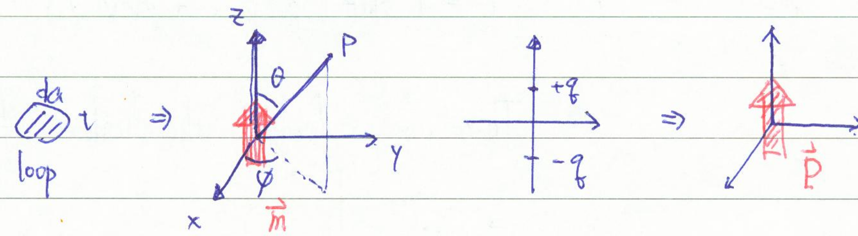
and this equation for B-field reduces to

$$\vec{B} = \nabla \times \vec{A} = -\frac{\mu_0}{4\pi}\left[\frac{m}{r^3} - \frac{3(\vec{m}\cdot\vec{r})}{r^5}\vec{r}\right] \quad \text{for small loop}$$



Home work : 5.23 , 5.26 , 5.35 , 5.36

* Magnetic Vector Potential for small current loop. 4/8

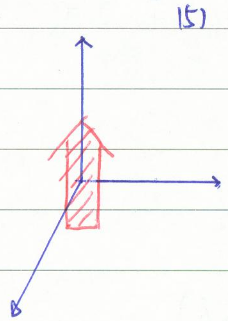
(1) $\vec{A} = \frac{\mu_0 I}{8\pi}\cdot\frac{1}{r^3}\cdot\oint(\vec{r}'\times d\vec{r}')\times\vec{r}$

(2) magnetic moment $\vec{m} = \frac{1}{2}\oint \vec{r}'\times d\vec{r}'$

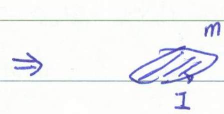
(3) $\vec{A} = \frac{\mu_0 I}{4\pi}\cdot\frac{1}{r^3}\,\vec{m}\times\vec{r} = \frac{\mu_0 I}{4\pi}\cdot\frac{1}{r^2}\,\vec{m}\times\hat{r}$

(4) $\vec{B} = \nabla\times\vec{A} = -\frac{\mu_0}{4\pi}\left[\frac{\vec{m}}{r^3} - 3\frac{\vec{m}\cdot\vec{r}}{r^5}\vec{r}\right] \sim \frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}$

(5) Magnetic dipole moment



$\vec{m} = \int \vec{I}\cdot da$



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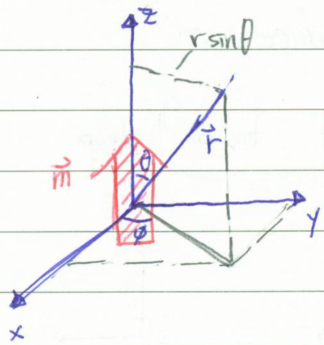

The magnetic field of a pure dipole is easiest to calculate if we put $\vec{m}$ at the origin.

where $\vec{m} = m_z \hat{z}$

$$\vec{r} = r\sin\theta\,[\cos\phi\,\hat{x} + \sin\phi\,\hat{y}] + r\cos\theta\,\hat{z}$$

Then we calculate the value of

$$\vec{m}\times\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & m \\ r\cos\phi\sin\theta & r\sin\phi\sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cdot m\sin\theta\,(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})$$

$$\vec{A}(r) = \frac{\mu_0}{4\pi}\cdot\frac{m\sin\theta}{r^2}\,(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) \rightarrow A\,\hat{\phi}.$$

Chapter 1. Note $(x.y.z) \leftrightarrow (r.\theta.\phi)$

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}.$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Then the finial form of $\vec{A}$ is

$$\vec{A}(r) = \frac{\mu_0}{4\pi}\,\frac{m\sin\theta}{r^2}\,\hat{\phi}$$ ⇒ A & m 簡單式

The next step : $\vec{B} = \nabla\times\vec{A}$

So the B-field is

$$\vec{B} = \nabla\times\vec{A} = -\frac{\mu_0 m}{4\pi r^3}\,(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

* How to expand to real world of sphere shell of solid sphere ?

$\vec{m} \equiv$  or  = solution of combine the spheric shell $\vec{A}$ rotating ω.

§5.58. A uniform changed solid sphere of radius R carries a total charge Q and its set spinning with angular velocity ω about the z-axis.

Example 5.11

$$\vec{A}(r,\theta,\phi) = \frac{1}{3}\mu_0 R \omega \sigma \cdot r\sin\theta \,\hat{\phi} \qquad (r \le R)$$

$$= \frac{1}{3}\mu_0 R^4 \omega \sigma \frac{\sin\theta}{r^2} \hat{\phi} \qquad (r \ge R)$$

If we know the dipole moment of $\vec{A}$ can be rewritten as

$$\vec{A} = \frac{\mu_0}{4\pi} \cdot \frac{m\sin\theta}{r^2} \hat{\phi}$$

if $r \ge R$, comparing two equations of

$$\frac{\mu_0 \sin\theta\, m}{4\pi r^2} = \frac{\mu_0 R^4 \omega \sigma \sin\theta}{3r^2}$$

$$\Rightarrow \vec{m} = \frac{4}{3}\pi R^4 \omega \sigma$$

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$$\vec{m} = \int I \, da \quad \Rightarrow \quad \vec{m}$$

$\vec{m} = \frac{4}{3}\pi R^4 \omega \sigma$ changing $R \to r$

$\sigma \to \rho \, dr$

$\rho \to Q / (\frac{4}{3}\pi R^3)$

So the magnetic dipole moment is

$$\vec{m} = \frac{4}{3}\pi \omega \int_0^R \rho r^4 \, dr = \frac{4}{3}\pi \omega \rho \frac{1}{5} R^5 \hat{z}$$

$$\vec{m} = \frac{1}{5} Q \omega R^2 \hat{z}$$ solid sphere.

* Find the magnetic field within the sphere.

$$\vec{B}_{ave} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{R^3} \quad (P5.57)$$

$$\vec{m} = \frac{1}{5} Q \omega R^2$$

$$\vec{B}_{average} \equiv \frac{1}{\frac{4}{3}\pi R^3} \int B \cdot dv = \frac{\mu_0 2m}{4\pi R^3} = \frac{\mu_0 2 Q \omega}{4\pi \cdot 5R} \hat{z}$$

* Find the approximation vector potential at a point (r, θ)

Note : $r \geq R$ & small current loop.

$A(r, \theta, \phi) = \frac{\mu_0 R^4}{3} \omega \sigma \frac{\sin\theta}{r^2} \hat{\phi}$ → spheric shell

$\vec{A} = \frac{\mu_0}{4\pi} \frac{1}{r^3} \vec{m} \times \vec{r}$ (條件是 : 在 small loop)

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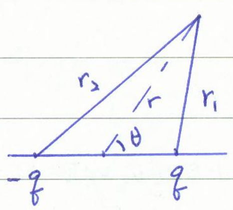
Reverse changing $A(r.\theta.\phi)$

$R \to r.$ $\quad \sigma \to \rho dr$

$$A(r.\theta.\phi) = \frac{\omega}{3}\frac{\mu_0 \sin\theta}{r^2}\int r^4 \rho dr' \hat{\phi} = \frac{\mu_0}{4\pi}\cdot\frac{Q\omega R^2}{5}\cdot\frac{\sin\theta}{r^2}\hat{\phi}.$$

$$= \frac{\mu_0}{4\pi}\cdot m_{sphere}\cdot\frac{\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\cdot\vec{m}\times\vec{r}$$

Electric dipole moment $\Leftrightarrow$ related with magnetic dipole moment.

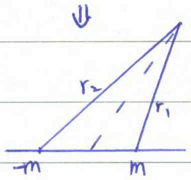


$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r-\frac{l}{2}\cos\theta} - \frac{q}{r+\frac{l}{2}\cos\theta}\right]$$

$$\cong \frac{1}{4\pi\varepsilon_0}\frac{q\,l\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$\vec{m} = \int I\, da$ $\quad$ Let us review this old concepts briefly. Its help us to explain the equivalence of a small current loop and a magnetic dipole moment.

$\Downarrow$



If there are two magnetic poles positive and negative the force between the two poles m and m' obeyed a columb's law.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{mm'}{r^2}$$

The force $\vec{F}$ on a pole m placed in a magnetic field $\vec{B}$ is given by $\vec{F} = \vec{m}\cdot\vec{B}$. combing this with $\vec{F} = \vec{\mu}\cdot\vec{H}$ columb's law.

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Then the magnetic field $\vec{B}$ at a distance r from m is

$$\vec{B} = \frac{m'}{4\pi\mu r^2} \quad , \quad V = \frac{m}{4\pi\mu_0 r}$$

1931. Dirac introduces the concept of monopole into quantum mechanics, the relation is

$$g_e = n\frac{h}{\mu_0} \quad (n = 1, 2, \cdots)$$

g : monopole of magnetic charge
e : electron , h : Plank's constant.

Then the equation can be changed as

$$e = n\left(\frac{h}{\mu_0 g}\right)$$

$e = 1.6 \times 10^{-19} C$, $h = 6.6 \times 10^{-34} J \cdot s$ $\mu_0 = 4\pi \times 10^{-7} \cdot \frac{T \cdot m}{A}$

n=1. Then g can be obtained as

$g = 3.3 \times 10^{-9}$ magnetic columb (磁庫)

* The magnetic scalar potential : $V_m = \lambda$

* $\vec{B} \Rightarrow \vec{A} \rightarrow \vec{m}$ (Magnetostatics) $\Rightarrow \vec{A} = A_0 + \nabla\lambda$

* V_m scalar potential?

Because $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ is not a conservative field and so it makes no sense to introduce a magnetic scalar potential.
But if $\nabla \times \vec{B} = 0$ is whatever the current density $\vec{J}$ is zero, the magnetic field $\vec{B}$ is such regin (say 2) can be written as the gradient of a scalar potential V_m.

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then $\vec{B}$ is given as

(1) $\vec{B} = -\mu_0 \nabla V_m$ $\quad\quad$ $\nabla \times \vec{E} = 0 \rightarrow \vec{E} = -\nabla V$

(2) $\nabla \cdot \vec{B} = -\mu_0 \nabla^2 V_m \Rightarrow$ driven a Laplace's equation.

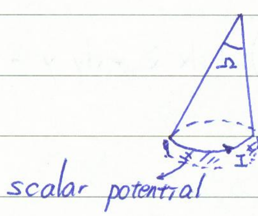
$\nabla^2 V_m = 0$ $\quad\quad$ Air / $\rightarrow$ conductor

↳ E-field Ch.3.

* Thus if we have a current-carrying conductor them outside the conductor $\nabla \times \vec{B} = 0$ & we have $\vec{B} = -\mu_0 \nabla V_m$

* Then the B-field represented as

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \hat{r}}{r^2}$$



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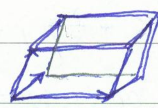

$(\vec{A} \times \vec{B}) \cdot \vec{C} = $ volume

If we displace P by a small distance $d\lambda$ the solid angles by a amount of $d\Omega$

The solid angles equal to its area.

$$d\Omega = \frac{dA}{r^2} = \frac{|d\lambda \times d\vec{s}|}{r^2} \quad \Leftrightarrow \quad \Omega = \frac{A}{r^2}$$

In general, the solid angle then is $(-d\vec{\lambda} \times d\vec{s}) \cdot \hat{r}_1 r^2$ where $\hat{r}_1$ is a unit-vector in the direction $\vec{r}$.

Integration of the expression

$$d\Omega = -d\vec{\lambda} \cdot \oint \frac{d\vec{s} \times \hat{r}}{r^2}$$

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where $\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \hat{r}}{r^2}$

So $d\Omega = -d\vec{A} \cdot \vec{B} \left(\frac{4\pi}{\mu_0 I}\right)$

$$\vec{B} \cdot d\vec{\lambda} = -\frac{\mu_0 I}{4\pi} d\Omega$$

* The minus sign means that we take Ω to be positive on that side of the current loop.

$$d\Omega = \frac{\partial\Omega}{\partial x} dx + \frac{\partial\Omega}{\partial y} dy + \frac{\partial\Omega}{\partial z} dz$$
$$= \nabla\Omega \cdot d\vec{\lambda}$$
$$= \left(\frac{\partial\Omega}{\partial x}\hat{x} + \frac{\partial\Omega}{\partial y}\hat{y} + \frac{\partial\Omega}{\partial z}\hat{z}\right) \cdot \left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)$$

then we can rewrite

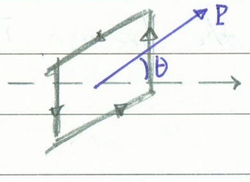
$$\vec{B} \cdot d\vec{\lambda} = -\frac{\mu_0 I}{4\pi} d\Omega \quad \text{as}$$

$$\vec{B} \cdot d\vec{\lambda} = -\frac{\mu_0 I}{4\pi} (\nabla\Omega \cdot d\lambda)$$

$$\vec{B} = -\frac{\mu_0 I}{4\pi} \nabla\Omega \Leftrightarrow -\mu_0 \nabla V_m$$

magnetic scalar potential is $V_m = \frac{I}{4\pi}\Omega$

Ex: Find the magnetic field at a point P at a distance from the small current loop.



* A current I flow in a small plane circuit of area A.
* The radius vector $\vec{r}$ of point P makes an angle θ with the normal to the loop.

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Solid angle is $\Omega = A\cos\theta / r^2$

then $V_m = \frac{I}{4\pi} \cdot \frac{A\cos\theta}{r^2}$

The B-field is $\vec{B} = \vec{B}_r + \vec{B}_\theta + \vec{B}_\phi$

$$\vec{B} = -\mu_0 \nabla V_m$$

$$\vec{B}_r = -\mu_0 \frac{\partial}{\partial r} V_m = \frac{-2\mu_0 I A}{4\pi} \cdot \frac{\cos\theta}{r^3}$$

$$\vec{B}_\theta = -\mu_0 \frac{1}{r} \cdot \frac{\partial}{\partial \theta} V_m = \frac{\mu_0 I A}{4\pi} \cdot \frac{\sin\theta}{r^3}$$

$$\vec{B}_\phi = 0$$