{dip} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}\int r'\cos\theta'\,\rho(r')\,dv'$$

The expression of vector dot vector

$$r'\cos\theta' = \vec{r}'\cdot\hat{r}$$

then $V_D = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\boxed{\hat{r}}\boxed{\int r'\rho(r')\,dv}$

observed $P(r)$ Object $\vec{r}'$

* So we can define the dipole moment term

$$p \equiv \int r'\rho(r')\,dv'$$

and the dipole contribution to the potential simplifies to $\boxed{V_D \cong \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot\hat{r}}{r^2}}$

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Legendre polynomials

$$P_\ell = \frac{1}{2^\ell \cdot \ell!}\left(\frac{d}{dx}\right)^\ell \cdot (x^2-1)^\ell$$

$x = \cos\theta'$ $\quad \ell=0 \,.\; P_0 = 1$

$\ell=1 \,.\; P_1 = \cos\theta'$

$\ell=2 \,,\; P_2 = (3\cos^2\theta' - 1)/2$

Dipole moment of potential

$$p = \int r' \rho(r')\, dv'$$

$$\Rightarrow V(r) = \frac{1}{4\pi\varepsilon_0}\, \frac{\hat{r}}{r^2} \cdot \int \vec{r}' \rho(r')\, dv'$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\hat{r}\cdot\vec{p}}{r^2} \sim \frac{1}{r} \text{ (outside)}$$

Example: The electric field of a dipole.

If we choose coordinates, so that p lies at origin and points in the $\boxed{z\text{-axis}}$

the the potential is

$$V_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\hat{r}\cdot\vec{p}}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2}$$

then we can get the negative gradient of V_D
Because we known the electric field is

$$\vec{E} = -\nabla V_D$$

$$(1) \quad E_r = -\nabla_r V_D = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p\cos\theta}{r^3}$$

$$(2) \quad E_\theta = -\nabla_\theta V_D = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sin\theta}{r^3}$$

$$(3) \quad E_\phi = -\nabla_\phi V_D = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} = 0$$

The total E-field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right] \Rightarrow x.y.z$$

The electric field of an electric dipole, if not along easy-axis. (z-axis), then the dipole moment may be expressed in the following way.

$$\vec{E} = -\nabla V_D = -\frac{1}{4\pi\varepsilon_0}\nabla\left[\frac{\vec{p}\cdot\vec{r}}{r^3}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{1}{r^3}\nabla(\vec{p}\cdot\vec{r}) + (\vec{p}\cdot\vec{r})\nabla\left(\frac{1}{r^3}\right)\right]$$

Now we known:

$$\nabla(\vec{p}\cdot\vec{r}) \Rightarrow \left[\begin{array}{l}\vec{p} = p_x\hat{x} + p_y\hat{y} + p_z\hat{z} \\ \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}\end{array}\right.$$

$$\vec{p}\cdot\vec{r} = p_x\cdot x + p_y\cdot y + p_z\cdot z$$

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$$\nabla(\vec{p}\cdot\vec{r}) = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)(p_x x + p_y y + p_z z)$$
$$= p_x\hat{x} + p_y\hat{y} + p_z\hat{z}$$
$$= \vec{p}$$

2) The second terms of

$$\nabla\left(\frac{1}{r^3}\right) = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)(x^2+y^2+z^2)^{-\frac{3}{2}}$$
$$= \frac{-3}{2}(x^2+y^2+z^2)^{-\frac{5}{2}}\cdot(2x\hat{x} + 2y\hat{y} + 2z\hat{z})$$

$$\Rightarrow \nabla\left(\frac{1}{r^3}\right) = \frac{-3}{r^5}(x\hat{x} + y\hat{y} + z\hat{z}) = -\frac{3}{r^5}\vec{r}$$

So the total E-field can be rewritten as

$$\vec{E} = -\nabla V_D = -\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r^3}\nabla(\vec{p}\cdot\vec{r}) + (\vec{p}\cdot\vec{r})\nabla\left(\frac{1}{r^3}\right)\right]$$
$$= -\frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3} + (\vec{p}\cdot\vec{r})\frac{-3\vec{r}}{r^5}\right]$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right]}$$

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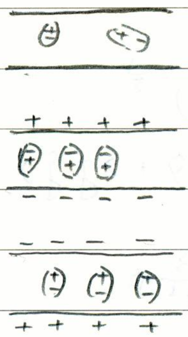
Chapter 4 Electric field in matter

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right] \Leftrightarrow \vec{p} = \underline{\underline{\alpha}}\vec{E}$$



α_1

α_2

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}$$