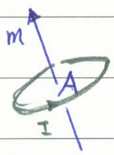
No.
Date. 4.15

Chapter 6.

§6.1.2 Torques : 力矩

* From the expression for the torque on a small current loop.

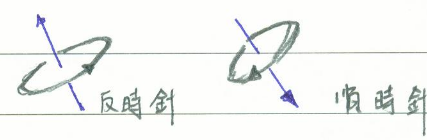


$$\vec{m} = \int I \, d\vec{a}$$

are summarized in its magnetic moment $m = IA$.

* The magnetic moment can be considered to be a vector quantity with direction perpendicular to the current loop in the right-hand rule.

$$\vec{N} = \vec{m} \times \vec{B}$$
$$= mB\sin\theta$$



* The potential energy associated with the magnetic moment is

$$U(\theta) = -\vec{m}\cdot\vec{B} = -mB\cos\theta$$

So that the difference in energy between aligned and anti-aligned is

$$\Delta U = 2mB$$

These relationship for a finite current loop extend to the magnetic dipoles of electron-obrit. (s-L) and to be intrinsic magnetic moment associated with electron spins (s-S).

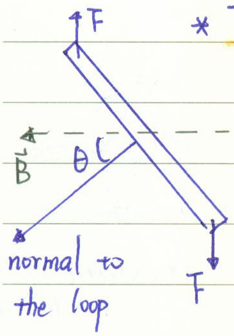


Figure 6.2(b)

* The Torque on a current carrying coil, as in a DC-motors, The torque exerted by the magnetic force is given by

$$N = BILW \sin\theta$$

The coil characteristics can be grouped as

$$m = IA \quad \text{for } n \text{ loop.}$$

* Rotational work

If you exerted the necessary torque to overcome the magnetic torque and rotate the loop from zero to 180 degress.

$$W = -\int_0^{\pi} N\, d\theta = -\int_0^{\pi} mB \sin\theta\, d\theta = +mB\cos\theta\Big|_0^{\pi} = -2mB$$

* Net force

In a uniform field, the net force on a current loop is zero.

$$\vec{F} = I\oint (d\vec{l} \times \vec{B}) = I\Big(\oint_{loop} d\vec{l}\Big) \times \vec{B} = 0.$$

No.
Date.

Considering a magnetic dipole moment the B-field can be reduced as (5.87)

$$\vec{B} = \nabla \times \vec{A} = \frac{-\mu_0}{4\pi} \frac{1}{r^3} \left[\vec{m} - 3(\vec{m} \cdot \hat{r})\hat{r} \right]$$

$$= \frac{-\mu_0}{4\pi} \left[\frac{\vec{m}}{r^3} - \frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5} \right]$$

In particular, because we have shown that the forces on small coils are of the same type as the forces on electric dipoles discussed in electrostatics (3.104)

$$\vec{E}_{dip} = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^3} \left[3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p} \right]$$

* a small coil of magnetic dipole moment $\vec{m}$ has potential U

$$U = - \vec{m} \cdot \vec{B}_{dip} \quad \text{and} \quad \vec{N} = - \vec{m} \times \vec{B}_{dip}$$

Likewise, the mutual potential energy W, because two coils of magnetic dipole moment m_1 & m_2 separated by a distance $\vec{r}$.

$$W = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\vec{m}_1 \cdot \vec{m}_2 - 3 \frac{(\vec{m}_1 \cdot \vec{r})(\vec{m}_2 \cdot \vec{r})}{r^2} \right]$$

$$U_{12} = - \vec{m}_1 \cdot \vec{B}_{dip}.$$

No.
Date.

$$\vec{B}_{dip_2} = \frac{\mu_0}{4\pi}\left[\frac{-\vec{m}_2}{r^3} + \frac{3(\vec{m}_2\cdot\vec{r})\vec{r}}{r^5}\right]$$

$$U_{12} = -\vec{m}_1 \cdot \left[\frac{\mu_0}{4\pi}\left(-\frac{\vec{m}_2}{r^3} + \frac{(\vec{m}_2\cdot\vec{r})\vec{r}}{r^5}\right)\right]$$

$$= \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2 - 3\frac{(\vec{m}_1\cdot\vec{r})(\vec{m}_2\cdot\vec{r})}{r^2}\right]$$

$$= \frac{\mu_0}{4\pi r^3}\left[-3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r}) + \vec{m}_1\cdot\vec{m}_2\right] \Rightarrow F_{12} = -\nabla U_{12}$$

For the left case

$$U = \frac{\mu_0}{4\pi r^3}\left[-3m_1m_2\cos\theta + m_1m_2\cos\theta\right] = \frac{-\mu_0 m_1 m_2 \cos\theta}{2\pi r^3}$$

the total force is related to the gradient of their potential energy.

$$F = -\nabla U_{12} = -\nabla(\vec{m}\cdot\vec{B})$$
$$= -(\vec{B}\cdot\nabla)\vec{m} + (\vec{m}\cdot\nabla)\vec{B} + \vec{B}\times(\nabla\times\vec{m}) + \vec{m}\times(\nabla\times\vec{B})$$

if $\vec{m}$ is fixed $\nabla\times B = 0$

$$(\vec{B}\cdot\nabla)\vec{m} = \left(B_x\frac{\partial}{\partial x} + B_y\frac{\partial}{\partial y} + B_z\frac{\partial}{\partial z}\right)\vec{m}_{\text{constant}} = 0$$

$$\nabla\times\vec{m} = 0 \qquad \nabla\times\vec{B} = 0$$

 ZINKEEN